METRIC FLUCTUATIONS: THE MACROSCOPIC EQUATIONS OF GRAVITY

AND

CHAOS VERSUS ANTI-CHAOS

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1 - INTRODUCTION

Although some relativists sho a rather sceptical attitude toward the uniqueness of the geometry of the Universe, it is generally accepted that the main goal of relativistic cosmology is to exhibit the true metric of the global structure of space-time.

The use of such Principle of Uniqueness, which seems to be guaranteed by a crude application of the Einstein's Theory of Classical General Relativity (CGR), transforms the so-called Cosmological Question into the trivial problem of choosing some special riemannian geometry.

Parallel to this development some physicists have tried alternative ways and investigated more realistic models of the Universe with an endless richness of structure. They have been asking for some elementary questions like, for instance: In what sense the assumption of a unique geometry to represent the global gravitational effects is a consequence of the principles of CGR? Is it possible that the very stuff of space-time can undergo fluctuations in such way as to make geometry an average property of inmost structure?

There is no doubt that for an arbitrary observer, in a region free of any matter, the geometrical properties are described by a unique set of tensors: the electric ($E_{\mu\nu}$) and the magnetic ($H_{\mu\nu}$) parts of Weyl conformal tensor. We also know how matter of any kind influences metrical properties. However one usually does not consider the role of vacuum fluctuations of space time in the evolution of metric structures. Or, in a rather generic way, how vacuum fluctuations of matter fields affect geometry. How could this be described inside a geometric theory of gravity?

There are many possible answers to this question. We present in section II a specific model which makes appeal to the similitude of classical long range fields (gravity and electrodynamics).

We will follow Maxwell's ideas on electrodynamics and Lorentz' theory of electrons, which recognize that we can unify the description of microscopic and macroscopic electromagnetic properties of matter introducing a set of two pairs of fundamental vectors $(\vec{E},\vec{B})$ and $(\vec{D},\vec{H})$. Intensive and inductive quantities.

In analogy we require gravity to deal with two fundamental pairs of tensors $(E_{\mu\nu},B_{\mu\nu})$ and $(D_{\mu\nu},H_{\mu\nu})$. Specific equations for those quantities are model dependent and can be treated in an unified manner by the formalism we will present here.

Let me point out that I do not believe in the somewhat curious slogan which claims that "if it works for Electrodynamics, it must work for Gravity". However, I do believe that it is worthwhile, as a general tactics, to deeply explore similitudes of theories as physicists usually do, in order to make explicit any possible internal inconsistency of the model. This is the precise meaning we must give to section II: to understand it as a tactics and not as the laid down of some basic fundamental principles of gravidynamics "outside the vacuum".

Section III presents a very distinct approach although its main leitmotiv remains the same. We want to criticize the idea of unique fixed configurations of geometry in some basic circumstances which are intended to be the singular events from which all the rest is coming. We then consider the model by which the initial state of the Universe is of highest symmetry. We call such stage an anti-chaos configuration. We show that this phase represents a non-stable configuration of the Universe which tends very rapidly to a general Friedmann stage.

II - MODIFIED THEORY OF GRAVITY(*)

Relativistic Electrodynamics, in a material medium, is described by four 4-vectors: E_α, B_α, D_α and H_α. They are unified in two second order anti-symmetric tensors $F_{\mu\nu}$, $P_{\mu\nu}$ by setting

$$F_{\mu\nu} = - V_\mu E_\nu + V_\nu E_\mu + N_{\mu\nu}{}^{\rho\sigma} V_\rho B_\sigma \tag{1}$$

$$P_{\mu\nu} = - V_\mu D_\nu + V_\nu D_\mu + N_{\mu\nu}{}^{\rho\sigma} V_\rho H_\sigma \tag{2}$$

which relates $F_{\mu\nu}$ to the pair (E_μ, B_μ) and $P_{\mu\nu}$ to the pair (D_μ, H_μ) for an arbitrary observer with four-velocity V^μ. We normalize this vector by setting $g_{\mu\nu} V^\mu V^\nu = 1$.

