NEUTRINOS IN THE UNIVERSE

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SUMMARY

We discuss the two very important neutrino problems in astrophysics today: (1) Neutrinos and binding of galaxies, clusters of galaxies, and the closure of the Universe; and (2) the solar neutrino problem. The evidence that the dominant mass-energy density (DMED) in the universe is neither stellar nor interstellar gaseous matter, and the identification of DMED with massive or massless neutrinos, are discussed. It is difficult to directly detect the large predicted fluxes of low energy cosmic neutrinos, and p-p solar neutrinos, due to their small cross sections. It was shown some time ago, however, that low energy neutrinos can be detected by coherent scattering (Opher, 1974) and a recent reanalysis (Lewis, 1980) confirms this conclusion. Superconducting electrons has been suggested for the coherent detector (Opher, 1982) and the sensitivity of this detector to predicted fluxes of cosmic neutrinos and p-p solar neutrinos is evaluated. For the abundant p-p solar neutrino flux, such a detector has the advantages compared to nuclear reaction detectors of: (1) directionality (important for background problems, nuclear reaction detectors (except perhaps ^{155}In) are isotropic), and (2) sensitivity (a relatively small detector (volume < 10 cm^{-3}) can give a detectable signal in a day). For the cosmic neutrinos, the coherent detector is the only means present available for their direct detection.

I - EVIDENCE THAT THE DOMINANT MASS-ENERGY DENSITY (DMED) IN THE UNIVERSE IS NEITHER STELLAR NOR INTERSTELLAR GASEOUS MATTER

We are interested in determining

$$\Delta M \equiv M - M_{SI} \qquad (1)$$

where M is the mass of a galaxy or cluster of galaxies, and M_{SI} is the mass contained in stellar or interstellar gaseous matter. Most of the stellar mass resides in intrinsically faint stars which can be determined only in the immediate vicinity of the sun. The stellar mass density near the sun lies in the range (Faber and Galagher, 1979) $0.05 < \rho_S < 0.09\ M_\odot\ pc^{-3}$ (where $M_\odot$ is the mass of the sun and 1 pc = 3.26 light years). The mean interstellar gaseous matter, primarily hydrogen, is $\rho_I \approx 0.03\ M_\odot\ pc^{-3}$ (Savage et al., 1977). The mass-to-light ratio (in units of $M_\odot/L_\odot$, where $L_\odot$ is the luminosity of the sun) in the solar neighborhood is (Faber and Gallagher, 1979)

$$\frac{M_{SI}}{L_B} \approx 2.3 - 3.3 \qquad (2)$$

(The subscript B signifies the use of a "B" filter).

The mass of our galaxy, the Milky Way, is determined from its rotation curve. Hartwick and Sargent (1978) found from a study of the velocities of globular clusters of stars and nearby dwarf galaxies that within an effective radius of 60 kpc from the center of our galaxy there is a mass of $8 \times 10^{11}\ M_\odot$. Sandage and Tammann's (1976) value of the luminosity of our galaxy is $L_B \approx 2.0 \times 10^{10}\ L_\odot$.

We obtain from the above

$$M_{Our\ Galaxy} >> M_{SI}$$

The most striking feature discovered in recent studies of the rotation curves of spiral galaxies, using the 21 cm line of neutral atomic hydrogen, is that the curves are flat at large radii. Since $GM/R \propto v^2$ for a constant rotational velocity we have $M \propto R$. The surface brightness of spiral galaxies, however, declines exponentially (Freeman, 1970; Schweizer, 1976). Large mass-to-light ratios are thus predicted. Roberts and Whitehurst (1975) obtain

$M/L_B \sim 100 - 300$. From the above

$$M_{\text{Spiral Galaxies}} >> M_{SI}$$

Faber and Gallagher (1979) summarize the data on spiral galaxies: "It seems most unlikely that flat rotation curves are an artifact of observational errors. No generally valid alternative explanation has been put forward for these flat rotation curves".

The lack of visible mass in clusters of galaxies has been known for many years. Zwicky (1933) and Smith (1936), in studying the Virgo cluster, pointed out the problem of the large discrepancy between M/L_B and M_{SI}/L_B. In 1937, Zwicky (1937) derived a value of $M/L_B \approx 500$ for the Coma cluster and $M_{SI}/L_B \sim 3$ for the solar neighborhood.

Although there is an apparent large discrepancy between M/L_B and M_{SI}/L_B in a large cluster, the evaluation of M/L_B has a number of problems connected with it. In the use of the virial theorem we assume a steady-state, which is questionable, when the crossing time is comparable to the Hubble time. The uncertainty of the method used for obtaining a properly averaged velocity squared, $\overline{v^2}$, also introduces an error. If $\overline{\sigma^2}$ is the radial velocity dispersion, we can have $\overline{\sigma^2} < \overline{v^2} < 3\overline{\sigma^2}$, depending upon the types of orbits that exist. Rood(1970) for example, suggests for the Coma cluster $\overline{v^2} \approx 2.1\ \overline{\sigma^2}$.

Knowing $\overline{v^2}$, we also need to know $\bar{R}$ in order to evaluate M from the virial theorem. Schwarzschild (1954) suggested using the formula

$$\bar{R} = 2\ \frac{\left[\int S(q)\,dq\right]^2}{\int S^2(q)\,dq} \tag{3}$$

where S(q) is the number of galaxies per unit area along a strip at a distance q from the center of the cluster. The problem with the above formula is that there is a large correction for background galaxies in the important outer region of the galaxy. For seven large clusters of galaxies, Faber and Gallagher (1979) obtain values for $M/L_V = 165\text{-}800$ with a median $M/L_V \approx 290$. (The subscript V signifies the use of a "V" filter) (for the Coma cluster, we have the ratio $L_V/L_B \approx 1.3$).

In order to avoid the problem of the outer region of the cluster of galaxies, another method to evaluate M/L is to concentrate just on the core of the cluster, using a mean core radius $\bar{R}_c$ and a mean radial velocity dispersion $\overline{\sigma_c}$. Some of the values of M/L_V for the cores of clusters that have been obtained are: $M/L_V \approx 280$ for Coma (Rood et al., 1972), $M/L_V \approx 350$ for Perseus (Bahcall, 1974), and $M/L_V \approx 250$ for Virgo (van den Bergh, 1977).

We thus have

$$M_{\text{Clusters of Galaxies}} >> M_{SI}$$

In their review article on masses and mass-to-light ratios of galaxies, Faber and Gallagher (1979) conclude: "The case for invisible mass in the universe is very strong and getting stronger".

