FIVE LECTURES ON PARTICLE PHYSICS AND COSMOLOGY

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I. INTRODUCTION TO THE BIG BANG MODEL[1.1]

The standard big bang model describes a homogeneous and isotropic Universe. The best observational evidence is that the Universe is indeed homogeneous and isotropic on large scales. Although structure is observed in the Universe on very large scales, the structure seems to be superimposed on a smooth homogeneous background. Galaxies are not distributed randomly in the Universe, but they are correlated. The correlation may be quantified in the form of a two-point correlation function for galaxies, $\xi(r)$, which gives the excess probability of finding a galaxy a distance r from another.[1.2] If $\xi(r) \gg 1$, galaxies are strongly correlated on a scale r and are not distributed smoothly. If $|\xi(r)| < 1$, galaxies can be well described as spread homogeneously throughout the Universe on the scale r. If $\xi(r) \ll -1$, galaxies are anti-correlated. The observations show that $\xi(r)$ decreases with increasing r and that $|\xi(r)| \lesssim 1$ on a scale of $5h^{-1}$Mpc,[(2)] i.e. on

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(2) The constant h reflects the uncertainty in the Hubble constant H_0, $H_0 = 100h\ \mathrm{kms}^{-1}\ \mathrm{Mpc}^{-1}$. It is expected that h is in the range $1 \lesssim h \lesssim 1/2$.

distance scales greater than $5h^{-1}$ Mpc the galaxy distribution is smooth to a good approximation.[1.2] If we assume that galaxies are a fair indication of mass, on scales greater than $5h^{-1}$ Mpc mass should be distributed in a homogeneous manner throughout the Universe, and the Universe becomes smoother on larger scales.

The photons in the 3K microwave background[1.3] give us a sample of the Universe at large distances. Even if the photons are not truly primordial, the cosmic photosphere, or the surface of last scattering, is certainly at cosmological distances. The mean free path for the microwave photons, λ, is related to the electron density, n_e, and the scattering cross section σ by

$$\lambda^{-1} = n_e \sigma \quad , \tag{1.1}$$

where the relevant cross section is the Toupson cross section, $\sigma = 8\pi\alpha^2/3m_e{}^2 = 6.65 \times 10^{-25} cm^2$. The electron density, n_e, is roughly half the baryon density, n_B. The baryon density is not well determined, but its value can be bracketed. It is convenient to express the baryon density in terms of a critical density, ρ_c

$$\rho_c = 3H_o{}^2/8\pi G = 1.88 \times 10^{-29} h^2 g cm^{-3}, \tag{1.2}$$

where G is Newton's constant. The baryon density in terms of Ω_B is

$$n_B = 1.12 \times 10^{-5} \Omega_B h^2 cm^{-3} \quad , \tag{1.3}$$

where Ω_B is the ratio of the baryon density to the critical density,

$$\Omega_B = \rho_B/\rho_c \quad . \tag{1.4}$$

The mean free path of the microwave photons is then

$$\lambda = \frac{1.3\times10^{29}}{\Omega_B h^2}\ \text{cm} = \frac{4.2\times10^{4}}{\Omega_B h^2}\ \text{Mpc} \quad . \tag{1.5}$$

Now this estimate for λ is a gross overestimate, since most electrons will be bound in neutral atoms. If we assume $\Omega_B h^2 \lesssim O(1)$, then the mean free path of the microwave photons is huge, and the photons must have had an origin at a very great distance in order to scatter and relax to a thermal distribution.

The microwave background is very nearly isotropic, i.e. the temperature is very nearly the same in all directions. On angular scales of about 4.5 arc minutes, a recent observation of Uson and Wilkinson[1.4] gives $\Delta T/T \leq 2.4\times10^{-5}$, where ΔT is a difference of the background temperature. On an angular scale of 180° there is a detected $\Delta T/T$ of about 10^{-3}, which could be the result of our galaxy having a peculiar velocity of $10^{-3}c$. The observed isotropy of the microwave background suggests that out to cosmological distances the Universe is isotropic about us. If we believe that we do not live in a special place in the Universe, then the Universe should be isotropic about every point in the Universe. A space that is isotropic about every point is homogeneous, so the microwave background implies that the Universe is homogeneous on large scales.

It should be stressed that a homogeneous, isotropic Universe is not the only possibility. There are many anisotropic cosmologies that can be constructed. In this paper I will only consider homogeneous isotropic cosmologies. There are several advantages for considering only such cosmologies. The foremost reason as discussed above is that our Universe seems to be homogeneous and isotropic. Another reason is that the symmetries of a homogeneous, isotropic space allow a reduction of parameters in the metric. The fewer parameters in the theory, the better chance to interpret data. If the data can be understood by the simple homogeneous, isotropic model, then we have accomplished something truly remarkable, we have constructed a simple model for the large scale structure of the Universe. If the data cannot be understood by a homogeneous, isotropic model, then either the Copernican principle or General Relativity is incorrect, which would be an even more remarkable discovery.

If we assume the Universe is homogeneous and isotropic, it is possible to choose coordinates (r,θ,ϕ,t) for which the metric takes the form

$$ds^2 = dt^2 - R^2(t) \left[dr^2/(1-kr^2) + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right] \quad . \qquad (1.6)$$

where the cosmological scale factor $R(t)$ is a function only of time. In the metric k is a constant, and it is possible to scale r such that $k = \pm 1,0$. The spatial curvature scalar, ${}^3R$, is related to k and R by

$${}^3R = k/R^2(t) \quad . \qquad (1.7)$$

If $k = 0$ the three space is flat, if $k = +1$ the three space has constant positive curvature, and if $k = -1$ the three space has constant negative curvature. The cosmological scale factor determines the proper distance between two fixed coordinates. The proper distance from the origin to coordinate r_1 is given by

$$d_{PROP} = R(t) \int_0^{r_1} dr\ (1-kr^2)^{-1/2} \tag{1.8}$$

$$= R(t) \begin{cases} \sin^{-1} r_1 & k = \pm 1 \\ r_1 & k = 0 \\ \sinh^{-1} r_1 & k = -1 \end{cases} .$$

If $r_1 < 1$ (r_1 is dimensionless and scaled to R) then $d_{PROP} = R(t)r_1$ for any k. The proper distance between any two comoving points scales with R(t).

The time evolution for R(t) is found by solving the Einstein field equations. Non-zero components of $R_{\mu\nu}$ for the metric of Eq. (1.6) are

$$R_{oo} = -3\ddot{R}/R$$
$$R_{ij} = -(\ddot{R}/R+2\dot{R}^2/R^2+2k/R^2)g_{ij}. \tag{1.9}$$

Of course the metric is only half of the problem, the other half of the problem is the dreaded right hand side, $T_{\mu\nu}$. Again, we can use the symmetry of the problem to greatly restrict the form of $T_{\mu\nu}$. A particularly simple choice for $T_{\mu\nu}$ is the perfect fluid form

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - pg_{\mu\nu} \tag{1.10}$$

where ρ is the energy density, p is the pressure, and U_μ is the fluid velocity four vector. In the fluid rest frame $U_\mu = \delta^o_\mu$.