From (1) and (2) we can easily obtain the expression of the 4-vectors in terms of the tensors $F_{\mu\nu}$ and $P_{\mu\nu}$:

$$E_\mu = F_{\mu\nu} V^\nu \tag{3a}$$

$$B_\mu = F^*_{\mu\nu} V^\nu \tag{3b}$$

$$D_\mu = P_{\mu\nu} V^\nu \tag{4a}$$

$$H_\mu = P^*_{\mu\nu} V^\nu \tag{4b}$$

The general equation of motion for linear and non-linear Electrodynamics (1) can be obtained from a variational principle by means of action S:

$$S = - \frac{1}{2} \int \sqrt{-g}\, P^{\mu\nu}(A_{\mu,\nu} - A_{\nu,\mu}) d_y x + \int \sqrt{-g}\, H\, (P,Q) d_y x - e \int \sqrt{-g}\, j^\mu A_\mu\, d_y x \tag{5}$$

in which the invariants P and Q are defined by

$$P \equiv \frac{1}{4} P_{\mu\nu} P^{\mu\nu} \tag{6a}$$

$$Q \equiv \frac{1}{4} P_{\mu\nu} \overset{*}{P}{}^{\mu\nu} \tag{6b}$$

The scalar H, which is the structural function, distinguishes different theories. For Maxwell's linear Electrodynamics we have

$$H = P \tag{7}$$

Indeed, variation of (5) with respect to A_μ yields

$$P^{\mu\nu}{}_{;\nu} = e\, j^\mu \tag{8}$$

Variation with respect to $P_{\mu\nu}$ gives

$$F_{\mu\nu} = 2 \frac{\delta H}{\delta P_{\mu\nu}} \tag{9}$$

in which we used the fact that

$$F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu} = A_{\mu,\nu} - A_{\nu,\mu} \tag{10}$$

which is relatated to the property of the free divergence of the dual

(*) This section is based on a joint work in collaboration with J.Plebansky.

$$\overset{*}{F}{}^{\mu\nu}{}_{;\nu} = 0.$$

Equation (9), the constitutive material relations can be written

$$F_{\mu\nu} = 2\,\frac{\partial H}{\partial P^{\mu\nu}} = H_P\,P_{\mu\nu} + H_Q\,P^*_{\mu\nu} \tag{12}$$

in which $H_P \equiv \frac{\partial H}{\partial P}$ and $H_Q \equiv \frac{\partial H}{\partial Q}$.

In the special case of (7) relation (12) reduces

$$F_{\mu\nu} = P_{\mu\nu} \tag{13}$$

and we obtain the theory of linear Electrodynamics.

Using (12) and definitions (3,4) we have

$$E_\mu = H_P\,D_\mu + H_Q\,H_\mu \tag{14}$$

$$B_\mu = -\,H_Q\,D_\mu + H_P\,H_\mu \tag{15}$$

Traditionally (e.g. Lorentz' theory of Electrons), one takes B_μ and D_μ as basic variables. This led us to consider the possibility of inverting formula (12) and set $P_{\mu\nu}$ as a function of $F_{\mu\nu}$. This can be done, following Plebansky, by defining a function L of invariants F and G. By analogy to (6), they are defined as:

$$F \equiv \frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} \tag{16a}$$

$$G \equiv \frac{1}{4}\,F_{\mu\nu}\,\overset{*}{F}{}^{\mu\nu} \tag{16b}$$

Function L is given by

$$L = \frac{1}{2}\,P^{\mu\nu}\,F_{\mu\nu} - H \tag{17}$$

or, using previous results

$$L = 2\,H_P\,P + 2\,H_Q\,Q - H(P,Q) \tag{18}$$

From this definition of L we have

$$P_{\mu\nu} = 2\,\frac{\delta L}{\delta F^{\mu\nu}} = L_F\,F_{\mu\nu} + L_G\,F^*_{\mu\nu} \tag{19}$$

In order to obtain the inverse relation (19) we must be able to get P and Q as functions of F and G.