II - IDENTIFICATION OF DMED WITH MASSIVE NEUTRINOS

There is apparently no "basic" elementary particle physics reason why the spin - 1/2 neutrino must have zero rest mass, in contrast to the case of photons (Taylor, 1976).

Nakagawa et al. (1963) were probably the first to discuss the possibility that a neutrino might have a mass. Pontecorvo (1968) discussed the possibility of massive neutrinos by drawing an analogy with neutral kaons. De Rejula and Glashow (1980) argued that neutrinos of mass $\sim$ 10 - 100 eV/c^2 would offer simultaneous explanations for the magnitude of the missing mass and its location in galactic halos. Neutrinos should be as copius as the photons of the 2.7 K background which outnumber baryons by a factor $\sim 10^8$ but contribute to the universal energy density only 10^{-4} that of baryons. Neutrinos would contribute a comparable energy density as the baryons if they have a mass of a few eV/c^2. Such a mass gives them a velocity on the order of the escape velocity of the galaxy if their kinetic temperature ($\equiv E_{k\nu}/k_B$, where $E_{k\nu}$ is the kinetic energy of the neutrinos) is a few degrees K. Their number density is limited by Fermi statistics. In order to supply the observed halo mass density they should have a mass of at least $\sim 24\ eV/c^2$.

It is interesting to note that Shapiro et al. (1980) recently showed that massive cosmic neutrinos of mass $\sim 10\ eV/c^2$ do not alter previous predictions of the standard Big Bang model for the primordial ^{4}He abundance.

The shape of the spectrum near the end-point in β decay is sensitive to the mass of the neutrino. In order to study the mass of the neutrino, we wish a β spectrum with the lowest possible energy end-point, which is that of tritium. Until 1979, the best limit of the neutrino mass was $< 55\ eV/c^2$ (at the 90% confidence limit) obtained by Bergkvist (1972).

Similar to the Cabibbo mixing of quarks, the operators ν_e and ν_μ in the lepton currents of the Weinberg-Salam theory of weak interactions has been suggested to be orthogonal superpositions of ν_1 and ν_2 neutrino fields with nonzero masses. This leads to oscillations $\nu_e \rightleftarrows \nu_\mu$ provided we exclude the trivial case $\Delta \equiv |m^2_{\nu_1} - m^2_{\nu_2}| = 0$ and a zero mixing angle (Bilenkii and Pontecorvo, 1977).

Reines, Sobel and Pasierb (1980) presented evidence for neutrino instability by comparing the neutral current (nc) reaction $\overline{\nu} + d \rightarrow n + p + \overline{\nu}$ which allows all neutrino types, with the charged current (cc) reaction $\overline{\nu_e} + d \rightarrow 2n + e^+$ which allows only the $\bar{\nu}_e$ type. They used the 2000 MW Savannah reactor and placed the detector 11.2 meters from the core. If the $\bar{\nu}_e$ changed its type on the way to the detector, the counting rate for the two neutron reaction events would be reduced. They measured the ratio

$$R = \frac{(\sigma_{cc}/\sigma_{nc})_{exp}}{(\sigma_{cc}/\sigma_{nc})_{theor}} \tag{4}$$

If no oscillations occurred we would have R = 1. They found

$$R = 0.40 \pm 0.22 \tag{5}$$

The implied value for the mass squared difference is

$$\Delta \approx 1\ eV^2/c^4 \tag{6}$$

Over the years, other reactor experiments had been performed that can, in principle, give evidence of neutrino oscillations. Analyzing four reactor experiments that had been done in the past, Silverman and Soni (1981) find that the experiments are consistent with neutrino oscillations.

The experiments are difficult, however, and physicists (including Richard Feynman) have doubt whether existing data show clear evidence of neutrino oscillations.

De Rujula and Glashow (1980) pointed out that massive neutrinos, ν_H, could, in principle, radiatively decay into a lighter neutrino, ν_L

$$\nu_H \rightarrow \nu_L + \gamma \tag{7}$$

Shipman and Cowsik (1981) argue that existing data of clusters of galaxies place lower limits of a few times $10^{23} - 10^{25}$ seconds on the neutrinos' radiative lifetimes depending on the energy of the emitted photon ($E_\nu > 1$ eV).

It is to be noted that a massive neutrino, like the electron, may be stable, not radiate, and not oscillate into other neutrinos. We would like experiments that are not based on instability and measure directly the neutrino mass.

Joshi and Chitre (1981) used existing data on the age of the universe to set limits on the mass of the neutrino. Symbalisty et al. (1980), using independent methods for calculating the age of the universe T_U, obtained the limit:

$$1.38 \times 10^{10} \text{ yrs} \leq T_U \leq 2.4 \times 10^{10} \text{ yrs}$$

The value of T_U in Friedmann models of the universe is $T_U = H_o^{-1} f(q_o)$, where $f(q_o)$ is a monotonically decreasing function of q_o (Weinberg, 1972). Using Einstein's equations we have $\rho_o = 3 H_o^2 q_o / 4\pi G$, or

$$T_U = \left[\frac{3}{4\pi G \rho_o}\right]^{1/2} F(q_o) \qquad (8)$$

or

$$\rho_o \propto (F/T_U)^2 \qquad (9)$$

where $F(q_o) \equiv q_o^{1/2} f(q_o)$ is monotonically increasing function of q_o. Assuming $T_U \simeq 1.4 \times 10^{10}$ yrs, and a number density $n_\nu \simeq 330 \text{ cm}^{-3}$ of neutrinos of mass m_ν, we obtain (Joshri and Chitre, 1981)

$$m_\nu \simeq 6.9 \text{ eV/c}^2 \qquad q_o = 1/2 \qquad (10)$$

The tritium β decay experiment of Bergkvest (1972) has recently been improved by Lyubimov et al. (1980) to obtain a value for the neutrino mass

$$14 \text{ eV/c}^2 \leq m_\nu \leq 46 \text{ eV/c}^2 \qquad (11)$$

at the 99% confidence level. The above is probably the best existing evidence today of a nonzero neutrino mass.

The capture of orbital electrons

$$Z \rightarrow (Z - 1) + \gamma + \nu_e \qquad (12)$$

gives promise of being a means in the future of obtaining a value for the neutrino mass. Similar to β decay, the mass of the neutrino can be obtained from the shape of the photon spectrum at the end-point. De Rujula (1981) pointed out that the probability of radiative L or M electron capture is very much higher then that of K capture and we can greatly improve the end-point statistics if we use a nucleus where K capture is supressed.

