Influence of the Cosmological Background on the Mutual Interaction of Quantum Fields*

JÜRGEN AUDRETSCH

Fakultät für Physik
Universität Konstanz
Postfach 5560
D-7750 Konstanz
W.-Germany

1. INTRODUCTION

The inflationary universe has renewed the interest in the semiclassical approach to quantum gravity which in this case is characterized by a classical description of the cosmological gravitational field by means of a Robertson-Walker space-time and a quantum field theoretical treatment of the matter content, taking into account gravitation as an influencing external field

In this context we restrict ourselves to the discussion of the quantum field theory of *mutually* interacting quantum fields in a given unquantized curved space-time. The intention is thereby i.) to contribute to the development of an appropriate conceptual framework (what are the measurable quantities, what is the related claculation scheme ?); ii.) to work out the physical characteristics of such a theory (including a discussion of advantages and deficiencies of the respective schemes); iii.) to answer qeustions like: How are the minkowskian cross sections and decay rates modified ?

The latter has been done in case studies[1-4] assuming a cosmic expansion law and particular types of mutual interaction which allow an *exact* treatment of the quantum field theoretical effects up to a certain

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order of the coupling parameter. Exact solvability is thereby given preference over physical applicability of the results, because at present, when conceptual framework and physical understanding still need improvement, examples first of all have to support intentions i.) and ii.)

The details of the exact calculations will not be described here. For them and also for additional details regarding the following text see references 1-4. All notations and abbreviations which are used here without definition can be looked up there. For a survey of the literature see also the reviews in references 5,6, and 7.

2. SEMI-CLASSICAL APPROACH

We study the interaction given by $\mathcal{L}_I(\lambda, g_{\alpha\beta}, \Phi, \Psi)$ between two types of neutral scalar particles described by the massive Klein-Gordon field Φ and the massless field Ψ. This may be regarded as a first step towards the discussion of a scalar quantum electrodynamics. λ is a coupling parameter. The fact that the curved space-time background acts quantum field theoretically already in zeroth order $\mathcal{L}_I = 0$ (for example in creating particles out of the vacuum), severely influences the outcome of the mutual interaction and its registration.

We restrict ourselves to a treatment in the *interaction picture* using an *in-out scheme* based on the S-matrix $S = \lim_{\alpha \to 0} \hat{T} \exp\{i \int \mathcal{L}_I \, e^{-\alpha|\eta|} d^4x\}$. The gravitational background is thereby always exactly taken into account. α is the switch-off parameter.

We consider a 3-flat open Robertson-Walker universe

$$ds^2 = a^2(\eta)\left(d\eta^2 - d\underline{x}^2\right) \tag{1}$$

which is *conformally flat*. Apart from the examples, the expansion law is left unspecified, but the in- and out- region ($\eta = -\infty$, $\eta = +\infty$) must allow the definition of free particles.

The Klein-Gordon particles are assumed to be *conformally coupled* to the background $(\nabla_\mu \nabla^\mu + m^2 + R/6)\Phi = 0$ (R = scalar curvature, for $\Psi : m = 0$). The general results can easily be transcribed for other fields an other interactions.

The physical specifications above have the following consequences: i.) Energy is not conserved. ii.) There is a conserved 3-momentum parameter called $\underline{p}$, $\underline{q}$, ... for Φ-fields and $\underline{k}$, $\underline{l}$, ... for Ψ-fields. The measured 3-momentum is $\underline{p}/a$ and $\underline{k}/a$ respectively. iii.) There is creation of massive Φ-pairs out of the vacuum in every mode $\underline{p}$ with total number $N^{(o)}(\underline{p}^\Phi|0) = |\beta_{\underline{p}}|^2$. iv.) Because of the conformal situation there is no corresponding creation of Ψ-particles: $N^{(o)}(\underline{k}^\Psi|0) = |\beta_{\underline{k}}|^2 = 0$.

The information regarding the influence of the gravitational background is essentially contained in the Bogoliubov coefficients relating in- and out- particle solutions of the Klein-Grodon equation $\alpha_{\underline{p}} = (u^{in}_{\underline{p}}, u^{out}_{\underline{p}})$, $\beta_{\underline{p}} = (u^{in}_{\underline{p}}, u^{out*}_{-\underline{p}})$ with $|\alpha_{\underline{p}}|^2 - |\beta_{\underline{p}}|^2 = 1$.