Using (12) and definitions (6) and (16) we have

$$\begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} H_P^2 - H_Q^2 & 2H_P\,H_Q \\ -2H_P\,H_Q & H_P^2 - H_Q^2 \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix} = \Omega \begin{pmatrix} P \\ Q \end{pmatrix} \tag{20}$$

Thus, if the matriz Ω is invertible we can have $\binom{P}{Q} = \Omega^{-1}\binom{F}{G}$ and using (18) we obtain $L = L(F,G)$ which can be put into equation (19) in order to have $P_{\mu\nu}$.

We have

$$D_\mu = L_F\,E_\mu + L_G\,B_\mu \tag{21}$$

$$H_\mu = -L_G\,E_\mu + L_F\,B_\mu \tag{22}$$

Then, for the pair D_μ and B_μ we write

$$D_\mu = \frac{1}{H_P} E_\mu - \frac{H_Q}{H_P} H_\mu \tag{23}$$

$$B_\mu = \frac{1}{L_F} H_\mu + \frac{L_G}{L_F} E_\mu \tag{24}$$

We recognize then that the present model, which contains Lorentz constitutive equations for a polarized medium, goes further by the introduction of generic non-linear terms. We do not intend to proceed in this analysis. This was done by Plebansky in his examination of Born-Infeld Electrodynamical Theory. The reason for such rather long review of Electrodynamics here is just this: it will serve as a guide in the manipulation of corresponding quantities for the gravitational field.

Indeed, we will use it in order to set up a theory of gravitation for distinct "media".

What is the meaning of the term "medium" applied with respect to a gravitational field?

In the present lectures we speak of a medium as being associated to (vacuum) fluctuations of the gravitational field and/or as distinct responses of matter in case of fluctuations of the geometry. In other words, the medium represents, phenomenologically, the fact that we are supposing the existence of an unique overall space time manifold with an unique geometry. This uniqueness property is a characteristic of an (abstract) average geometry which happens to satisfy equations of motion distinct from Einstein's equations (2) (which are strictly valid in the zero point fluctuation) and which are thus described by constitutive relations of the generic type similar to equations (23, 24) as we shall see.

Let us start by setting the fundamental variables which we will deal with in the case of gravidynamics.

The geometric variables consist of the irreducible parts of the curvature tensor, that is

$W_{\alpha\beta\mu\nu}$ (Weyl conformal tensor)

$C_{\mu\nu}$ (Trace-free Ricci contracted tensor)

R (Scalar of curvature)

Besides these there are the I-quantities (inductive quantities) $J_{\alpha\beta\mu\nu}$, $J_{\mu\nu}$ and U.

Both tensors $J_{\alpha\beta\mu\nu}$ and $J_{\mu\nu}$ are trace-free. $J_{\alpha\beta\mu\nu}$ has all symmetries of Weyl tensor and $J_{\mu\nu}$ is symmetric.

The metrical objects can be used to construct 14 invariants. We will limit our interest here only on three of them, in order to simplify our exposition. They are:

$$A \equiv \frac{1}{8} W_{\alpha\beta\mu\nu} W^{\alpha\beta\mu\nu} \tag{25a}$$

$$B \equiv \frac{1}{8} W_{\alpha\beta\mu\nu} \overset{*}{W}{}^{\alpha\beta\mu\nu} \tag{25b}$$

$$C = \frac{1}{4} C_{\mu\nu} C^{\mu\nu} \tag{25c}$$

There are three scalars which can be constructed correspondigly with the I-quantities:

$$\tilde{A} = \frac{1}{8} J_{\alpha\beta\mu\nu} J^{\alpha\beta\mu\nu} \tag{26a}$$

$$\tilde{B} = \frac{1}{8} J_{\alpha\beta\mu\nu} \overset{*}{J}{}^{\alpha\beta\mu\nu} \tag{26b}$$

$$\tilde{C} = \frac{1}{4} J_{\mu\nu} J^{\mu\nu} \tag{26c}$$

Let us start by discussing a simple model in which only the fourth order tensors appear .