Probably the best candidate is

$$^{163}Ho_{67} \rightarrow {}^{163}Dy_{66} + \Delta E \qquad (13)$$

where $\Delta E \sim 2 - 10$ keV. CERN and other laboratories are now involved in trying to perform the above experiment.

If massive neutrinos are identified with DMED, we can examine some of the implications. Landau and Lifschitz (1969) calculated the relation between m_ν and the mass, M, and radius, R, of a selfgravitating spherical Fermi gas:

$$m_\nu = \left[\frac{91.9 \ \hbar^6}{16 \ G^3 R^3 M}\right]^{1/8} \qquad (14)$$

Using the core radius of the Coma cluster $R \sim 0.7$ Mpc and a core mass $M \sim 4 \times 10^{49}$ g, Cowsik and Mc Clelland (1973) obtain

$$m_\nu \sim 2\ eV/c^2 \qquad (15)$$

Neutrinos decouple from other leptons at a temperature $T \sim 1\ MeV/k_B$ when time scales of weak interactions become long compared with the expansion time of the universe. Because the neutrinos decouple while they are still relativistic ($m_\nu << kT/c^2$), they will remain roughly as abundant as photons. After decoupling the neutrinos are a collisionless gas with only gravitational interactions.

If the early universe was turbulent we are faced with the problem of the avoidance of the creation of a large amount of black holes. On the other hand, if the universe was homogeneous, which we like to assume, we have the problem of the creation of galaxies.

Galaxies are large inhomogeneities in the universe. The mean density within 10 kpc of the center of our galaxy is $\sim 10^{-24}$ g/cm^3 which can be compared with the mean density of the visible mass $\rho_{mo} \sim 3 \times 10^{-30}$ g/cm^3. The condensation process undoubtedly began in the early Big Bang and not only effected the present distribution of matter but also induced fluctuations of the relic microwave radiation. If $\delta\rho$ is the difference between the average density of the universe, ρ, and the average density in a galaxy, we obviously now have

$$\left(\frac{\delta\rho}{\rho}\right)_o >> 1 \qquad (16)$$

For a static universe, the collapse solution for $\delta\rho/\rho$ is (Opher, 1981b)

$$\frac{\delta\rho}{\rho} \propto e^{\left[2\pi st \sqrt{\left(\frac{1}{\lambda_J}\right)^2 - \left(\frac{1}{\lambda}\right)^2}\right]} \propto e^{\left[2\pi \frac{s}{\lambda_J}\right]t} \qquad \lambda >> \lambda_J \qquad \text{(STATIC UNIVERSE)} \qquad (17)$$

where

$$\lambda_J \equiv s\left(\frac{\lambda}{G\rho_o}\right)^{1/2} \qquad (18)$$

is the "Jean's length, s is the speed of sound, and λ is a characteristic dimension of the inhomogeneity, $\delta\rho$. In an expanding universe, the exponential increase with time of $\delta\rho/\rho$ changes to the dependence (for $q = 1/2$)

$$\frac{\delta\rho}{\rho} \propto t^{2/3} \propto (1+z)^{-1} \qquad q = 1/2 \qquad \text{(EXPANDING UNIVERSE)} \qquad (19)$$

where z is the cosmological red shift.

Until the recombination era ($z \sim 10^3$, $T \sim 0.3\ eV/k_B$), the Jeans length is approximately equal to the particle horizon (Opher, 1981b). Since the dimension of a galaxy is smaller than the particle horizon, without the aid of cosmic neutrinos, galaxies could only begin to be formed after the recombination era with a growth rate $\delta\rho/\rho \propto (1+z)^{-1}$. If the z when galaxies formed $\left[\frac{\delta\rho}{\rho}\right] \sim 1$ is z_{gal}, we have for the minimum value of $\frac{\delta\rho}{\rho}$ at $z = z_{rec}$

$$\left(\frac{\delta\rho}{\rho}\right)_{min} \simeq \frac{z_{gal}}{z_{rec}} \simeq z_{gal} \times 10^{-3} \qquad (20)$$

The above relation implies that we must have primordial density fluctuations

$$\left(\frac{\delta\rho}{\rho}\right)_{Primordial} > 0.1\% \qquad (21)$$

Whereas matter could only begin to collapse into galaxies after decoupling from radiation in the recombination era ($T \sim 0.3\ eV/k_B$), we have that neutrinos could begin to collapse after they decouple, which is much earlier ($T \sim 1\ MeV/k_B$). Matter is thus able to fall into the enhanced density fluctuations already created by the neutrinos.

III - IDENTIFICATION OF DMED WITH MASSLESS NEUTRINOS

Weinberg (1962) argued some time ago that in an oscillating universe the neutrino density is determined by the requirement that there is a perfect balance between emission and absorption per cycle. In an oscillating cosmo ogy, Weinberg (1962) proved

$$n_\nu = 1/2 \qquad (22)$$

where n_ν is the occupation number of neutrinos plus antineutrinos. For example, the neutrino levels may be 1/2 full and the antineutrinos 1/2 full, or the neutrino levels empty and the antineutrino levels full.

Let us assume that the oscillating universe does not contract into a singularity but contracts only to a minimum scale factor R_m, where R_o is the present scale factor. Weinberg (1962) showed that the resulting Fermi energy is

$$E_F \simeq \left(\frac{R_m}{R_o}\right) 5 \times 10^6 \text{ eV} \qquad (23)$$

The corresponding E_F for a neutrino energy density sufficient to close the universe is

$$E_F \simeq 0.02 \text{ eV} \qquad (24)$$

We thus have

$$\frac{R_m}{R_o} \simeq 4 \times 10^{-9} \qquad (25)$$

IV - THE SOLAR NEUTRINO PROBLEM

Various non-standard solar models have been suggested over the years to explain the low experimentally observed solar neutrino emission. Standard solar models predict neutrino emission > 4 SNU (1 SNU ≡ 10^{-36} solar neutrinos captured per target atom (^{37}CI) per second) but the observed experimental upper limit at the 90% confidence limit is 1.5 SNU (Davis,1972; Bahcall and Sears, 1972; Davis and Evans, 1973; Bahcall and Davis, 1976).