3. IN-OUT TRANSITIONS AMPLITUDES

An in-out transition amplitude relating in- and out-states which contain only a finite number of particles can always be written in the form

$$\langle \text{out } d^\Phi s^\Psi | S^{(z)} | c^\Phi r^\Psi \text{ in}\rangle = \sum_{g,t} \langle \text{out } d^\Phi s^\Psi | g^\Phi t^\Psi \text{ in}\rangle\langle \text{in } g^\Phi t^\Psi | S^{(z)} | c^\Phi r^\Psi \text{ in}\rangle, \tag{2}$$

$(z) \leftrightarrow$ order in λ.

The zeroth order part thereby reduces to

$$\langle \text{out pairs}^\Phi \hat{e}^\Phi s^\Psi | \text{pairs}^\Phi \hat{h}^\Phi r^\Psi \text{ in}\rangle = f(\alpha,\beta)\delta_{\hat{e},\hat{h}}\,\delta_{s,r}\langle \text{out } 0|0 \text{ in}\rangle\ , \tag{3}$$

where we have introduced the notation $|\text{general state}^\Phi\rangle$ $|\text{pairs}^\Phi \hat{e}^\Phi\rangle$. The hat denotes a state consisting of unpaired states only. $f(\alpha,\beta)$ is a function of the Bogoliubov coefficients of the modes involved.

On the other hand, it has been shown by Parker [8] for the universe in question that

$$|\langle \text{out } 0|0 \text{ in}\rangle|^2 \xrightarrow{V \to \infty} 0 \quad , \tag{4}$$

where V is a normalization volume. Accordingly we have for zero and all higher orders of the mutual interaction the fact that *any in-out amplitude becomes arbitrarily small for increasing normalization volume V.* It is zero in the limit $V \to \infty$. A single in-out amplitude with a finite number of outgoing particles loses its physical meaning for $V \to \infty$ because the background fills all modes.

Another consequence of (3) for the zeroth order is, that *it is impossible to find unpaired (!) outgoing particles in a mode which has initially been unoccupied.*

4. ADDED UP PROBABILITY

To construct a physically and mathematically reasonable transition probability, we note that in our case a massless particle in the out-region has not come out of the background, but has solely been created or influenced by the mutual interaction: *massless particles are good indicators*. The added up probability is based on this:

$$w^{\text{add}}(s^\Psi|c^\Phi r^\Psi) = \sum_{\text{all } d} |\langle \text{out } d^\Phi s^\Psi|S|c^\Phi r^\Psi \text{ in}\rangle|^2 \tag{5}$$

It answers the question: What is the probability that a particular state of massless particles $|S^\Psi \text{out}\rangle$ will be found in the out-region regardless of what has happened to the massive states ? w^{add} can be obtained in working out in-in amplitudes[1]:

$$w^{\text{add}}(s^\Psi|c^\Phi r^\Psi) = \sum_{\text{all } d} |\langle \text{in } d^\Phi s^\Psi|S|c^\Phi r^\Psi \text{ in}\rangle|^2 \tag{6}$$

This permits a *Feynman diagram technique* which differs from the one in flat space-time only by the replacement of the plane waves for massive particles by the exact solution $u^{\text{in}}_{\underline{p}}$.

In reference [1)] we have applied the concept w^{add} to the *decay of a massive particle* according to the interaction $_I = \sqrt{-g}\,(\lambda\,/a(\eta))\,\Phi^2\Psi^2$ for an expansion law $a(\eta) = 1 + e^{2b\eta}$. The life time is worked out exactly up to the second order in λ. The main gravitational effect consists of *finite additive (!)* corrections outside energy conservation. The result reflects the existence of two time scales, the infinite one of mutual interaction and the finite one of the gravitational influence. See reference 1 for details and the resulting criticism. We turn to another example.

5. EXAMPLE: DECAY OF A MASSIVE PARTICLE IN UNIVERSES WITH STATICALLY BOUNDED EXPANSION LAWS

We discuss the decay of a massive Φ-particle into two massless Ψ-particles for the mutual interaction

$$\mathcal{L}_I = \sqrt{-g}\,(\lambda/a(\eta))\,\Phi\Psi^2 \tag{7}$$

with coupling parameter λ, which contains a factor $a^{-1}(\eta)$ to make the interaction conformally invariant, so that the exact calculation becomes less cumbersome. m in (7) breaks the conformal invariance. The diagrams which contribute to the added-up transition probability are shown in figure 1. The second reflects energy non-conservation.