Following the Electrodynamical theory we set for the action

$$S_g = \int \sqrt{-g}\ (- \frac{1}{4} W_{\alpha\beta\mu\nu} J^{\alpha\beta\mu\nu} + \mathcal{H})\ d_y x \tag{27}$$

in which $\mathcal{H} = \mathcal{H}(\bar{A},\bar{B})$.

Varying $J^{\alpha\beta\mu\nu}$ we have

$$W_{\alpha\beta\mu\nu} = 4 \frac{\partial \mathcal{H}}{\partial J^{\alpha\beta\mu\nu}}$$

$$W_{\alpha\beta\mu\nu} = \mathcal{H}_{\bar{A}} J_{\alpha\beta\mu\nu} + \mathcal{H}_{\bar{B}} J^{*}_{\alpha\beta\mu\nu} \tag{28}$$

From the symmetrics of $W_{\alpha\beta\mu\nu}$ and $J_{\alpha\beta\mu\nu}$ we can show that there are only 10 independent components for each tensor. This can be characterized by two pairs of symmetric trace-free second-order tensors $(E_{\mu\nu},B_{\mu\nu})$ and $(D_{\mu\nu},H_{\mu\nu})$ through the definitions

$$E_{\mu\nu} = - W_{\mu\rho\nu\sigma} V^{\rho} V^{\sigma} \tag{29a}$$

$$B_{\mu\nu} = - W^{*}_{\mu\rho\nu\sigma} V^{\rho} V^{\sigma} \tag{29b}$$

$$D_{\mu\nu} = - J_{\mu\rho\nu\sigma} V^{\rho} V^{\sigma} \tag{30a}$$

$$H_{\mu\nu} = - J^{*}_{\mu\rho\nu\sigma} V^{\rho} V^{\sigma} \tag{30b}$$

Conversely, we can write

$$W^{\alpha\mu\beta\nu} = (\eta^{\alpha\mu\lambda\sigma} \eta^{\beta\nu\tau\varepsilon} - g^{\alpha\mu\lambda\sigma} g^{\beta\nu\tau\varepsilon}) V_{\lambda} V_{\tau} E_{\sigma\varepsilon} + (\eta^{\alpha\mu\lambda\sigma} g^{\beta\nu\tau\varepsilon} + g^{\alpha\mu\lambda\sigma} \eta^{\beta\nu\tau\varepsilon}) V_{\lambda} V_{\tau} B_{\sigma\varepsilon} \tag{31}$$

and

$$J^{\alpha\mu\beta\nu} = (\eta^{\alpha\mu\lambda\sigma} \eta^{\beta\nu\tau\varepsilon} - g^{\alpha\mu\lambda\sigma} g^{\lambda\nu\tau\varepsilon}) V_{\lambda} V_{\tau} D_{\sigma\varepsilon} + (\eta^{\alpha\mu\lambda\sigma} g^{\beta\nu\tau\varepsilon} + g^{\alpha\mu\lambda\sigma} \eta^{\beta\nu\tau\varepsilon}) V_{\lambda} V_{\tau} H_{\sigma\varepsilon} \tag{32}$$

in which $g_{\alpha\mu\beta\nu} \equiv g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\mu\beta}$. Using equation (28) and the above definitions we have

$$E_{\mu\nu} = \mathcal{H}_{\bar{A}} D_{\mu\nu} + \mathcal{H}_{\bar{B}} H_{\mu\nu} \tag{33}$$

$$B_{\mu\nu} = - \mathcal{H}_{\bar{B}} D_{\mu\nu} + \mathcal{H}_{\bar{A}} H_{\mu\nu} \tag{34}$$