The solar neutrino problem is a serious one and many non-standard solar models have been suggested to explain the problem such as: (1) there is a black hole in the sun (Clayton et al. 1975); (2) the sun is not in a steady state (Dilke and Gough, 1972; Fowler, 1972) and (3) the sun is inhomogeneous (Prentice, 1973; Faulkner et al., 1975; Bhavser and Harm, 1977).

A recent explanation of the solar neutrino problem is based on the properties of the plasmon-electron interaction (Opher, 1981a). Due to this interaction, the high energy electron distribution is distorted which decreases the effective radiation opacity and the high energy neutrino flux (E > 0.81 MeV) to which the ^{37}Cl experiment is sensitive (Bahcall and Davis, 1976). The theory predicts the normal low energy abundant p-p neutrino flux (E_ν < 0.42 MeV).

A standard solar model predicts a p-p flux at the earth's surface (Bachall et al., 1973; Bahcall, 1978)

$$F_{\nu S} = 6.1 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1} \qquad (26)$$

A number of nuclear reactions have been suggested for detecting the above abundant flux. The primary problem of the detection of this flux by incoherent nuclear reaction detectors is the large masses of relatively exotic materials required to obtain a significant signal. For example, the inverse β decay of ^{115}In to the 614 keV excited state of ^{115}Sn has been suggested (Raghavan, 1976). In order to achieve an event rate of 1/day requires 3 tons of material. The detector has the capability of directionality if the direction of the recoil electron is detected which, however, is not easy. The capture of p-p neutrinos by ^{71}Ga to produce ^{71}Ge has also been suggested. A feasible detector requires 20 tons of this material (Bahcall and Davis, 1976).

V - A COHERENT NEUTRINO DETECTOR USING SUPERCONDUCTING ELECTRONS

The index of refraction for neutrinos is (Goldberger and Watson, 1964)

$$n = 1 - 2\pi \lambda^2 A \tag{27}$$

where A is the total real part of the forward scattering amplitude per cm^3. In 1974 the neutrino scattering amplitude was not well understood and the critical angle of scattering, for example,

$$\Theta_c = 2\left[|n - 1|\right]^{1/2} \tag{28}$$

was given by Opher (1974) as

$$\Theta_c = 2^{1/4} \left[\frac{NG}{E_\nu}\right]^{1/2} |Z\, g_e + Z\, g_p + (A - Z)\, g_n|^{1/2}\, |\cos\phi| \tag{29}$$

where g_e, g_p and g_n are the neutrino coupling constants to the electrons, protons, and neutrons, respectively (all of order unity). N is the density of atoms, G (= 1.410×10^{-49} erg - cm^3) is the weak coupling constant, E_ν is the neutrino energy, ϕ is the phase angle of scattering, and A-Z (A ≡ atomic mass, Z ≡ atomic number) is the number of neutrons. In the evaluation of (3), because the values of ϕ, g_e, g_p and g_n were not well defined, Opher (1974) used $|\cos\phi| \approx 1$ and $|Z\, g_e + Zg_p + (A - Z)\, g_n| \approx A + Z$. Using the present knowledge of neutrino scattering, Lewis (1980) finds for ν_e neutrino scattering, for example, $n > 1$ and

$$\Theta_c = 2^{1/4} \left[\frac{NG}{E_\nu}\right]^{1/2} \left[2Z - (A - Z)\right]^{1/2} \tag{30}$$

(when necessary to be specific, massless ν_e neutrinos will be assumed in this section). Comparing (29) and (30) we note that the two expressions are identical if we take in (29) $|\cos\ \phi| = 1$, $g_e = 1$, $g_p = 1$ and $g_n = -1$.

Lewis (1980) showed that coherent scattering detectors based on refraction or critical scattering have comparable detection efficiencies. Due to its inherent simplicity, the coherent neutrino detector discussed in the present section is based on refraction.

The principle of the detector is shown in Figure 1. We have three types of layers,

$$n_P \approx 1 \tag{31}$$

and layers (S) and (C) have $n_S \approx n_C$ with

$$|n_S - 1| \approx |n_C - 1| >> |n_P - 1| \tag{32}$$

The layer (S) is superconducting while the layer (C) is not.

From Snell's law, we have from Figure 1 and (32)

$$n_S \cos(\Theta + \delta\Theta) = \cos\Theta \tag{33}$$

Since $\delta\Theta$ is very small, and assuming $\delta\Theta << \Theta$, we have