With Eq. (1.10) for $T_{\mu\nu}$ the (oo) and (ii) components of $R_{\mu\nu}$ - 1/2 $g_{\mu\nu}R = 8\pi GT_{\mu\nu}$ give

$$\begin{aligned} (\dot{R}/R)^2 + k/R^2 &= (8\pi G/3)\rho \quad &:(oo) \\ 2\ddot{R}/R + \dot{R}^2/R^2 &= -8\pi Gp \quad &:(ii) \end{aligned} \tag{1.11}$$

Conservation of energy momentum $T^{\mu\nu}{}_{;\nu} = 0$ implies

$$(d/dr)(\rho R^3) = -3pR^2. \tag{1.12}$$

Although $T^{\mu\nu}{}_{;\nu} = 0$ is not an independent equation (it is related to the others by the Bianchi identities) it is convenient to have the form in Eq. (1.12).

Note that the first equation in Eq. (1.11) can be written in the form

$$k/H^2R^2 = (8\pi G\rho/3H^2-1) \tag{1.13}$$

where H is the expansion rate (Hubble parameter)

$$H = \dot{R}/R . \tag{1.14}$$

The r.h.s. of Eq. (1.13) is simply $\Omega-1$. Therefore if $\Omega > 0$ ($\Omega < 0$), k is positive (negative), and if $\Omega = 1$, then $k = 0$.

We will consider two simple forms for the equation of state; "matter" with $p = 0$, and "radiation" with $p = \rho/3$. Equation 1.12 then gives

$$\rho_R \propto R^{-4};\ \rho_M \propto R^{-3} \tag{1.15}$$

for the radiation and matter energy densities. The generic behavior of R(t) for $k = +1$, $k = -1$, or $k = 0$ is shown in Fig. 1.1. If $k = +1$ the Universe is closed, if $k = -1$ the Universe is open, if $k = 0$, the Universe is at the borderline. The generic behavior obtains for any equation of state, so long as $3p+\rho > 0$.

From Eq. (1.15) we see that for sufficiently small value of R, the Universe was radiation dominated. The radiation energy density receives a contribution not only from photons, but from all species of particles with mass smaller than the temperature. Therefore the radiation energy density is given by

$$\rho_R = (\pi^2/30)g_* \, T^4 \tag{1.16}$$

where g_* counts all species of particles with masses less than T, weighted by their spin degeneracy factors and a factor that depends on whether the particle is a boson or fermion

$$g_* = \sum_{\text{bosons}} g_B + (7/8) \sum_{\text{fermions}} g_F \ . \tag{1.17}$$

We may also neglect the curvature term, k/R^2, relative to $G\rho(\propto R^{-4})$ in the early Universe, and solve Eq. (1.11) for the time since infinite temperature

$$t = (45/16\pi^3)^{1/2} g_*^{-1/2} m_{pl}\, T^{-2}, \tag{1.18}$$

where m_{pl} is the Planck mass, $m_{pl} = G^{-1/2}$.

The final ingredient in the standard model is conservation of entropy. The total entropy in a comoving volume is given by

$$S = s\, R^3 \tag{1.19}$$

where s is the entropy density defined by

$$\begin{aligned} s &= (\rho + p)/T \\ &= (2\pi^2/45)\, g_* T^3 \quad . \end{aligned} \tag{1.20}$$

Note that if g_* changes as the temperature of the Universe falls below the mass of some particle, the temperature of the Universe will not scale exactly as R^{-1}, since $g_*(T)T^3R^3$ is constant, rather than T^3R^3.

Application of the standard big bang model discussed above gives a good description of the present day Universe. It can explain the observed redshift. It also relates the age of the Universe to the Hubble parameter (for a matter dominated Universe)

$$t_U = H_o^{-1} \int_o^1 \frac{dx}{[1-\Omega+\Omega/x]^{1/2}} . \qquad (1.21)$$

For $\Omega = 1$

$$t_U = 2/3\, H_o^{-1} = 6.5h^{-1} \times 10^9 y . \qquad (1.22)$$

If $h = 1/2$, then the Universe is 13×10^9y old, which is in agreement with most dating methods.

The isotropy of the microwave background suggests the Universe was smooth when the photons last scattered. This occurred when the Universe was hot enough to ionize hydrogen, at a temperature of about 4×10^3K, or about 10^{12}s after the big bang.

We firmly believe that we have a good model of the Universe starting at 10^{12}s. How early can we extrapolate the standard model? In the next section I will review primordial nucleosynthesis, which suggests that as far back as 1 second after the big bang the Universe was well described by the standard model.

II. PRIMORDIAL NUCLEOSYNTHESIS[2.1]

The observed isotropy of the microwave background radiation is evidence that the standard big bang model of the Universe can be believed as early as 10^{12}s after the big bang. In the mid 1960's Peebles[2.2]; Wagoner, Fowler and Hoyle[2.3]; and later Wagoner[2.4] demonstrated that at a few minutes after the big bang, a significant fraction of the neutrons and protons would be synthesized into ^{4}He. In addition to ^{4}He, interesting amounts of ^{2}H, ^{3}He, and ^{7}Li are also predicted to have been produced at the same time. To date there are no other models to account for the ^{4}He and ^{2}H observed, and big bang nucleosynthesis provides the best evidence that the Universe was once at temperatures in excess of 1 MeV at times about one second after the bang. In this section I will review the main features of primordial nucleosynthesis.

There are six weak reactions that can interconvert neutrons and protons

$$
\begin{aligned}
& n \leftrightarrow p + e^- + \bar{\nu}_e \\
& n + \nu_e \leftrightarrow p + e^- \\
& n + e^+ \leftrightarrow p + \bar{\nu}_e
\end{aligned}
\tag{2.1}
$$

It is convenient to combine the total rates for destroying neutrons [$\Gamma_{n\to p} = \Gamma(n\to pe\bar{\nu}) + \Gamma(n\nu\to pe) + \Gamma(ne^+\to p\bar{\nu})$] and protons [$\Gamma_{p\to n} = \Gamma(pe\bar{\nu}\to n) + \Gamma(pe\to n\nu) + \Gamma(p\bar{\nu}\to ne^+)$] in terms of the temperature T and neutrino temperature T_ν[3]

$$\Gamma_{n\to p} = \frac{G_F^2(1+3g_A^2)}{2\pi^3} \int \frac{[1-m_e^2/(Q+q)^2]^{1/2}\,(Q+q)^2\,q^2}{[1+\exp(q/T_\nu)][1+\exp(-(Q+q)/T)]}\,dq$$

$$\Gamma_{p\to n} = \frac{G_F^2(1+3g_A^2)}{2\pi^3} \int \frac{[1-m_e^2/(Q+q)]^{1/2}\,(Q+q)^2\,q^2}{[1+\exp(-q/T_\nu)][1+\exp((Q+q)/T_\nu)]}\,dq, \tag{2.2}$$