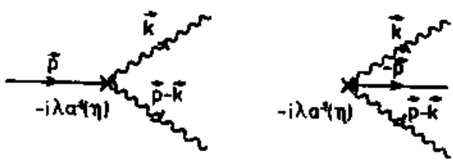


FIGURE 1: Diagrams contributing to the added-up transition probability (6)

We have to have approximately flat in- and out-regions to make a particle definition possible. This is guaranteed in the case of statically bounded monotonic expansion laws $a(\eta)$. In the following we discuss the structure of the gravitational influence on the particle decay for the scale factor representing a *step* at $\eta = 0$:

$$\hat{a}(\eta) = \Theta(-\eta)\, a_i + \Theta(\eta)\, a_o \tag{8}$$

with

$$a_i = (A-B)^{1/2}, \quad a_o = (A+B)^{1/2}, \quad A > B > 0 \tag{9}$$

We then add the corresponding rigorous calculation for the *tanh expansion law:*

$$\tilde{a}(\eta) = (A + B \tanh b\eta)^{1/2} \tag{10}$$

which is the prototype of a smoothed out step, leading to all the typical physical deviations from the step situation. The expansion law is bounded by a_i and a_o in the in- and out-region, respectively. In the following, quantities with hat and tilde will refer to the respective expansion laws above. The details of the calculation can be found in reference 4.

We note that for our expansion law there is creation of massive Φ-particles out of the vacuum (i.e. in zeroth order of the mutual interaction). It is given by the Bogoliubov coefficient β :

$$N^{(o)}(\underline{p}|0) = |\beta_{\underline{p}}|^2 \tag{11}$$

The *added-up probability* to find two massless Ψ-particles in the out-region if there has been one ingoing massive Φ-particle with momentum parameter $\underline{p}$, turns out to be

$$\hat{w}^{add}\,(1^{\Psi}_{\underline{k}}\; 1^{\Psi}_{\underline{p}-\underline{k}} | 1^{\Phi}_{\underline{p}}) = \frac{\lambda^2 \pi}{2k|\underline{p}-\underline{k}|V} \left[\frac{1}{2} \frac{1}{E_i} T_\eta\, \delta(\omega_{-i}) + \left(\frac{1}{2} + |\hat{\beta}_{\underline{p}}|^2\right) \frac{1}{E_o} T_\eta\, \delta(\omega_{-o}) \right] + \hat{\Delta}(k) \tag{12}$$

in the step-case. The result of the tanh-case is obtained from this in replacing hat by tilde and adding a finite term to T_η. See reference 4 for the exact expression:

$$\tilde{w}^{add} \doteq \hat{w}^{add} \, (\hat{} \to \tilde{} \,;\, T_\eta \to T_\eta + \text{finite}) \tag{13}$$

T_η is the infinite η-time between the in- and out-regions during which the quantum interaction takes place.

$\hat{\Delta}(k)$ and $\tilde{\Delta}(k)$ are of the same structure and become singular at $\omega_{-i} = 0$ and at $\omega_{-o} = 0$, comp. figure 2. With the definitions

$$E_{i/o} = (\underline{p}^2 + m^2 a_{i/o}^2)^{1/2} \tag{14}$$

$$\omega_{-i/o} = E_{i/o} - k - |\underline{k} - \underline{p}| \tag{15}$$

and because of (12, 13), this means that the decay becomes resonant if conservation of 3-momentum as well as conservation of *measured* energy is fulfilled. With regard to the asymptotic regions, the latter means $\omega_{-i} = 0$ for $\eta \leq 0$ and ω_{-o} for $\eta \geq 0$. Accordingly, we have two singularities at different values of the momentum parameter k. In addition we have a finite part of the spectrum outside of the resonances, because