$$\cos(\delta\Theta) \approx 1$$

$$\sin(\delta\Theta) \sim \delta\Theta \tag{34}$$

$$\delta\Theta << \Theta$$

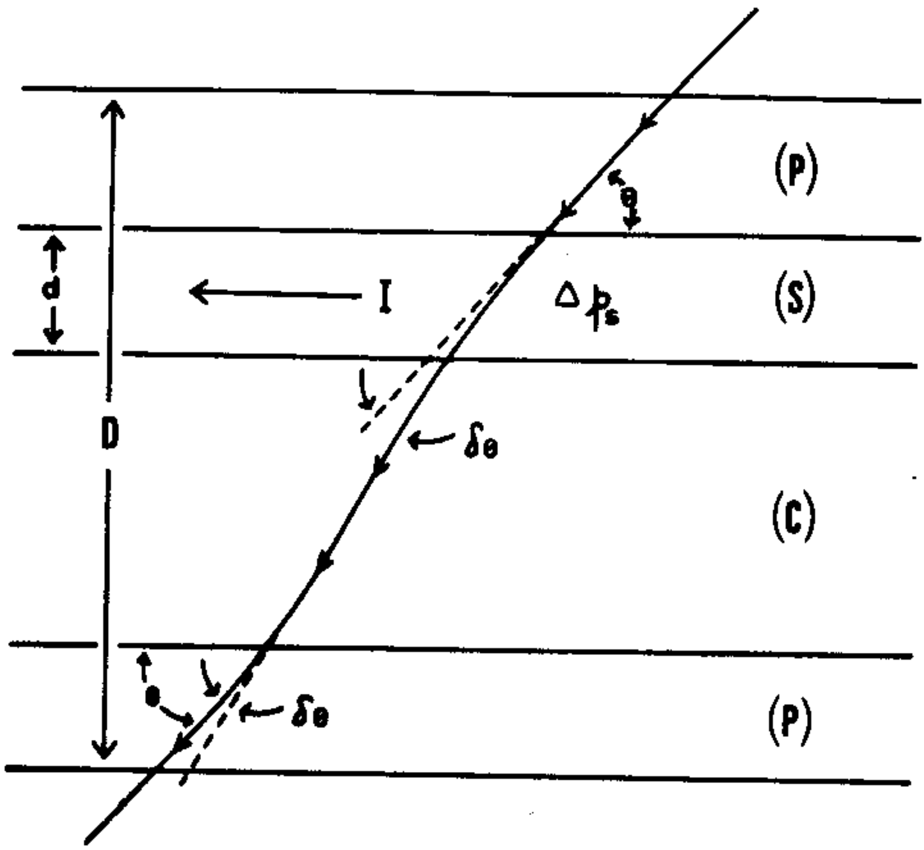

FIGURE 1 - Principle of neutrino detector. Neutrinos enter the detector at an angle θ with respect to the (P) layer which has $n_P = 1$. They are refracted at superconducting layer (S) by angle δθ. They continue quase unperturbed into the nonsuperconducting layer (C) which has $n_C \simeq n_S$ and $|n_C - 1| \simeq |n_S - 1| >> |n_P - 1|$. They are then refracted again at the (C) - (P) interface by the angle δθ.

From (28), (33) and (34)

$$\delta\theta = \frac{\theta_c^2}{2} \text{ ctn } \theta \qquad (35)$$

The deflection (35) is due to the coherent scattering from the particles in layer (S). For example, the θ_c in (36) is given by (30) for massless ν_e neutrinos, where N, Z and (A-Z) are for the (S) layer, and in (28) we have $n_S > 1$.

In Figure 2 we have a single section of the neutrino detector, producing an energy output δU_S, due to the refraction of the neutrino flux from the layer (S). The area of each (S) layer is LW and is at an angle θ with respect to the incoming neutrino flux. There are

$$\beta = \frac{H}{D} \qquad (36)$$

(S) layers.

From Figures (1) and (2), the momentum transfer Δp in a time τ to a single (S) layer due to a neutrino flux F_ν, is

$$\Delta p = (F_\nu\ \tau)(LW \sin\theta)\ (\frac{E_\nu}{c}\ \delta\ \theta \sin\theta) \qquad (37)$$

For ν_e neutrinos, for example, we have from (30), (35) and (37)

$$\Delta p = (\frac{NG}{c})\ (\frac{2Z - (A - Z)}{2\sqrt{2}}) \sin(2\theta)\ (F_\nu\tau)(LW) \qquad (38)$$

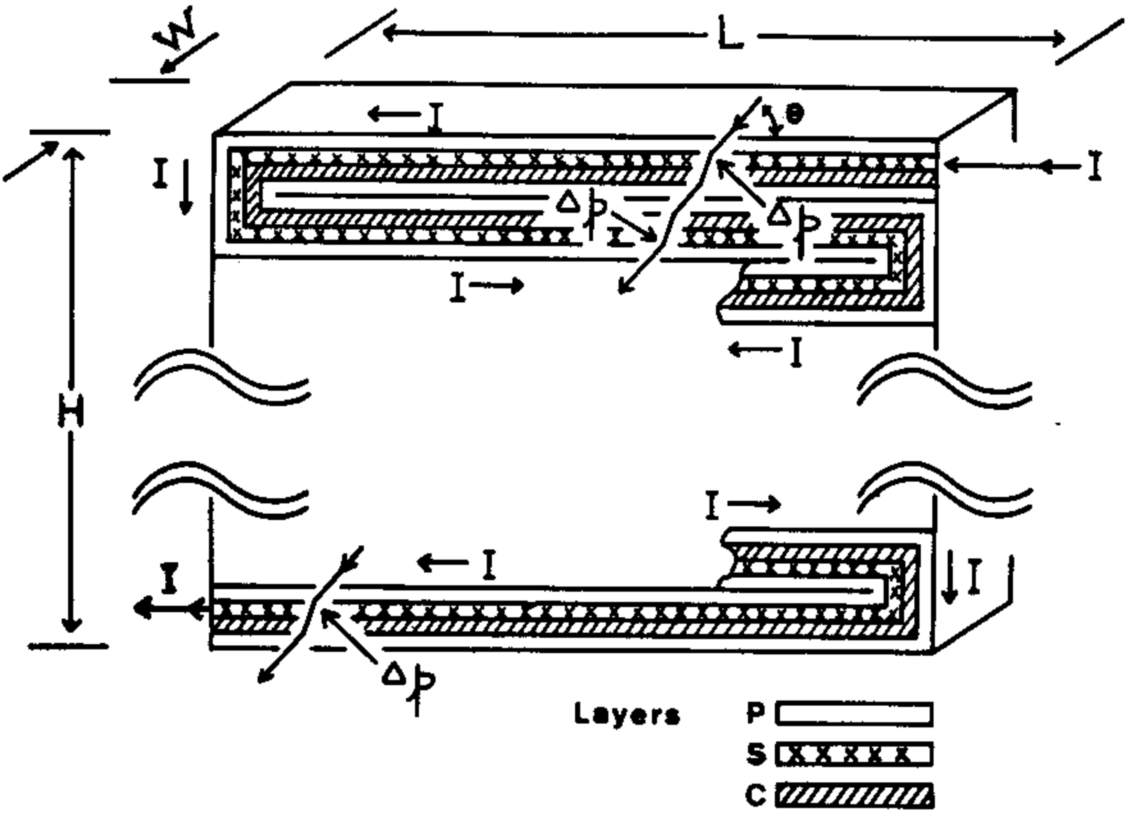

FIGURE 2 - Section of neutrino detector. It has length L, width W, and height H. The momentum transfer to the (S) layer is always in the direction of the current I.

Relation (38) describes coherent scattering from all of the scatterers in the (S) layer (electrons, protons and neutrons). From our previous discussion, we have in (29) $g_e = 1, g_p = 1$ and $g_n = -1$. The fraction Γ of the momentum transfer Δp in (38) which is transferred to the superconducting electrons is

$$\Gamma = \frac{\alpha_S\ Z\ g_e}{Z\ g_e + Z\ g_p + (A - Z)\ g_n} \qquad (39)$$

where α_S is the fraction of all of the electrons that are superconducting. We thus have for the momentum transfer to the superconducting electrons, Δp_S, from (38) and (39)

$$\Delta p_S = \Gamma\ \Delta p = \left[\frac{\alpha_S\ n_e\ G}{2\ \sqrt{2}\ \ c}\right]\ \sin\ (2\theta)\ \ (F_\nu \tau)(LW) \qquad (40)$$

where n_e is the total electron density.