Following the Lorentz choice of pairing we consider $B_{\mu\nu}$ and $D_{\mu\nu}$ as basic variables. This immediately leads to the problem of inverting formula (28) and setting $J_{\alpha\beta\mu\nu}$ as a function of Weyl conformal tensor. We will do this by defining a function $\mathcal{L}$ of the invariants A and B:

$$\mathcal{L} = \frac{1}{4} J^{\alpha\beta\mu\nu} W_{\alpha\beta\mu\nu} - \mathcal{H} \tag{35}$$

or, using previous results,

$$\mathcal{L} = 2\, \mathcal{H}_{\bar{A}} \bar{A} + 2\, \mathcal{H}_{\bar{B}} \bar{B} - \mathcal{H}(\bar{A},\bar{B}). \tag{36}$$

From this we have

$$J_{\alpha\beta\mu\nu} = 4 \frac{\delta \mathcal{L}}{\delta W^{\alpha\beta\mu\nu}} = L_A W_{\alpha\beta\mu\nu} + L_B W^{*}_{\alpha\beta\mu\nu} . \tag{37}$$

To obtain $\mathcal{L}$ as function of A and B we must be able to write $\bar{A}$ and $\bar{B}$ as functions of A and B.

From above relations we have

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \mathcal{H}_{\bar{A}}^2 - \mathcal{H}_{\bar{B}}^2 & 2\mathcal{H}_{\bar{A}} \mathcal{H}_{\bar{B}} \\ -2\mathcal{H}_{\bar{A}} \mathcal{H}_{\bar{B}} & \mathcal{H}_{\bar{A}}^2 - \mathcal{H}_{\bar{B}}^2 \end{pmatrix} \begin{pmatrix} \bar{A} \\ \bar{B} \end{pmatrix} \tag{38}$$

In case this matrix is invertible we can write

$$D_{\mu\nu} = L_A E_{\mu\nu} + L_B B_{\mu\nu} \tag{39}$$

$$H_{\mu\nu} = - L_B E_{\mu\nu} + L_A B_{\mu\nu} . \tag{40}$$

Finally, for the Lorentz pair $(D_{\mu\nu}, B_{\mu\nu})$ we have

$$D_{\mu\nu} = \frac{1}{H_{\bar{A}}} E_{\mu\nu} - \frac{H_B}{H_{\bar{A}}} H_{\mu\nu} \tag{41}$$

$$B_{\mu\nu} = \frac{1}{L_A} H_{\mu\nu} + \frac{L_B}{L_A} E_{\mu\nu} \tag{42}$$

These are the corresponding gravitational counterpart of the constitutive equations for a polarized medium in electrodynamics.

From the action (27) for these fields we obtain the equations of motion by independent variation of the connection $\Gamma^{\alpha}_{\mu\nu}$ and metric $g_{\mu\nu}$, using Palatini's technique. We obtain

$$J^{\alpha\beta\mu\nu}{}_{;\nu} = 0 \tag{43}$$

$$W_{\mu\alpha\beta\lambda} J_{\nu}{}^{\alpha\beta\lambda} + (2H - \frac{1}{2} W_{\alpha\beta\lambda\sigma} J^{\alpha\beta\lambda\sigma}) g_{\mu\nu} = 0 \tag{44}$$

We remark that, in an analogous way as in the previous Electrodynamical case, if the structural function depends linearly on invariant $\bar{A}$ only, equation (43) reduces to Bianchi identity (in the abscence of source) and (44) reduces to the well-known identity (3)

$$W_{\alpha\beta\mu}{}^{\nu} W^{\alpha\beta\mu}{}_{\tau} = \frac{1}{4} W_{\alpha\beta\mu\lambda} W^{\alpha\beta\mu\lambda} \delta^{\nu}_{\tau} \tag{45}$$

From the constitutive equation (28), in this case the inductive tensor $J_{\alpha\beta\mu\nu}$ coincides with Weyl tensor

$$W_{\alpha\beta\mu\nu} = H_{\bar{A}} J_{\alpha\beta\mu\nu} + H_{\bar{B}} J^{*}_{\alpha\beta\mu\nu} = J_{\alpha\beta\mu\nu} \tag{46}$$

Another special case is given by

$$H = \bar{B} \tag{47}$$

(or any multiple of this invariant).