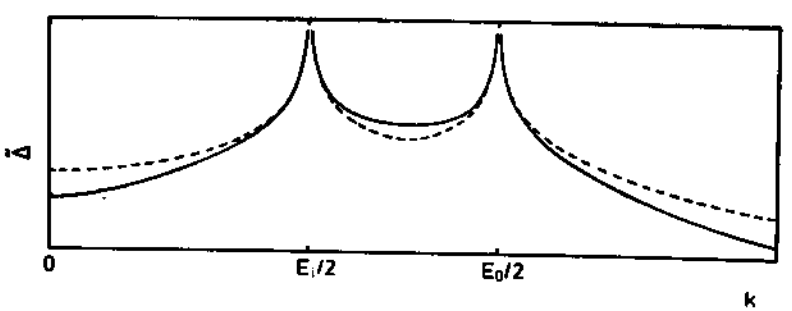


FIGURE 2: The correction term $\tilde{\Delta}(k)$ in (13) as function of the energy of the massless particles for different values of the expansion parameter b (for decaying particle at rest : $\underline{p} = 0$). Logarithmic scaling of the $\tilde{\Delta}$-axis. The correction term $\hat{\Delta}(k)$ shows a similar spectrum.

there is no energy conservation. Note the particular appearance of $|\beta|^2$ and the factors 1/2 in (12) and (13). A physical explanation of all this will be given below.

From w^{add} we can obtain the *total decay probability*

$$w^{tot} = \sum_{\underline{k}} w^{add}(1^{\Psi}_{\underline{k}}\, 1^{\Psi}_{\underline{p}-\underline{k}} | 1^{\Phi}_{\underline{p}}) \tag{16}$$

for a massive particle at rest ($\underline{p} = 0$). It turns out to be in the step case:

$$\hat{w}^{tot} = \frac{\lambda^2}{4\pi m}\left[\frac{1}{2\,a_i} T_\eta + \frac{1}{2\,a_o}\left(\frac{1}{2} + |\hat{\beta}|^2\, T_\eta\right)\right] . \tag{17}$$

In the tanh-case it is of the same structure but contains a finite correction R^{fin}:

$$\tilde{w}^{tot} = \hat{w}^{tot}(\hat{\ }\to\tilde{\ }) + R^{fin}(a_i, a_o, |\tilde{\beta}|^2) . \tag{18}$$

Both results are to be compared with the minkowskian probability

$$w^{tot}_{Mink} = \frac{\lambda^2}{4\pi m} T_t . \tag{19}$$

We turn to the interpretation of the two w^{tot}:

1. How is the result (17) related to (19) ? In our case one half of the particles has the chance to decay in the Minkowsky-region $\eta \leq 0$ with $a(\eta) = a_i$ (and the other half in the region $\eta \geq 0$ with $a(\eta) = a_o$). This explains the factors 1/2. The parameters a_i and a_o appear in the result, because $\mathcal{L}_I$ of (7) contains λ/a instead of λ. Introducing in (19), according to $T_t = a_i T_\eta$, the conformal time T_η, we end up with the first term of (17).

2. With regard to the second term of (18) we have to take into account additionally that at $\eta = 0$ massive particles are created out of the background (zeroth order). These particles are decaying in the region $\eta \geq 0$. Therefore, as compared to the first term, the factor 1/2 has to be replaced by $(1/2 + |\hat{\beta}|^2)$.

3. In the tanh-case, the gravitational influence is not localized at $\eta=0$, but smeared out over a finite time interval T^{grav} around $\eta = 0$. This fact must be related to the appearance of the finite correction R^{fin}.

Our result, although suggestive from the physical point of view, still contains the infinite duration T_η of the mutual interaction. Can we process it further ? In Minkowski space-time, the division of (19) by T_t would lead to the reciprocal *liefetime* of massive particles at rest:

$$\frac{1}{\tau_{Mink}} = \frac{\lambda^2}{4\pi m} . \tag{20}$$

The corresponding procedure (division by T_η)gives in the step-case:

$$\frac{1}{\hat{\tau}} = \frac{\lambda^2}{4\pi m}\left(\frac{1}{2a_i} + \frac{1}{2a_o}\right) + \frac{\lambda}{4\pi E_o}|\beta|^2 . \tag{21}$$

To work out the η-lifetime $\tilde{\tau}$ in the tanh-case, we have to take into account that the infinite duration T_η of the mutual interaction on one hand, and the finite duration (gravitational time) of the influence of the curved background around $\eta = 0$ on the other, introduce two different time scales. This is a characteristic trait of an S-matrix approach in a universe with an asymptotically flat expansion law (the curved space-time is localized in time). The natural consequence is, to divide R^{fin} by an appropriate T^{grav}:

$$\frac{1}{\tilde{\tau}} = \frac{1}{\hat{\tau}} + \frac{R^{fin}}{T^{grav}} . \tag{22}$$

(22) seems to be a physically reasonable result which, as far as numerical values are concerned, will give at least a certain mean lifetime. Nevertheless it must be stressed that T^{grav} cannot be defined exactly. Because of the underlying two time-scales there is no direct way of further evaluating w^{tot} exactly. Additional attempts are necessary to obtain a localized picture of the interaction process.