Due to the current I in Figure (2), the superconducting electrons have an average velocity

$$v_S = \frac{I}{e(\alpha_S\ n_e)\ Wd} \qquad (41)$$

It is noted from Figure (2), that the momentum transfer Δp to the (S) layer is always in the direction of v_S.

The energy of the neutrinos scattering from the moving electrons will be Doppler shifted.Since the momentum of a massless neutrino for example, is E_ν/c,we have for the β layers in the section of Figure (2).

$$\delta U_S = \beta\ \Delta p_S\ v_S \quad (42)$$

where δU_S is the energy transfer due to Doppler scattering. Substituting (36), (40) and (41) into (42) we obtain

$$\delta U_S = \frac{G}{2\sqrt{2}\ c} \sin\ (2\Theta)(F_\nu \tau)\ (\frac{LH}{dD})(\frac{I}{e}) \quad (43)$$

From its derivation, δU_S is the energy of the signal produced by a single section of the neutrino detector shown in Figure (2). In a volume V there are N = V/(LWH) sections connected in series. The total energy of the signal produced by the neutrino detector $U_S = N\ \delta U_S$ is, from (43).

$$U_S = \frac{G}{2\sqrt{2}\ c}\ \sin(2\Theta)\ (F_\nu \tau)\ (\frac{V}{dDW})\ (\frac{I}{e}) \quad (44)$$

The current I in (44) need not be constant. For example, if the detector is connected to a resonant circuit of frequency f, we have

$$I = I'\ \sin\ (2\pi f t) \quad (45)$$

where I' is time independent. If we rotate the neutrino detector section in Figure (2) at a frequency f/2, we have

$$\Theta = \pi\ f\ t \quad (46)$$

The time average of I sin (2Θ) in (44) is then, from (45) and (46)

$$<I\ \sin\ (2\Theta)> = \frac{I'}{2} \quad (47)$$

and (44) becomes

$$<U_S> = \frac{G}{4\sqrt{2}\ c}\ (F_\nu \tau)\ (\frac{V}{dDW})\ (\frac{I'}{e}) \quad (48)$$

Although the angle Θ does not appear explicitely in (48), the neutrino detector is directional. A maximum $<U_S>$ will occur when I in (44) is maximum which occurs when sin (2Θ) is maximum and $\Theta = \pi/4$. Similarly, an isotropic source with respect to the neutrino detector has $<U_S> = 0$.

As seen in Figure (2), a superconducting layer (S) containing a current I is immediately followed by an (S) layer with a current in the opposite direction. The neighboring sections to Figure (2) (not shown) have the current I going in the opposite direction. In general, the neutrino detector is constructed such that it has an approximate zero net magnetic field and a minimum magnetic inductance.

Let us connect the neutrino detector to a loop which has an inductance much larger than that of the neutrino detector. Let the cross sectional area be ℓ^2 and the length 10 ℓ. We then have

$$\Delta H_\ell = \frac{<U_S>}{\ell^3\ H_\ell}\ \frac{4\pi}{10}\ \text{gauss} \quad (49)$$

$$H_\ell = \frac{I'}{\ell}\ \frac{4\pi}{100}\ \text{gauss} \quad (50)$$

where ΔH_ℓ is the change in the magnetic field, H_ℓ, of the loop due to the change in energy $<U_S>$ and I' is the current in (48). The units of $<U_S>$, I' and ℓ in (49) and (50) are ergs, amperes and centimeters, respectively.

Combining (48)-(50), we obtain for the change in the magnetic field ΔH_ℓ (in gauss) and the signal temperature T_S ($\equiv <U_S>/k_B$) (in degrees K)

$$\Delta H_\ell = 5.2 \times 10^{-40}\ (\frac{F_\nu \tau}{\ell^2})\ (\frac{V}{dDW})\ \text{gauss} \quad (51)$$

$$T_S \simeq 5.8 \times 10^{15} \, (\Delta H_\ell)^2 \, \ell^3 \, K \quad (52)$$

where

$$\gamma \equiv \frac{H_\ell}{\Delta H_\ell} \quad (53)$$

and F_ν, τ, and ℓ in (51) and (52) are in units of $cm^{-2}\,s^{-1}$, s, and cm, respectively.

In order for T_S in (52) to be a detectable signal, we require

$$\frac{T_S}{T_N} \gtrsim 1 \quad (54)$$

where T_N is the noise temperature ($\equiv k_B^{-1}$ x noise energy) of the measuring circuit. In the above description of the neutrino detector, the ΔH_ℓ in (52) is measured by a sensitive magnetometer which is intimately coupled to the loop of the circuit.

The most sensitive magnetometers today are SQUIDS (Superconducting Quantum Interference Devices) which have already attained values for T_N

$$T_N \simeq \frac{hf}{2k_B} \quad (55)$$

for f in the tens of kHz range (Mc Donald, 1981). For $f \sim 10^4$ we obtain from (55)

$$T_N \simeq 2.4 \times 10^{-7} \, K \quad (56)$$

It is noted that the temperature in (56) is seven orders of magnitude less than the ambient thermal temperature in which the superconductors operate.

The minimum value for T_N, given by (55), has been shown to be generally valid (Heffner, 1962; Tesche and Clark, 1977). Still lower values of T_N in (56) should be attainable at lower frequencies. Of particular importance, SQUIDs have already shown the capability of measuring magnetic fields as small as (Mc Donald, 1981)

$$\Delta H_\ell \simeq 10^{-10} \quad \text{gauss} \quad (57)$$

The characteristic dimension ℓ_Q of SQUIDs is, in general, very small. They can be constructed today with values as small as

$$\ell_Q \sim 10^{-3} \quad (58)$$

We use, in the following, the criteria that a neutrino flux is detectable if it produces a ΔH_ℓ greater than ten times (57) in a loop with ℓ in (51) ten times (58).

Let us now examine some simples properties of superconductors to obtain approximate values of d in (51) and $\alpha_S \, n_e$ in (40) and (41).