In this case, the constitutive relation implies that $J_{\alpha\beta\mu\nu}$ is the dual of $W_{\alpha\beta\mu\nu}$, consequently the action S_g reduces to a topological invariant

$$S_g = \sigma \int \sqrt{-g}\ W_{\alpha\beta\mu\nu} \overset{*}{W}{}^{\alpha\beta\mu\nu} d_y x \tag{48}$$

There are many questions which one should investigate as a consequence of our present model, like for instance the following set (which does not intend to exhaust all possible interesting questions):

(i) The generalization of Bel's super energy tensor.
(ii) The velocity of light in the present theory of gravity (for different structural functions).
(iii) The evolution of weak perturbations.
(iv) Gravitational waves (for different values of H).
(v) Characteristic surfaces.

We intend to analyse some of those questions elsewhere.

Just in order to give a simple example of how one can construct a slight modification of standard gravity theory in case $R_{\mu\nu} \neq 0$ let us assume that $H = H(\bar{A}, \bar{B}, C, U)$.

We set

$$S_{II} = \int \sqrt{-g} \left[- \frac{1}{4} \left(W_{\alpha\beta\mu\nu} J^{\alpha\beta\mu\nu} - 2 C_{\mu\nu} J^{\mu\nu} + \frac{1}{6} RU \right) + H \right] \quad (49)$$

The constitutive relations obtained by independent variations of $J^{\alpha\beta\mu\nu}$, $J^{\mu\nu}$ and U are

$$W_{\alpha\beta\mu\nu} = H_A J_{\alpha\beta\mu\nu} + H_B J^{*}_{\alpha\beta\mu\nu} , \quad (50)$$

$$C_{\mu\nu} = - H_C J_{\mu\nu} \quad (51)$$

$$R = 24 H_U \quad (52)$$

The very special case in which we choose H as given by

$$H = A - C + \frac{1}{48} U^2 \quad (53)$$

gives

$$W_{\alpha\beta\mu\nu} = J_{\alpha\beta\mu\nu} \quad (54)$$

$$C_{\mu\nu} = J_{\mu\nu} \quad (55)$$

$$R = U \quad (56)$$

Correspondingly, action S_{II} reduces to a topological invariant.

III - CHAOS X ANTI-CHAOS

"Some say that it is easy to make order from disorder. Others believe that order exists for all time. For me, I am still asking for the meaning of order".

Cosmologists have conceived two main scenarios for the early times of our Universe, which have been named chaos and anti-chaos.

In the chaotic era all possible initial states of the Universe were equally probable. We have thus to deal with a very arbitrary mixture of configurations which, by a series of physical phenomena (yet to be known), gave origin to the present apparently highly homogeneous and isotropic phase.

In the other case, of an anti-chaos, the initial configuration corresponds to the highest state of symmetry.

Distinct cosmologists adhere either to one or other of these hypothesis with different leitmotives and with different consequences. Others adhere to a whole spectrum of conditions bounded by these two limiting situations.

In the sixties, the chaotic program, was made popular by Misner. However it provoked a lot of new questions which were far from being understood. This has diminished greatly the interest of cosmologists in such somehow extravagant program.

The opposite case, anti-chaos, has its roots in the thirties, due mainly to the work of de Sitter and Lemaître.

Although this model is older, its interest has been recently renewed for many different reasons.

In this lecture I intend to present a model by means of which anti-chaos can be presented as an initial (although non-stable) configuration of the Universe.

III.1 - CONFORMAL COUPLING AND THE FUNDAMENTAL SOLUTION

Let us assume that the Lagrangian of the metric $g_{\mu\nu}(x)$ and of the scalar field $\phi(x)$ is given by

$$L = \sqrt{-g}\ \{R + \partial_\mu\phi^*\partial_\nu\phi\ g^{\mu\nu} - m\ \phi^*\phi - \frac{1}{6} R\ \phi^*\phi + 2\ \Lambda\} \tag{1}$$

in which we have introduced an additional constant term (Λ is the so-called cosmological constant).