(3) The integrals run from $(-\infty,+\infty)$ with the interval $(-Q-m_e, -Q+m_e)$ removed.

where G_F is Fermi's constant $g_A = 1.2$, and Q is the neutron-proton mass difference

$$Q = m_n - m_p = 1.293 \text{ MeV} . \tag{2.3}$$

Note that at high temperature, $T > Q$, (assuming $T=T_\nu$)

$$\begin{aligned} \Gamma_{n\to p} = \Gamma_{p\to n} &= \frac{7\pi}{30} \quad G_F^2 \ (1+3g_A^2)T^5 \\ &= 0.8T_{MeV}^5 \quad sec^{-1} \end{aligned} \tag{2.4}$$

where T_{MeV} is the temperature in MeV. This is to be compared with the expansion rate $H = \dot{R}/R$

$$H = \left(\frac{8\pi G\rho}{3}\right)^{1/2} = 0.7 \ T_{MeV}^2 \quad sec^{-1} \tag{2.5}$$

where we have used $g_* = 43/4$ to account for $\gamma, e^\pm$, and 3 neutrinos. Therefore when $T_{MeV} \geq 1$, the $n \leftrightarrow p$ reactions occur on a timescale greater than the expansion rate and the Universe should consist of roughly equal amounts of neutrons and protons, $n/p = \exp(-Q/T)$.

If the heavier elements were in equilibrium, the number density of species i would be

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp[(\mu_i - m_i)/T] \quad , \tag{2.6}$$

where μ_i is the chemical potential for species i. The chemical potential for a species of Z_i protons and $A_i - Z_i$ neutrons is

$$\mu_i = Z_i \mu_p + (A_i - Z_i)\mu_n \quad . \tag{2.7}$$

It is convenient to express the abundance of the elements in terms of mass fractions, defined as

$$X_i = n_i A_i/n_N \tag{2.8}$$

where n_N is the total (bound plus free) nucleon density. Therefore in nuclear statistical equilibrium the mass fraction of species i is

$$X_i = \left[\frac{4\zeta(3)}{\sqrt{2\pi}} \frac{n_N}{n_\gamma}\left(\frac{T}{n_N}\right)^{3/2}\right]^{A_i-1} \frac{g_i}{2} A_i^{1/2} \times X_p^{Z_i} X_n^{A_i-Z_i} \exp(B_i/T) \quad , \tag{2.9}$$

where n_γ is the photon number density, and B_i is the binding energy

$$B_i = -m_i + Z_i m_p + (A_i - Z_i)m_n \quad . \tag{2.10}$$

Although the nuclear reaction rates proceed much faster than the expansion rate, nuclear statistical equilibrium will not be obtained. Starting with a gas of neutrons and protons, production of nuclei commences with deuterium production. However the binding energy of deuterium is only 2.22 MeV, and its mass fraction in nuclear statistical equilibrium is

$$X_{^2H} = 4 \times 10^{-13} \Omega_B h^2 \; T_{MeV}^{3/2} \exp(2.22/T_{MeV}) \tag{2.11}$$

where Ω_B is the fraction of critical density today in baryons, and we have assumed $X_n = X_p = 0.5$. For $T \geq 10^{-1}$ MeV deuterium acts like a bottleneck impeding the build up of heavier elements. Once the bottleneck is broken at $T = 10^{-1}$MeV $\simeq 10^9$K nuclear statistical equilibrium is obtained for light elements. Since the binding energy of ^{4}He is much larger than the binding energy of the other light elements, most of the available neutrons will be processed into ^{4}He. The lack of stable nuclei with A=5 or A=8 prevents build up of heavier elements via n+^{4}He, p+^{4}He reactions, and the Coulomb barrier prevents ^{4}He+^{3}He reactions.

It is possible to estimate the amount of ^{4}He produced. When the n↔p reactions freeze out (Γ_p, $\Gamma_n < H$) at T = 0.7 MeV the neutron-proton ratio is n/p = exp(-Q/0.7MeV) ≃ 0.16. After freeze out n/p changes only through neutron decay. At T ≃ 0.1 MeV when the ^{2}H bottleneck is broken some of the neutrons have decayed and n/p ≃ 0.14. At this point almost all of the neutrons are processed into ^{4}He. If all the neutrons are turned into ^{4}He

$$Y = X(^4He) = 2\,X_n = 2\,\frac{n/p}{1+n/p} \simeq 0.25\ . \tag{2.12}$$

In Figure 2.1 is given the ^{4}He abundance from primordial nucleosynthesis that results from a numerical calculation of Wagoner's code[2.3,2.4] for 2,3, or 4 light (m < 25MeV) neutrinos and τ_n = 10.61 min. Determination of the primordial ^{4}He abundance from observations of metal-poor systems suggests $Y = 0.24 \pm 0.01$[2.1]. For n_B/n_γ in the range $1-10 \times 10^{-10}$ primordial nucleosynthesis also predicts mass fractions of ^{2}H in the 10^{-3}-10^{-4} range, ^{3}He in the 10^{-4}-10^{-5} range and ^{7}Li in the 10^{-9} range. All these predictions are consistent with the best estimates of the primordial values.[2.1]

Primordial nucleosynthesis is remarkably successful in predicting the abundances of the light elements, and its success is our strongest evidence that we can extrapolate the standard big bang model of the Universe back to one second after the bang when the temperature of the Universe was about 1 MeV.

III. DARK MATTER OBSERVED IN THE UNIVERSE[3.1]

One of the fundamental cosmological parameters is the mean energy density of the Universe. In this section I will discuss determination of the contribution of galaxies to the mean energy density. In determining the masses of galaxies, clusters, etc., it will become obvious that most of the mass in the Universe is dark - invisible to us. The existence of dark matter is very exciting from a particle physics point of view, since the dark matter may be some elementary particle that was produced in the early Universe. In the next section I will review two possible candidates for dark matter. In this section I will briefly review the evidence for dark matter.