As is well known, the BCS (Bardeen, Cooper and Schrieffer, 1957) theory predicts well the eletrodynamic properties of a superconductor. Many of the electrodynamic properties, however, can be described by simpler equations (Fairbank and Fetter, 1967). At zero temperature the supercurrent density $\vec{J}_S$ obeys the London equations (London and London, 1935a, 1935b; London, 1937)

$$\text{curl} \, \vec{J}_S = - \left(\frac{c}{4\pi \, x'^2}\right) \vec{H} \quad (59)$$

$$\vec{J}_S = n_S^* \, e^* \, \vec{v}_s \quad (60)$$

where n_S^* (e^*) is the effective number density (charge) of the superconducting electrons and x' is a characteristic length called the penetration depth. Combining (59) with Maxwell's equations in a semi-infinite medium (x > 0), the solution of (59) is

$$H = H' \exp(-x/x') \quad (61)$$

with

$$x' = \left[\frac{m^* c^2}{4\pi \; n_S^* \; e^{*2}}\right]^{1/2} \quad (62)$$

where m* is the effective mass of the superconducting electrons. From the BCS theory we have

$$e^* = 2 \; e \quad (63)$$

$$m^* = 2 \; m_e \quad (64)$$

$$n^* = \frac{1}{2} \; \alpha_S \; n_e \quad (65)$$

where $\alpha_S \; n_e$ is the term appearing in (40) and (41). Pure superconductors have

$$x' \sim 10^{-5} \quad (66)$$

From (62) - (66) we then have

$$\alpha_S \; n_e \sim \frac{1}{2} \times 10^{23} \; cm^{-3} \quad (67)$$

Since (66) describes the thickness within which the supercurrent exists, the minimum value for d in the evaluation of (51) is

$$d_{min} \simeq 10^{-5} \; cm \quad (68)$$

The value of x' can be much larger than (66) and d larger than (68). At zero degrees, the superconducting ground state consists of bound pairs of electrons having a binding energy $2\Delta(0)$. The bound pairs of electrons spread out over a coherence length

$$\xi_o = \frac{\hbar \; v_F}{\pi \; \Delta(0)} \quad (69)$$

where v_F is the Fermi electron velocity. In pure superconductors we have

$$\xi_o \sim 10^{-4} \; cm \quad (70)$$

Defining

$$\kappa \equiv \frac{x'}{\xi} \quad (71)$$

we have for a pure superconductor (called Type I) from (70), (71) and (66)

$$\kappa \ll 1 \qquad \text{(Type I)} \quad (72)$$

The coherence length ξ characterizes the range of non-local influence between two points and the penetration depth x' characterizes the distance within which superelectrons are unable to screen the electromagnetic field. In Type II superconductors we have

$$\kappa \gg 1 \qquad \text{(Type II)} \quad (73)$$

The origin of Type II superconductors is the reduced electronic mean free path in alloys and thin films: as the electron mean free path decreases, x' increases to greater values than (66) and ξ decreases to smaller values than (70). As metals become increasingly imperfect, the percentage of magnetic flux ejection declines, reaching zero, or

$$x' \to \infty \quad (74)$$

in extreme cases of inhomogeneous or cold worked alloys (Hulm et al., 1981). We thus have from (74)

$$d_{max} \to \infty \tag{75}$$

Using superconducting electrons, very large values of τ in (51) are possible. Persistent currents greater than a year have been observed in superconductors, without evidence of diminution. Values for τ greater than a year could thus, in principle, be used in (51). In order to emphasize the great sensitivity of the detectors discussed, we will use instead a value of τ equal to a single day.

VI - DETECTION OF MASSIVE AND MASSLESS COSMIC NEUTRINOS

Relation (51) was derived assuming a point source of neutrinos. Cosmic neutrinos are not a point source and we use instead of F_ν in (51) the effective value

$$\langle F_\nu \rangle \equiv \varepsilon_\nu (F_\nu)_{av} \tag{76}$$

$$\varepsilon_\nu \equiv \frac{(F_\nu)_{max} - (F_\nu)_{min}}{(F_\nu)_{max} + (F_\nu)_{min}} \tag{77}$$

where ε_ν describes the magnitude of the isotropy of the flux. The average of the neutrino flux over solid angle is $(F_\nu)_{av}$, and $(F_\nu)_{max}(F_\nu)_{min}$ is the maximum (minimum) flux.

Relation (51) can be shown to be approximately valid for all types of neutrinos (massive and massless) even though it was derived assuming only massless ν_e neutrinos. In the following discussion, due to the large uncertainties involved, it will be used unmodified.

VI.1 - Massive Cosmic Neutrinos

Let us assume that the local density of massive neutrinos, $\rho_{M\nu}$ is approximately equal to the observed local mass density, ρ_{ML} and their velocity, v_ν, is on the order of the escape velocity of the galaxy, v_{ES}. Bahcall and Soneira (1980) give for the local density in a standard two component model of the galaxy.

$$\rho_{ML} \approx 0.15\ M_\odot\ pc^{-3} \tag{78}$$

In the same model, the total mass in the disc part of the galaxy is $5.6 \times 10^{10}\ M_\odot$ and in the spherical part $3.3 \times 10^9\ M_\odot$. The escape velocity from the sun in the model is give as

$$v_{ES} = 2.60 \times 10^7\ cm/s \tag{79}$$

Let us now estimate (F_ν) and ε_ν in (76) for massive neutrinos. Let us assume a neutrino mass $m_\nu \sim 30\ eV/c^2$, a local neutrino mass density $\rho_{M\nu} \sim \rho_{ML}$ given by (78), and a neutrino velocity $v_\nu \sim v_{ES}$ given by (79). We thus have

$$m_\nu \sim 30\ eV/c^2 \tag{80}$$

$$\rho_{M\nu} \sim 0.15\ M_\odot\ pc^{-3} \tag{81}$$

$$v_\nu \sim 2.6 \times 10^7\ cm/s \tag{82}$$

Using the values of (80-82), we obtain for the average value of the massive neutrino flux

$$(F_{\nu M})_{av} \equiv \frac{v_\nu\ \rho_{M\nu}}{m_\nu} \approx 5 \times 10^{15}\ cm^{-2}\ s^{-1} \tag{83}$$

The velocity of the sun with respect to the galactic center is (Lang, 1980, p. 532)

$$v_\odot \approx 2.5 \times 10^7\ cm/s \tag{84}$$

From (77) and (82)-(84) we obtain

$$\varepsilon_{\nu M} \approx 1 \tag{85}$$

From (85) we thus expect the massive neutrino flux to be highly anisotropic: the maximum $(F_{\nu M})_{max}$ occurring in the direction of motion of the sun with respect to the galactic center and the minimum $(F_{\nu M})_{min}$ occurring in the opposite direction.