The equations of motion which result from this Lagrangian are

$$\Box\ \phi + (m^2 + \frac{1}{6} R)\phi = 0 \tag{2}$$

$$(1-\frac{1}{6}\phi^2)(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}) = -\frac{1}{2}(\phi^*_{,\mu}\phi_{,\nu}+\phi^*_{,\nu}\phi_{,\mu})+\frac{1}{2}g_{\mu\nu}(\phi^*_{,\lambda}\phi_{,\varepsilon}g^{\lambda\varepsilon}-m^2\phi^*\phi)-\frac{1}{6}\Box(\phi^*\phi)g_{\mu\nu}+ \\ +\frac{1}{6}(\phi^*\phi)_{,\mu;\nu}+\Lambda\ g_{\mu\nu} \tag{3}$$

Taking the trace of equation (3) gives

$$R = m^2\ \phi^*\phi - 4\ \Lambda \tag{4}$$

Substituting this value into equations (2) gives

$$\Box\ \phi + (m^2 - \frac{2}{3}\ \Lambda)\ \phi + \frac{m^2}{6}\ \phi^2\phi = 0 \tag{5}$$

The form of the equation of ϕ shows explicitly that the final consequence of the introduction of the conformal term proportional to $R\phi^2$ in Lagrangian (?) is equivalent to a quartic self-coupling of ϕ.

This fact suggests a further generalization of the equation of ϕ by introducing *ab initio* the self-interacting term

$$\sigma(\phi^*\phi)^2$$

in the Lagrangian. Thus we set for L the form:

$$L=\sqrt{-g}\ \{R + \partial_\mu\phi^*\partial_\nu\phi g^{\mu\nu} - m^2\ \phi^*\phi - \frac{1}{6}\ R\ \phi^*\phi + \sigma(\phi^*\phi)^2 + 2\ \Lambda\} \tag{6}$$

The corresponding equation for ϕ:

$$\Box\ \phi + (m^2 - \frac{2}{3}\ \Lambda)\ \phi + (\frac{m^2}{6} - 2\sigma)\ |\phi|^2\ \phi = 0 \tag{7}$$

We see that the role of the cosmological constant Λ (for $\Lambda > 0$) is to reduce the value of the mass of ϕ to an effective mass m_{eff} given by:

$$(m_{eff})^2 \equiv m^2 - \frac{2}{3}\ \Lambda \tag{8}$$

We recognize here the possibility of induction, by Λ, of the mechanism of spontaneous breaking of symmetry.

Let us review briefly the main points of this mechanism in flat space-time. In this case the equation for ϕ is

$$\overset{o}{\Box}\ \phi + m^2\phi - 2\sigma\ \phi^2\phi = 0 \tag{9}$$

If we look for the simplest solution $\phi = \phi_o$ = constant we obtain

$$\phi_o^2 = \frac{m^2}{2\sigma} \tag{10}$$

This is possible only if the mass of the ϕ field is imaginary (once $\sigma < 0$). This value for ϕ_0 corresponds to the minimum of energy, which in this case is given by the minimum of the potential

$$V(\phi) = m^2 \phi^2 - \sigma \phi^4 \tag{11}$$

We remark that in the fundamental state ϕ_0 the system does not have the gauge symmetry (invariance under a change in the phase $\phi \to e^{i\alpha}\phi$) which is valid for the Lagrangian. Let us pass now to the curved geometry and ask for the modifications on this phenomenon introduced by the coupling with gravity.

Equation (7) shows that the new constant solution, corresponding to (10), is:

$$\phi_0^2 = 2\left(\frac{2\Lambda - 3m^2}{m^2 - 12\sigma}\right) \tag{12}$$

We take as definition of the energy-momentum tensor of the ϕ field the expression (Chernikov and Taginov, 1968)

$$T_{\mu\nu}[\phi] = t_{\mu\nu} - \frac{\phi^2}{6}\left(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}\right) \tag{13}$$

in which $t_{\mu\nu}$ is the energy tensor

$$t_{\mu\nu} \equiv \frac{1}{2}(\phi^*_{;\mu}\phi_{,\nu} + \phi^*_{;\nu}\phi_{,\mu}) - \frac{1}{2} g_{\mu\nu}\left[\phi^*_{;\lambda}\phi_{,\varepsilon} g^{\lambda\varepsilon} - m^2\phi^*\phi + \sigma(\phi^*\phi)^2\right] + \frac{1}{6}\Box(\phi^*\phi) g_{\mu\nu} - \frac{1}{6}(\phi^*\phi)_{,\mu;\nu}.$$

We can, however, re-write $T_{\mu\nu}$ in another form, using equation (3) $g_{\mu\nu}$, with the inclusion of the σ-term.