6. GRAVITATIONAL AMPLIFICATION IN ZEROTH ORDER OF THE MUTUAL INTERACTION

We turn to an important phenomenon which governs the mutual interaction of quantum fields in curved space-times. The *mean number* of outgoing massive ϕ-particles in the mode $\underline{p}$ if the in-state was $|a\ \mathrm{in}\rangle$ is:

$$N(\underline{p}^{\phi}|a) = \sum_{\mathrm{all}\, b} |\langle \mathrm{out}\ b|S|a\ \mathrm{in}\rangle|^2\, n(\underline{p}^{\phi}|b) \tag{23}$$

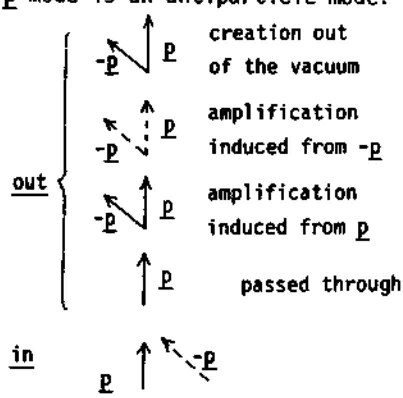
Parker [8,9] has shown that the respective zeroth order is of the structure

$$N^{(o)}(\underline{p}^{\phi}|a) = N^{(o)}(p^{\phi}|0) + n(\underline{p}^{\phi}|a) + N^{(o)}(p^{\phi}|0)\Big[n(\underline{p}^{\phi}|a) + n(-\underline{p}^{\phi}|a)\Big] \tag{24}$$

where $n(\underline{p}^{\phi}|a)$ is the number of ϕ-particles occupying the $\underline{p}$-mode of state $|a\rangle$. The meaning of the three terms in (24) is: particle creation out of the vacuum, particles which have passed through and finally, *gravitationally induced amplification* of the ingoing particle content. This amplification results in additional outgoing pairs. This is indicated by the appearance of $n(-\underline{p}^{\phi}\,|\,a)$. Ingoing particles in the mode $-\underline{p}$ induce therefore creation in the mode $\underline{p}$. See figure 3 for a schematic representation.

Fermions on the other hand show attenuation (negative third term). For complex fields the $-\underline{p}$-mode is an antiparticle mode.

FIGURE 3 :
Outgoing $\underline{p}$-mode in zeroth order



7. GRAVITATIONAL AMPLIFICATION IN HIGHER ORDER OF THE MUTUAL INTERACTION

For higher orders of the mutual interaction we obtain correspondingly [2)]

$$N^{(z)}(\underline{p}^{\phi}|a) = \sum_{\text{all } b} |\langle \text{in } b|S|a \text{ in}\rangle|^2_{(z)}\, n(\underline{p}^{\phi}\, b) + N^{(o)}(\underline{p}^{\phi}|0)\Big[\sum_{\text{all } b} |\langle \text{in } b|S|a \text{ in}\rangle|^2_{(z)} \big(n(\underline{p}^{\phi}|b) + n(-\underline{p}^{\phi}|b)\big)\Big] + \mathrm{Re}(\beta^*_{\underline{p}}\, \sigma_{\underline{p}}) \tag{25}$$

with

$$\sigma_{\underline{p}} = -2\alpha^*_{\underline{p}} \sum_{\text{all } b} \langle \text{in } a|S^{\dagger}|b \text{ in}\rangle \langle 1^{\phi}_{\underline{p}} 1^{\phi}_{-\underline{p}}\, b \text{ in}|S|a \text{ in}\rangle \tag{26}$$

The second term is again the *amplification*, now being part of the mutual interaction. The third term has no correspondence in the zeroth order formula.