The bulk of visible matter in the Universe is concentrated into galaxies. The mass density contributed by galaxies can be written as

$$\rho_G = L\left(\frac{M}{L}\right) \qquad (3.1)$$

where L is the luminosity per unit volume contributed by galaxies, and (M/L) is the mass to light ratio of galaxies. The luminosity density is given by integrating the luminosity of galaxies as a function of mass over the number of galaxies with that mass

$$L = \int dn/dM\, L(M)\, dM \simeq 3 \times 10^8 h\, L_\odot\, \mathrm{Mpc}^{-3} \qquad (3.2)$$

where the numerical estimate is from ref. 3.2. Throughout this discussion I will try to keep all uncertainties due to the Hubble constant. In Eq. (3.2) a factor of h^3 enters from dn, and a factor of h^{-2} enters from L(M). If we express (M/L) in solar units, then

$$\rho_G = 2 \times 10^{-32} h\, (M/L) \simeq 10^{-3} h^{-1} \rho_c\, (M/L)\ . \qquad (3.3)$$

The problem now is to determine the mass to light ratio. In the local stellar neighborhood M/L is 1-2, in the inner Milky Way M/L is 4-8 and in the outer Milky Way M/L is 40±30.[3.3] For the inner parts of elliptical or spiral galaxies M/L is (8-12)h.[3.3] Nowhere is the evidence for (M/L) large enough to make Ω_G close to one. If we use M/L = 10h, then Ω_G = 0.01.

The really remarkable observation fact is the extremely large (M/L) values. These large values suggest dark matter. However the best evidence for dark matter comes from rotation curves of galaxies.

In standard spiral galaxies the light is in highly flattened disks. The radial scale of the spiral galaxy is set by the Holmberg radius, where the surface brightness falls below 26.5 mag $arcsec^{-1}$. The average Holmberg radius for spirals is about $(10-15)h^{-1}$kpc. The scale height for the thickness of the disk is typically about 0.3 kpc. However Ostriker and Peebles suggest that cold stellar disks have an instability against "bar" modes. This bar instability can be removed if the galaxy is embedded in a massive "hot" halo. A massive hot halo is also suggested by studies of rotation curves of galaxies. In any model of the galaxy, the rotation velocity of stars, gas, etc. depends upon the mass interior to the orbit, and for objects "outside" most of the mass of the galaxy, the velocity must decrease with distance (Kepler's law). However studies of rotation curves of galaxies show no evidence of decreasing, even as far out as three times the Holmberg radius. This is very strong evidence that there is a massive halo surrounding galaxies that is non-luminous.

The identity of the non-luminous matter is unknown. If we include the contribution from the halo $\Omega_G \geq 0.01 - 0.1$. The agreement of the predictions of standard model nucleosynthesis with inferred primordial values suggests that the values of Ω_G in the above range easily can be in the form of baryons.[2.1]

The real evidence for non-baryonic matter comes from the inferred Ω due to clusters of galaxies. Again, using the velocities of galaxies in the cluster to measure the mass of the cluster it is possible to infer an Ω due to clusters. The (M/L) values for clusters of galaxies range as high as (100-500)h, giving a value of Ω in the range $\Omega = 0.1-0.5$. A value of Ω of 0.5 is too large to be allowed by primordial nucleosynthesis - therefore if Ω is greater than about 0.2 some of the mass density of the Universe must be non-baryonic.

Finally, as will be discussed in the final section, the only reasonable value for Ω from the theoretical point of view is $\Omega = 1$. Therefore we see that there seems to be dark matter in the Universe on scales as small as our local solar neighborhood, to scales as large as the entire Universe.

IV. CANDIDATES FOR DARK MATTER - NEUTRINOS AND AXIONS

As reviewed in Section III, the study of the structure of galaxies reveals the presence of a component of the total mass of the galaxy that is dark. Dark matter seems to be present not only in galactic halos, but also in the disk in the local vicinity of the solar system. Dark matter is also present in larger systems, such as binary galaxies, small groups of galaxies, clusters of galaxies, and perhaps in the Universe as a whole.

It is not clear if all the dark matter problems have the same solution. It may be that the dark matter in the disk is different than the dark matter in the halo, which in turn is different than the dark matter in clusters of galaxies, etc. It is also not clear whether baryons, either in the form of primordial black holes, jupiters, etc. could be some (or all) of the dark matter. Of particular cosmological interest is the possibility that some component of dark matter is non-baryonic, in the form of some elementary particle that is a remnant of the big bang.

In this section I will discuss the production of elementary particles in the big bang. Some proposed candidates for dark matter are given in Table I. Possible masses range from 10^{-5}eV for axions to 10^{28}eV for pyrgons or Kaluza-Klein monopoles. The relic abundances of the particles if they are to contribute a significant fraction of the mass of the Universe are also given in Table I. One striking fact from the table is that there is a range of about 10^{33} in possible ino masses and abundances. Another striking fact is that particle physicists have been remarkably generous in providing candidates for the dark matter.

TABLE I

SOME CANDIDATES FOR DARK MATTER

Candidate	Mass	Present Abundance
Axion	10^{-5}eV	10^{9}cm^{-3}
Neutrinos	10eV	10^{2}cm^{-3}
Gravitino/Photino	10^{3}eV	1 cm^{-3}
Baryons	10^{9}eV	10^{-6}cm^{-3}
Sneutrino/Photino	10^{11}eV	10^{-8}cm^{-3}
GUT Monopoles	10^{25}eV	10^{-22}cm^{-3}
Pyrgons/K.-K. Monopoles	10^{28}eV	10^{-25}cm^{-3}

Since the rate that particle theorists propose new candiates is faster than the rate for writing a comprehensive survey of possibilities, I will not attempt to discuss all possibilities. Rather, I will concentrate on what I consider to be the two most likely possibilities - neutrinos and axions.

<u>Neutrinos</u>

Neutrinos are neutral leptons, i.e. particles that only participate in weak interactions. In the early Universe neutrinos would have been produced in weak processes such as $e^+e^- \rightarrow \nu_i\bar{\nu}_i$ where the subscript i indicates the neutrino family, e, μ, or τ (or possibly more). If $E > m_e$, the cross section for neutrino production is

$$\sigma(e^+e^- \leftrightarrow \nu_i\bar{\nu}_i) \simeq G_F^2E^2 \qquad (4.1)$$

where G_F is Fermi's constant. When $E > m_e$, the number density of electrons is given by $n_e \sim T^3$, so the production rate of neutrinos is $(E \simeq T)$

$$\Gamma_P \sim n\sigma \sim T^3G_F^2E^2 \simeq G_F^2T^5 \ . \qquad (4.2)$$

This rate is to be compared with the expansion rate of the Universe $\Gamma_E = T^2/m_{pl}$

$$\begin{aligned} \frac{\Gamma_P}{\Gamma_E} &\sim G_F^2T^3m_{pl} \\ &\sim O(1)\ [T/1\text{MeV}]^3 \ . \end{aligned} \qquad (4.3)$$

When the temperature of the Universe is greater than about 1 MeV, Γ_P/Γ_E is much greater than one and neutrinos interact; they are created and they are destroyed. The neutrinos would then be in equilibrium with the rest of the matter in the Universe. When the temperature of the Universe is less than about 1 MeV, Γ_P/Γ is much less than one and neutrinos "freeze out." After freeze-out they no longer interact and cannot equilibrate with the rest of the Universe.