Substituting (83) and (85) into (77) we obtain

$$\langle F_{\nu M}\rangle \simeq 5 \times 10^{15} \text{ cm}^{-2} \text{ s}^{-1} \tag{86}$$

The wavelength of the massive neutrinos from (80) and (82) is

$$\lambda\!\!\!^{-}_{\nu}(M) = \frac{\hbar}{M_{\nu} v_{\nu}} \sim 5 \times 10^{-4} \text{ cm} \tag{87}$$

In order for the index of refraction analysis of the previous section to be valid,we require $\lambda\!\!\!^{-}_{\nu}$ to be less than all dimensions in Figs. (1) and (2)

$$\lambda\!\!\!^{-}_{\nu} < D, W, d \tag{88}$$

In order to satisfy (88) with the wavelength (87), we take $W \simeq 10^{-3}$ cm, $d \simeq 10^{-3}$, and $D \simeq 5 \times 10^{-3}$ cm. Let us evaluate the minimum valume, V, in (51) which produces a detectable signal ΔH_{ℓ} of magnitude ten times (57). Besides (86) we require knowing ℓ and τ. Values for these quantities are somewhat arbitrary. As discussed at the end of the previous section we use $\tau \sim 1$ day ($\sim 10^5$ s) although much larger values could be used. We also use ℓ equal to ten times (58). From the above we have

$$\begin{aligned} &\langle F_{\nu M}\rangle \simeq 5 \times 10^{15} \text{ cm}^{-2} \text{ s}^{-1}, && d \simeq 10^{-3} \text{ cm} \\ &D \simeq 5 \times 10^{-3} \text{ cm}, && W \simeq 10^{-3} \text{ cm} \\ &\ell \simeq 10^{-2} \text{ cm}, && \tau \simeq 10^{5} \text{ s}. \end{aligned} \tag{89}$$

Substituting (90) and ten times (57) into (51) we obtain for the volume of the massive cosmic neutrino detector

$$V(M) \simeq 2 \times 10^{-3} \text{ cm}^3 \tag{90}$$

VI.2 - Massless Cosmic Neutrinos

If massless neutrinos have an energy density $\rho_{E\nu}$ equal to the critical energy density ρ_{EC} required to close the universe, the average neutrino flux is

$$\langle F_{\nu 0}\rangle_{av} \simeq \frac{\rho_{EC} c}{\bar{E}_{\nu}} \tag{91}$$

where $\bar{E}_{\nu}$ is the average energy of the neutrinos. We use $\rho_{EC}(\text{ergs}) = 1.9 \times 10^{-29}\, h^2 \text{ g cm}^{-3}\, c^2$ (Lang, 1980, p. 522), where the Hubble constant is $H_o = 100$ h km s^{-1}. We take $h \simeq 0.5$.

In order for the neutrinos to have an energy density ρ_{EC} and not be degenerate, we require $\bar{E}_{\nu} \gtrsim 10^{-2}$ eV. Except for this requirement, essentially nothing is known about $\bar{E}_{\nu}$. We choose

$$\bar{E}_{\nu} \simeq 1 \text{ eV} \tag{92}$$

with the understanding that there is a great uncertainty involved in this choice. We obtain from (92) and (91)

$$\langle F_{\nu 0}\rangle_{av} \simeq 8 \times 10^{13} \text{ cm}^{-2} \text{ s}^{-1} \tag{93}$$

A reasonable assumption for the factor ϵ_{ν} in (76) for massless neutrinos is that it is comparable to the corresponding factor $\epsilon_{\gamma R}$ for the relic photon radiation:

$$\epsilon_{\nu 0} \simeq \epsilon_{\gamma R} \tag{94}$$

The factor $\varepsilon_{\gamma R}$ was recently measured (Muller, 1980)

$$\varepsilon_{\gamma R} \simeq 1.3 \times 10^{-3} \tag{95}$$

with the maximum flux in the direction $\alpha = 11.23 \pm 0.46$ hr, $\delta = 19 \pm 7.5^{\circ}$.

Substituting (93) - (95) into (76), we obtain

$$\langle F_{\nu 0} \rangle \simeq 1 \times 10^{11} \text{ cm}^{-2} \text{ s}^{-1} \tag{96}$$

The wavelength of a massless neutrino of energy (92) is

$$\lambda_{\nu 0} = \frac{\hbar c}{\bar{E}_\nu} = 2 \times 10^{-5} \text{ cm} \tag{97}$$

In order to satisfy (88) with the wavelength (97), we take $d \simeq 10^{-4}$ cm, $W \simeq 10^{-3}$ cm, and $D \simeq 10^{-3}$ cm. As in (89) we use $\ell \simeq 10^{-2}$ cm and $\tau \simeq 10^5$ s. From the above and (96)

$$\begin{aligned} &\langle F_{\nu 0} \rangle \simeq 1 \times 10^{11} \text{ cm}^{-2} \text{ s}^{-1}, && d \simeq 10^{4} \text{ cm} \\ &D \simeq 10^{-3} \text{ cm}, && W \simeq 10^{-3} \text{ cm} \\ &\ell \simeq 10^{-2} \text{ cm}, && \tau \simeq 10^{5} \text{ s} \end{aligned} \tag{98}$$

Substituting (98) and ten times (57) into (51), we obtain for the volume of the massless cosmic neutrino detector

$$V(0) \simeq 2 \text{ cm}^3 \tag{99}$$

VII - DETECTION OF P-P SOLAR NEUTRINOS

The solar neutrinos have a very small wavelength. Taking $E_\nu \sim 100$ keV as a characteristic energy for the neutrinos, the corresponding wavelength is

$$\lambda_\nu(\text{p-p}) \sim 0.02 \text{ Å} \tag{100}$$

For (100), condition (88) is easily satisfied.

Let us evaluate the minimum volume, V, in (51) which produces a detectable signal ΔH_ℓ of magnitude ten times (57). We take

$$\begin{aligned} &F_\nu = 6.1 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}, && d \simeq 10^{-4} \text{ cm} \\ &D \simeq 10^{-3} \text{ cm}, && W \simeq 10^{-3} \text{ cm}, \\ &\ell \simeq 10^{-2} \text{ cm}, && \tau \simeq 10^{5} \text{ s} \end{aligned} \tag{101}$$

Substituting (101) and ten times (57) into (51), we obtain for the volume of the solar neutrino detector

$$V(S) \simeq 3 \text{ cm}^3 \tag{102}$$

Obviously, from (51), large volumes produce large signals ΔH_ℓ. From (102), (99) and (90), however, present technology indicates the possibility of constructing a low energy coherent neutrino detector of volume < 10 cm^3; this very small volume emphasizes the very great sensitivity of the above described detector.

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