Keeping in mind that the calculation is in fact based on one single coherent in-out process, the structure of (26) may be visualized using the following diagrammatic rule: "Let the mutual interaction happen completely within the in-region only and describe it accordingly by in-in transition amplitudes. Process now the corresponding particle outcome as in zeroth order in a twofold way: At first the particles in the $\underline{p}$-mode pass through into the out-region as in (24) to obtain the first term of (25). Secondly, these particles are amplified in the same way as in (24) whereby a possible outcome in the $-\underline{p}$ - mode contributes in a symmetric manner. This leads to the second term in (25). Finally the σ-term of (25) is to be added. See figure 4 for a schematic representation.

Writing the particle number according to

$$N^{(z)}(\underline{p}^{\Phi}|a) = \sum_{\text{all } b} |<\text{in } b|S|a \text{ in}>|^2_{(z)}\, n(\underline{p}^{\Phi}|b)\left\{1 + N^{(o)}(\underline{p}^{\Phi}|0)\left[1+\frac{n(-\underline{p}^{\Phi}|b)}{n(\underline{p}^{\Phi}|b)}\right]\right\} \qquad (27)$$

$$+ \text{Re}\,(\beta^*_{\underline{p}}\, \sigma_{\underline{p}})\,.$$

we can read off that *amplification acts as a mode-dependent amplifica-*

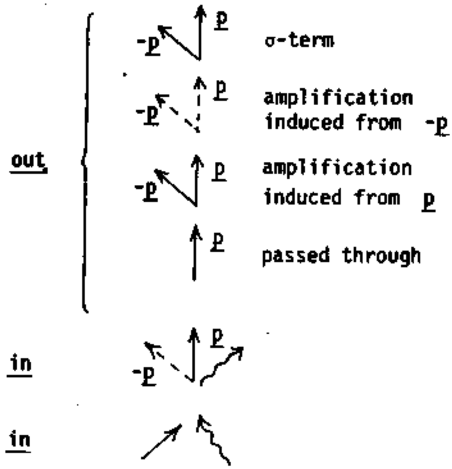


FIGURE 4: Outgoing p-mode in higher order

tion factor (!) This means that the minkowskian contributions contained in the in-in transition amplitude are altered in a multiplicative way, that may lead to considerable modification.

Fermions show again attenuation instead of amplification. See the appendix of reference 2 for details.

An intermediate consequence of (24) and (25) is:

$$N(\underline{p}^{\Phi}|a) - N(-\underline{p}^{\Phi}|a) = \sum_{\text{all } b} |<\text{in } b|\,S\,|\,a \text{ in}>|^2\left[n(\underline{p}^{\Phi}|b) - n(-\underline{p}^{\Phi}|b)\right]. \qquad (28)$$

Assymmetry in the particle content of the outgoing $\underline{p}$- and $-\underline{p}$ -mode can therefore be solely caused by the structure of the mutual interaction as represented by S. *Amplification and the process leading to the σ-*

term in (25) all happen as creation of $(\underline{p},-\underline{p})$ pairs, thus reflecting 3-momentum conservation. In the charged case we would find particle-antiparticle pairs.

In reference 2 we have studied *particle creation out of the vacuum* for the interaction $\mathcal{L}_I = -(\lambda/a^2(\eta))\Phi\Psi$ and the expansion law $a^2(\eta) = 1 + e^{2b\eta}$. This is done exactly up to the order λ^2. The multiplicative amplification and the role of the σ-term are discussed in detail.

8. PAIR-INCLUDING TRANSITION PROBABILITY

We can improve the predictive power of the concepts introduced above in specifying the massive part of the end state too and in using less extensive summations [3]. *Good massive indicators* are all configurations without massive particle pairs, because they consist of massive particles which originate from the interaction only. The corresponding probability that such a transition has occurred regardless of a creation of massive pairs out of the background or interaction is the *pair-including probability*:

$$w^{inc}(\hat{d}^\Phi s^\Psi | c^\Phi r^\Psi) = \sum_{\text{all } Q} |\langle \text{out } Q^\Phi \hat{d}^\Phi s^\Psi | S | c^\Phi r^\Psi \text{in}\rangle|^2 . \quad (29)$$

The sum thereby goes over all possible states Q which consist only of massive pairs. Such pair states are indicated by capital letters.

w^{inc} can again be built up out of in-in amplitudes:

$$w^{inc}(\hat{d}^\Phi s^\Psi | c^\Phi r^\Psi) = \sum_{\text{all } Q} |\langle \text{in } Q^\Phi \hat{d}^\Phi s^\Psi | S | c^\Phi r^\Psi \text{ in}\rangle|^2 . \quad (30)$$

This guarantees that for a finite order of mutual interaction the sum over Q stops, ending with states Q which contain only a particular finite number of massive particles. A perturbation scheme based on Feynman rules, which are again as simple as in flat space-time, may therefore be establis hed. In the framework sketched above, this probability w^{inc} seems to be a concept as close as it can be to what we are used to in flat space-time.