If we assume that $m_\nu \ll 1$ MeV, the neutrinos will be relativistic at freeze out, and the number density of neutrinos (plus antineutrinos) at freeze out ($T=T_F$) would be

$$n_\nu = (\zeta(3)/\pi^2)\ (3/4)\ T_F^3 = (3/4)\ n_\gamma \tag{4.4}$$

where we have assumed 2-component neutrinos and n_γ is the number density of photons at freeze out.

The neutrinos decouple before e^+e^- annihilation. The e^+e^- annihilation increases the neutrino temperature by a factor of $(11/4)^{1/3}$ because the entropy in e^+e^- pairs is converted into photons but not neutrinos (since neutrinos have decoupled). Therefore the number density of neutrinos today, $n_{\nu o}$, is (per family)

$$n_{\nu o} = (3/4)\ (4/11)\ n_{\gamma o} = 110\text{cm}^{-3}\ . \tag{4.5}$$

If the neutrino has a mass $m_{\nu i}$, then the relic neutrinos would contribute a fraction of the closure density

$$\Omega_{\nu i} = 0.01\ (m_{\nu i}/eV)h^{-2}\ . \qquad (4.6)$$

If $h = 1/2$, $m_{\nu i}$ as low as 25eV could close the Universe. If we require $\Omega_{\nu i} \leq 1$, then $m_{\nu i} \lesssim 100h^2$eV. The limit $m_{\nu i} \lesssim 100$eV is much better than the present bounds on $m_{\nu\mu}$ ($\lesssim 0.5$ MeV) and $m_{\nu\tau}$ ($\lesssim 164$ MeV).

The bound on m_ν has assumed that there are two degrees of freedom for ν in equilibrium at 1MeV, that there is a single species with a large mass, that the neutrinos are stable, and that $m_\nu < 1$MeV. If there are more than two degrees of freedom for neutrinos, and the other degrees of freedom interact with normal matter more weakly than usual, the bound on the mass has been studied by Olive and Turner.[4.1] If the neutrino is unstable with a lifetime less than the age of the Universe, for a sufficiently short lifetime the massless decay products of the neutrino will give $\Omega \leq 1$, as pointed out by Dicus, Kolb and Teplitz.[4.2] Finally if the neutrino is very massive its number density at freeze out will be exponentially supressed. In this case a neutrino with mass greater than about 2 GeV will give $\Omega \leq 1$, even if stable.[4.3]

From the particle physics point of view neutrinos are the most likely candidate ino to be important for galaxy formation. We know neutrinos exist! The standard Weinberg-Salam model has massless neutrinos, but there is no deep understanding (e.g. a symmetry principle) to explain why they should be massless. If neutrinos are stable ($\tau > t_u$) and have a mass in the 25-100eV range they will play an important role in the dynamics of galaxy formation.

Axions[4.4]

In the theory of strong interactions, QCD, it is possible to add to the usual Lagrangian

$$L_0 = -(1/4)\, G^a_{\mu\nu} G^{a\mu\nu} \quad , \tag{4.7}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$, a term of the form

$$L_\theta = (\theta/32\pi^2)\ \mathrm{tr}\ G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \tag{4.8}$$

where $\tilde{G}^{a\mu\nu}$ is the dual of $G^a_{\mu\nu}$, $\tilde{G}^{a\mu\nu} = G^a_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}$. It is possible to express L_θ as a total divergence, but unlike QED (where a similar term can be discarded as a surface term) it can have physical effects due to instantons. Since L_θ has the form $\sim \theta\vec{E}\cdot\vec{B}$, it violates P and T, hence it is odd under CP. One physical effect of the L_θ term would be a contribution to the neutron electric dipole moment. The fact that the neutron electric dipole moment is less than of order 10^{-19}e cm requires $(\theta/32\pi^2) \leq 10^{-8}$. There is an additional contribution to θ. The quarks receive a mass when a Higgs field receives a vacuum expectation value $\langle\phi\rangle$. In general the coupling of ϕ to the quarks is neither real nor diagonal. When a rotation is performed to have the mass matrix real and diagonal, θ receives a contribution

$$\theta = \arg \det \underline{M} \quad , \tag{4.9}$$

where $\underset{\sim}{M}$ is the quark mass matrix. Therefore the relevant parameter for CP violation is

$$\bar{\theta} = \theta + \arg \det \underset{\sim}{M} \quad . \tag{4.10}$$

The two terms in Eq. (4.10) have quite different origins, and it is necessary that they cancel to give $\bar{\theta} \leq 10^{-8}$. In order to understand this cancellation, Peccei and Quinn[4.5] introduced a global U(1) symmetry such that $\theta = -\arg \det \underset{\sim}{M}$ when $\phi = \langle\phi\rangle$. Thus ϕ is determined dynamically, and at the minimum of the Higgs potential, $\bar{\theta} = 0$. Weinberg[4.6] and Wilczek[4.7] pointed out that the spontaneous breaking of the $U(1)_{PQ}$ symmetry would lead to the appearance of a pseudo Nambu-Goldstone particle, called the axion. The axion is a pseudo Nambu-Goldstone particle since the $U(1)_{PQ}$ symmetry is not exact and is broken by instanton effects. Therefore the axion is not exactly massless, but picks up a mass

$$m_a = f_\pi m_\pi / v = 30\text{keV}\ (250\text{GeV}/v) \tag{4.11}$$

where the factor $f_\pi m_\pi$ comes from instanton effects, and v is the magnitude of the vacuum expectation of the Higgs field, $\langle\phi\rangle = ve^{i\alpha}$. In the original axion models ϕ was the Higgs responsible for the $SU_2 \times U_1$ weak breaking, but Kim[4.8] pointed out that it is not necessary to tie ϕ to the weak breaking. Throughout this section I will keep v arbitrary. For the axion to suppress strong CP violation v is undetermined, but cosmological arguments will be able to bracket v to be in the range $10^8 - 10^{12}$ GeV.

The axion couplings to fermions is

$$L_{ffa} = (m_f/v) i \bar{f}\gamma_5 f a \tag{4.12}$$

where a is the axion field (a-Im ϕ) and f is some fermion with mass m_f. The axion also couples to photons through the anomaly

$$L_{\gamma\gamma a} = (\alpha/3) v^{-1} F_{\mu\nu}\tilde{F}^{\mu\nu} a \tag{4.13}$$

where the F's are for the electromagnetic field. If $m_a < 2m_e$, the axion will decay to two photons with a lifetime

$$\tau(a \to \gamma\gamma) = (v/f_\pi)^5 \tau_\pi \tag{4.14}$$

where τ_π is the neutral pion lifetime. In order to hide the axion from detection, it is necessary to make v large. The properties of the invisible* axion are shown in Fig. (4.1).