9. SPECIFIED MEAN NUMBER

Specifying again not only the in- but also the unpaired part of the out-state and allowing for the production of pairs as above (thus isolating the particular transition process as far as possible) we are led [3] to the following concept of a *specified mean number* N(←):

$$N(\underline{p}^{\Phi}|\hat{d}^{\Phi}s^{\Psi} \leftarrow c^{\Phi}r^{\Psi}) = \sum_{\text{all } Q} |\langle \text{out } Q^{\Phi}\hat{d}^{\Phi}s^{\Psi}|S|c^{\Phi}r^{\Psi} \text{ in}\rangle|^2 \, n(\underline{p}^{\Phi}|Q^{\Phi}\hat{d}^{\Phi}s^{\Psi}) \tag{31}$$

which can be transcribed into

$$\begin{aligned} N(\underline{p}^{\Phi}|\hat{d}^{\Phi}s^{\Psi} \leftarrow c^{\Phi}r^{\Psi}) &= N^{(o)}(\underline{p}^{\Phi}|0)\, w^{inc}(\hat{d}^{\Phi}s^{\Psi}|c^{\Phi}r^{\Psi}) + \\ &+ \sum_{\text{all } Q} |\langle \text{in } Q^{\Phi}\hat{d}^{\Phi}s^{\Psi}|S|c^{\Phi}r^{\Psi} \text{ in}\rangle|^2 \{n(p^{\Phi}|Q^{\Phi}\hat{d}^{\Phi}s^{\Psi}) + N^{(o)}(p^{\Phi}|0) \cdot \\ &\cdot \left[n(\underline{p}^{\Phi}|Q^{\Phi}\hat{d}^{\Phi}s^{\Psi}) + n(-\underline{p}^{\Phi}|Q^{\Phi}\hat{d}^{\Phi}s^{\Psi})\right]\} + \text{Re}\,(\beta^{*}_{\underline{p}}\,\tilde{\sigma}_{\underline{p}}) \,. \end{aligned} \tag{32}$$

Summation over $\hat{d}$ reproduces (24) and (25). The first term therefore is a weighted particle creation out of the vacuum. The second term is again the amplification which shows its specific structure already on this level.

10. EXAMPLE: COMPTON EFFECT IN THE $\Phi^2\Psi^2$-MODEL REFLECTS GRAVITATIONALLY INDUCED AMPLIFICATION

We study the Compton scattering in the interaction $\mathcal{L}_I = \sqrt{-g}\,\lambda\,\Phi^2\,\Psi^2$ outside of forward scattering [3]. We disregard the contribution resulting from pair creation out of the vacuum and call the specified mean numbers which refer to the mutual interaction only $N_{int}(\leftarrow)$. Discussion of the amplitudes in (32) then leads directly to

$$N_{int}(p^{\Phi}|1^{\Phi}_{\underline{p}}\, 1^{\Psi}_{\underline{k}} + 1^{\Phi}_{\underline{q}}\, 1^{\Psi}_{\underline{l}}) = N_{int}(\underline{k}^{\Phi}|1^{\Phi}_{\underline{p}}\, 1^{\Psi}_{\underline{k}} + 1^{\Phi}_{\underline{q}}\, 1^{\Psi}_{\underline{l}}) \left[1 + N^{(0)}(p^{\Phi}|0)\right] + O(\lambda^3) \quad (33)$$

where $1^{\Phi}_{\underline{p}}\, 1^{\Psi}_{\underline{k}}$ is the end- and $1^{\Phi}_{\underline{q}}\, 1^{\Psi}_{\underline{l}}$ is the initial state of Compton scattering. (33) clearly demonstrates for the Compton effect the gravitationally induced amplification: *Massive Φ- and massless Ψ-particles leaving the mutual interaction are not going out in pairs, as one would expect from the situation in flat space-time. Rather the number of the massive particles is amplified by a momentum-dependent factor*

$$1 + N^{(0)}(\underline{p}^{\Phi}|0) = 1 + |\beta_{\underline{p}}|^2 .$$

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