The possible values of v can be limited by consideration of stellar evolution, in particular energy loss in red giant stars.[4.9] If axions can be produced in the core, they would escape the star causing an energy loss, and the nuclear fuel would have to be burned at a greater rate to compensate. If the loss is great enough the evolution of the red giant star would be too rapid to account for the observed numbers. Note that the mass of the axion is proportional to v^{-1}. If v is small enough, the axion would be too massive to be produced in the star. The

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*Note that as $v \to \infty$, the axion decouples from the low energy theory. A model with a large v will decouple a, thus make it "invisible."

axions are produced either through "Compton" emission, $\gamma+e\to a+e$, or through "Primakoff" emission, $\gamma+e\to\gamma+a$. In the first case the cross section for axion production depends upon the axion-electron coupling, while in the second case the cross section for axion production depends upon the axion-photon couplings. From Fig. (4.1) it is seen that both couplings are proportional to v^{-1}, so if v is <u>large</u> enough the axion production cross section will be small enough that axion emission is not a problem. Stellar evolution rules out $10^2\text{GeV} \lesssim v \lesssim 10^8\text{GeV}$.

Axions may play an important role in galaxy formation, since for certain values of v, Ω_a could be close to one, where Ω_a is the fraction of closure density in axions today.[4.10] When the temperature of the Universe was $T > v$, the finite temperature effects should have restored the symmetry,[4.11] and $\langle\phi\rangle=0$. When the temperature crops below $T\approx v$, a phase transition occurs and $|\langle\phi\rangle| = v$. However for temperatures $v \gtrsim T \gtrsim \Lambda_{QCD}$, where $\Lambda_{QCD} \approx 100\text{MeV}$, instanton effects are not important,[4.12] and the axion is a true Nambu-Goldstone particle. In this temperature regime the phase of $\langle\phi\rangle$ is irrelevant. When the temperature drops to $T \lesssim \Lambda_{QCD}$, the degenerate minima in $\langle\phi\rangle$ become noticeable and the axion field will evolve to one of the minima. The equation describing its evolution is (assuming the minimum at $\alpha=0$).

$$\ddot{\alpha} + 3H\dot{\alpha} + (\partial V/\partial\alpha) + \Gamma_a\dot{\alpha} = 0 \qquad (4.15)$$

where $\partial V/\partial\alpha = m_a^2\alpha$. We can ignore the Γ_a term in (4.15) for the invisible axion. The axion mass in $\partial V/\partial\alpha$ is a function of temperature[4.10,4.12]

$$m_a(T) \approx (\Lambda^2/v)(\Lambda/T)^4\,[\ln(T/\Lambda)] \qquad (4.16)$$

for temperatures $T \geq O(\Lambda)$.

The potential energy in the axion field due to the misalignment of α is

$$V(\alpha) = \alpha^2 m_a^2 v^2 = \alpha^2 \delta V. \tag{4.17}$$

If we assume m_a is a constant, then Eq. (4.15) implies

$$\alpha = \alpha_o A \cos(m_a t) \quad , \tag{4.18}$$

where $\alpha_o = \alpha(t_{QCD})$ and $A = (T/T_{QCD})^{3/2}$. This would correspond to an energy density today of

$$\rho_a = \alpha_o^2 10^{-22} g\ cm^{-3} \quad , \tag{4.19}$$

or about $10^7 \rho_c$ if $\alpha_o \approx 1$. However, m_a is not a constant and during the period that it changes the amplitude of the oscillation, A, is damped to keep the adiabatic invariant $A^2(t)m(t)$ constant. Even if $\alpha \approx 1$ at high temperatures α is damped to $\alpha_o^2 = 10^{-7} v_{12}{}^{7/6}$. Therefore, the true ρ_a today is[4.10]

$$\rho_a \approx 10^{-29} v_{12}^{7/6} g\ cm^{-3}. \tag{4.20}$$

From Eq. (4.20) we see that if $v_{12} > O(1)$, Ω_a would be greater than 1, and if $v_{12} = O(1)$, the Universe today would be dominated by a condensate of zero momentum axions.

Hot and Cold Particles

Neutrinos and axions are examples of hot and cold dark matter respectively. The particle is hot or cold depending upon the velocity of the particle when the Universe becomes matter dominated.

Recall that the radiation energy density today is

$$\rho_{RO} = (\pi^2/30)g_* T^4 \qquad (T = 2.7) \tag{4.21}$$

with g_* today given by

$$g_* = 2 + 2 \cdot 3 \cdot (7/8) \cdot (1/1.401)^4 \tag{4.22}$$

where the 2 is for photons, and the second term comes from three neutrinos with two degrees of freedom at a temperature $T_\nu = (1.401)^{-1} T_\gamma$. Since $\rho_R \sim R^{-4} = (1+z)^4$, the energy density in radiation at redshift z is

$$\rho_R = \rho_{RO}(1+z)^4 \tag{4.23}$$

From Eq. (4.21) and Eq. (4.22)

$$\Omega_{RO} = 3.9 \times 10^{-5} h^{-2} \tag{4.24}$$

is the present fraction of the critical density in the form of radiation. If we assume there is some massive particle with $\Omega_M = \Omega_{MO}$ today, then

$$\rho_M = (1+z)^3 \Omega_{MO} \rho_c \tag{4.25}$$

and

$$\frac{\rho_M}{\rho_R} = \frac{2.6\times10^4 h^2 \Omega_{MO}}{1+z} \tag{4.26}$$

The crucial observation is that in deriving Eq. (4.26) we have not specified the identity of the dark matter, only that it gives a total Ω_{MO} today (we expect $\Omega_{MO} \approx 0.9$ if the total $\Omega = 1$).

The importance of the velocity has to do with the damping of perturbations by free-streaming. Perturbations of the particles will suffer collisionless phase mixing on scales up to[4.13]

$$\ell_{DAMP} = H^{-1} v \tag{4.27}$$

where H^{-1} is the horizon and v is the particle velocity. Note that $H^{-1} \approx t_u$, and that the damping scale increases until the particle becomes non-relativistic. The Universe becomes matter dominated at a temperature O(10eV), and if a particle has mass $m \lesssim O(3T) = 30\text{eV}$ (for example neutrinos), structure up to $H^{-1}(T=10\text{eV})$ will be wiped out. This corresponds to a mass of about $3 \times 10^{15} M_\odot$.[4.13] If a particle is cold, $v \ll 1$, the damping scale will be much smaller.[4.14] The coldest particle is the axion, since it is a condensate of zero momentum particles.

The numerical simulations of the clustering[4.15] of relic particles depend upon the cosmological parameters (such as Ω, H_o, and the spectrum and magnitude of the initial perturbation spectrum) and upon how "hot" the particle is. Therefore, from a particle physics viewpoint it is only necessary to specify Ω and determine how hot the particle is. The identity of the particle is irrelevant, as long as it is dissipationless.

V. INFLATION

The standard big bang model is an accurate description of the Universe at least as far back as the time of primordial nucleosynthesis, one second after the big bang. If we believe that the observed baryon asymmetry was generated dynamically in the early Universe, then the standard big bang model is an accurate description of the Universe as far back as 10^{-36} s after the bang.

Despite the success of the standard model there are some fundamental problems. The first problem has to do with the "flatness" of the Universe. Today we know that $\Omega - 1 = O(1)$, that the Universe is close to critical density. If we use Eq. (1.13) we can express Ω-1 as

$$\Omega-1 \propto H^{-2}R^{-2} \quad . \tag{5.1}$$

In an isentropic expansion $R \sim T^{-1}$. The Hubble parameter $H^2 \sim \rho \sim T^m$, where m = 4 or 3 depending upon whether the Universe is radiation dominated or matter dominated. Therefore Eq. (5.1) implies

$$\Omega - 1 = (\Omega-1)_{TODAY}(T/T_o)^{-n} \tag{5.2}$$

where $n=2$ for the radiation dominated epoch and $n=1$ for the matter dominated epoch, and T_o is the present temperature, $T_o = 2.7$ K. At primordial nucleosynthesis, $T = 10^9$ K, $\Omega - 1 = O(10^{-17})$, and at the Planck temperature, $T = 10^{32}$ K, $\Omega - 1 = O(10^{-63})$. Unless $\Omega - 1$ was fine tuned to zero to an accuracy of one part in 10^{63}, the Universe would have recollapsed, or become curvature dominated long before today. The extraordinary fine tuning is referred to as the flatness problem.

A second problem of the standard cosmology has to do with particle horizons. A massless particle emitted from a radial coordinate r_H reaches the origin (r=0) in a time t, given by

$$\int_o^t \frac{dt}{R(t)} = \int_o^{r_H} \frac{dr}{(1-kr^2)^{1/2}} \quad . \tag{5.3}$$

The physical distance travelled by the massless particle is given by

$$d_H = R(t)r_H = R(t) \int_o^t dt/R(t) \ , \tag{5.4}$$

where we have ignored the kr^2 term in the r.h.s. of Eq. (5.3). If the expansion of the Universe is such that $R \sim t^{1/2}$ (radiation dominated) or $R \sim t^{2/3}$ (matter dominated) then $d_H \approx t$. It should be stressed that $R(t)$ depends upon the equation of state, and for sufficiently bizarre equations of state, the integral in Eq. (5.4) might diverge, removing horizons. The existence of particle horizons makes the observed isotropy and homogeneity of the Universe hard to understand. At the surface of last scattering for the microwave photons, the horizon distance corresponds to an angular size of less than 1°. In the standard cosmology the observed smoothness cannot be due to microphysical processes.

These two fundamental problems can be solved in a model with a creation of a large amount of entropy. The most popular way to create the entropy is a method proposed by Guth -- Inflation. The Inflationary Universe is based upon the Universe going through a phase transition with a period of time during which the Universe is dominated by vacuum energy. At the end of the inflationary period the vacuum energy is converted to radiation, increasing the entropy by a large (exponential) amount.

I will illustrate inflation in the general form of "new inflation." New inflation models are based upon phase transitions associated with the spontaneous breaking of symmetry. As discussed in the axion section, at sufficiently high temperature the effects of the ambient background gas should restore the symmetry, giving $\langle\phi\rangle=0$. As the Universe cooled below the critical temperature, the potential minimum is no longer at $\phi=0$, but at say $\phi=\sigma$. If the Higgs field is away from the minimum the potential energy of the Higgs field $V(\phi)$ may dominate the radiation energy density. If $V(\phi)$ dominates, then

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} = \frac{8\pi GV(\phi)}{3} \qquad (5.5)$$

which has the solution

$$R/R_0 = \exp[(8\pi V(\phi)/3m_{pl}^2)^{1/2}t] \ . \tag{5.6}$$

The Universe is in a deSitter phase while it is dominated by potential energy. If we assume that at some temperature $T < T_c$ a smooth spatially homogeneous region of the Universe starts to evolve to the zero temperature minimum, the equation of motion for the ϕ field is

$$\ddot{\phi} + 3H\dot{\phi} + \partial V/\partial\phi + \Gamma_\phi\dot{\phi} = 0 \tag{5.7}$$

where Γ_ϕ is the ϕ decay width. Equation (5.7) is only for the zero momentum component of the Higgs field. The $H\dot{\phi}$ part of the equation of motion is present because the expansion of the Universe redshifts away any ϕ momentum. The expansion rate, H, is determined by the total energy density ρ

$$\rho = \dot{\phi}^2/2 + V(\phi) + \rho_R \tag{5.8}$$

with

$$\dot{\rho}_R = -4H\rho_R + \Gamma_\phi\dot{\phi}^2. \tag{5.9}$$

The first key to new inflation is a flat region of the potential that will result in a slow evolution of the Higgs field. During the slow evolution of the Higgs field $\rho \approx V(\phi)$ and the scale factor expands exponentially $R \propto \exp(Ht)$, with $H = (V(\phi)/m_{pl}^2)^{1/2}$. If the slow evolution phase lasts say $100H^{-1}$, then $R/R_o \sim \exp(100)$ where R_o was the scale factor at the start of inflation. The second key to new inflation is a steep region around the minimum of the potential. The steep region results in a large $\Gamma_\phi \propto m_\phi \propto (d^2V/d\phi^2)^{1/2}$. If reheating is good, the inflation region can be reheated to a temperature comparable to the temperature at the start of inflation $T \simeq T_o$. Therefore the entropy in the region has increased by a factor of [assuming $R/R_o \sim \exp(100)$]

$$\Delta = S/S_o = R^3T^3/R_o^3T_o^3 \sim \exp(300). \qquad (5.10)$$

The homogeneity/isotropy of the Universe is guaranteed since the observed entropy in the Universe, $S \sim 10^{88}$, can easily be produced in a smooth way in a single inflation region. The flatness problem is solved, since after inflation H is the same, so $\Omega - 1 \propto H^{-2}R^{-2}$ has decreased by exp(-200). Although inflation smooths the Universe, quantum mechanical fluctuations produce density perturbations. The magnitude of the perturbations are model dependent, but in all models they are "scale free," i.e. a Harrison-Zel'dovich spectrum.

At present inflation is extremely compelling, and although a truly attractive particle physics model with successful inflation has not been constructed, there are some models that do work.[5.3]

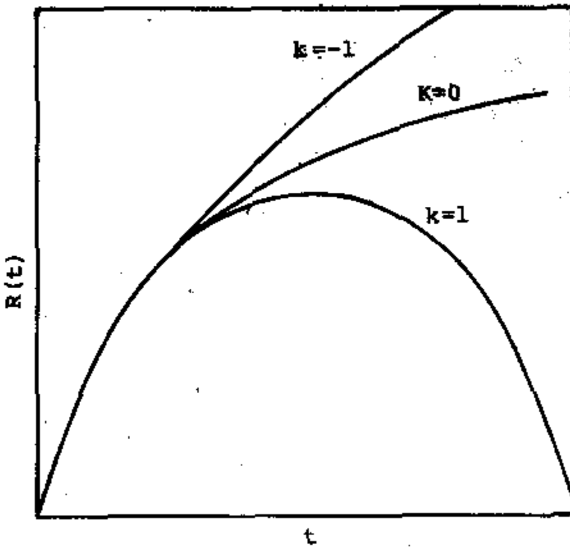


Figure 1.1

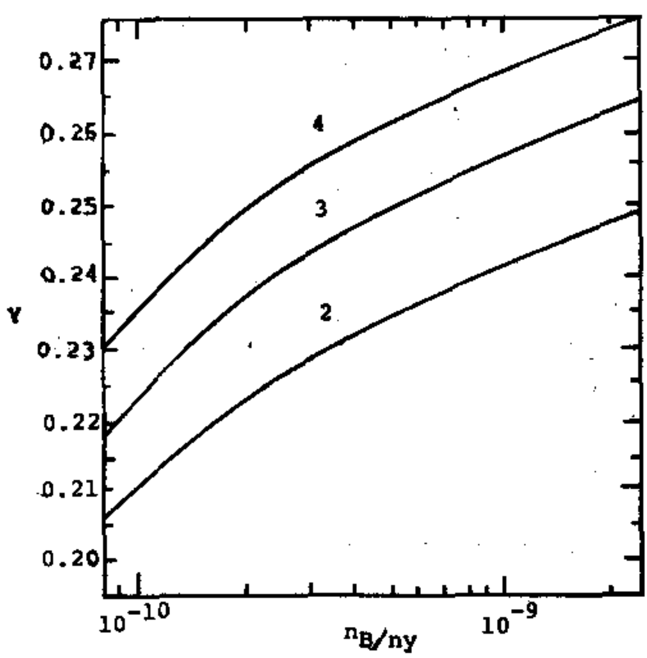
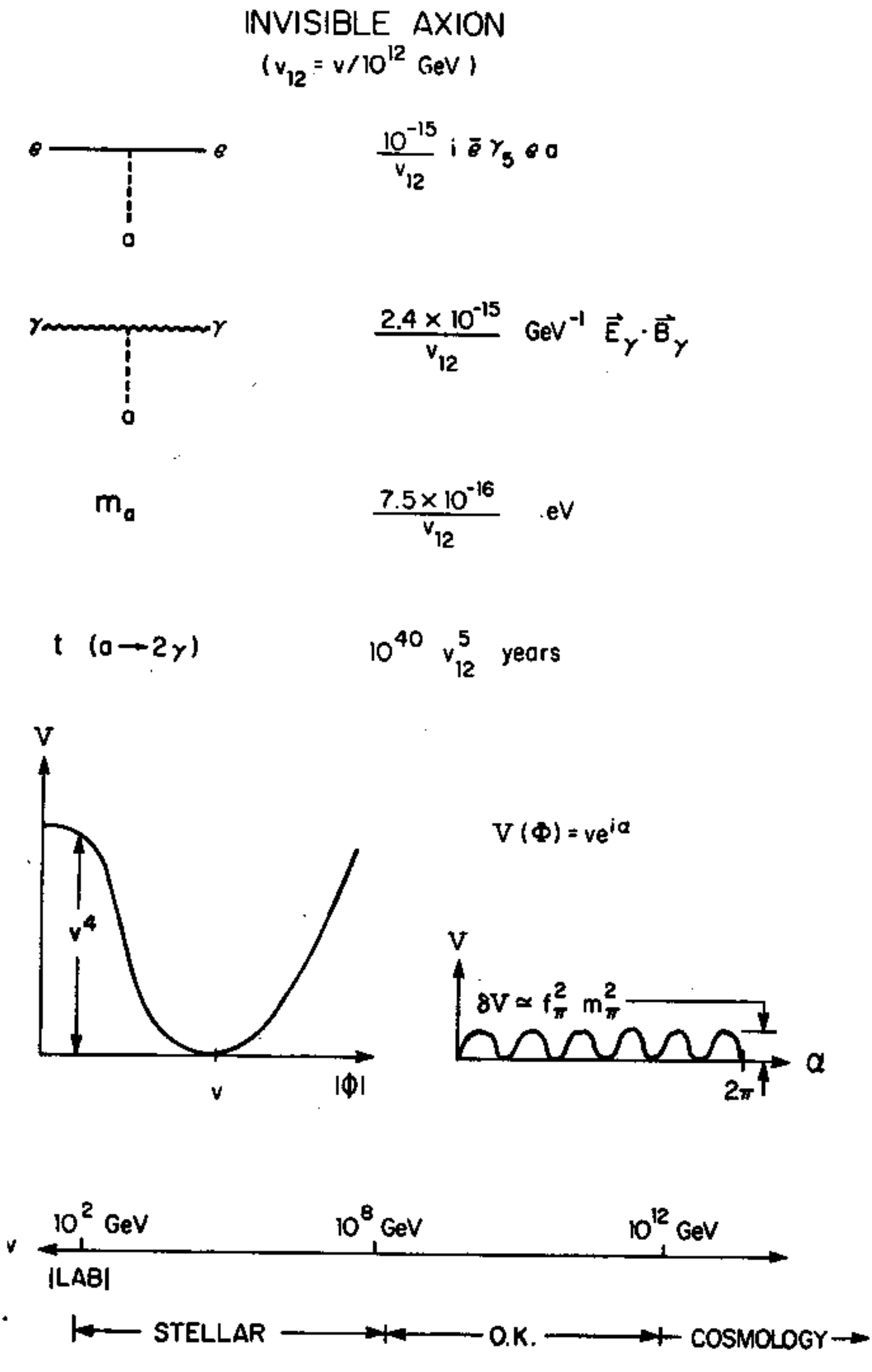


Figure 2.1


INVISIBLE AXION
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m_a
7.5 × 10^-16 / v12 eV
t (a → 2γ)
10^40 v12^5 years
V
v^4
v
|Φ|
V (Φ) = ve^iα
V
δV ≃ f_π^2 m_π^2
α
2π
10^2 GeV
10^8 GeV
10^12 GeV
v
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COSMOLOGY

Figure 4.1

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