Formally we have

$$T_{\mu\nu}[\phi] = t_{\mu\nu} - \frac{\phi^2}{6}\left(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}\right) = t_{\mu\nu} - \frac{\phi^2}{6}\left[-\frac{t_{\mu\nu}}{1 - \frac{\phi^2}{6}} + \frac{\Lambda\, g_{\mu\nu}}{1 - \frac{\phi^2}{6}}\right]$$

or

$$T_{\mu\nu}[\phi] = \frac{t_{\mu\nu}}{1 - \frac{\phi^2}{6}} - \frac{\phi^2}{6 - \phi^2}\Lambda\, g_{\mu\nu} \tag{13'}$$

Thus, the energy $E(\phi)$ in the fundamental state ϕ_0 is

$$E(\phi_0) = \frac{(3m^2 - \Lambda)\phi^2 - 3\sigma\phi^4}{6 - \phi^2} \tag{14}$$

Let us add to E a constant, just to renormalize the energy of the zero point, that is, we have $E(0) = -\Lambda$. We write $\bar{E} = E - \Lambda$.

The extremum points of $\bar{E}$ are given by solutions of the equation

$$\sigma\phi_0^4 - 12\sigma\phi_0^2 + 6m^2 - 2\Lambda = 0 \tag{15}$$

In order to the fundamental states (12) to be an extremum they must satisfy simultaneously both equations (12) and (15). This is not possible in general, but it can occur if the cosmological constant Λ and the self-coupling constant σ are related by

$$\sigma = \frac{m^4}{8\Lambda}, \tag{16}$$

which implies, that

$$\phi_0^2 = \frac{m^2}{2\sigma}. \tag{17}$$

This is precisely the result, expressed in equation (24), we had obtained before in case of absence of gravity.

The interpretation we can give to such result is this: in general, the introduction of a cosmological constant to renormalize the value of E(0) does not allow the mechanism of spontaneous breaking of symmetry. However, if the constants σ and Λ are not completelly independent but satisfy relation (38), then the constant fundamental solution ϕ_0 constitutes the ground state of our system. As a consequence of this, the gauge symmetry is broken. We call this an induced broken mechanism symmetry. The extremum will be a minimum only if $3m^2 - 2\Lambda < 0$, in which case the effective mass is imaginary. We find here the same situation as in the flat space time. The role of the cosmological constant Λ is just to set the mass m of the ϕ field to be real and an imaginary effective mass to appear, which allows the existence of a ground state distinct from the trivial one ($\phi = 0$).

What is the role of the other constant, σ, which appears in this theory?

In order to understand this let us set $\sigma = 0$ and see what are the consequences of it.

This case is represented in figure(5.2). See that the point of constant solution ϕ_0 cannot be an extremum for any value of Λ. Instead of this, we recognize the instability of the constant solution which corresponds to $E(\phi_0) = -\frac{3}{2} m^2$.

The equation for the metric in this state, reduces to

$$R^{\mu}{}_{\nu} - \frac{1}{2} R\, \delta^{\mu}{}_{\nu} = \frac{3}{2} m^2\, \delta^{\mu}{}_{\nu} \qquad (18)$$

which corresponds to a negative effective cosmological constant $\Lambda_{eff} = -\frac{3}{2} m^2$. In case of homogeneous cosmological models it corresponds to closed de Sitter Universe. Figure (5.2) shows that this is a highly unstable situation, the whole system decaying to the asymptotically stable case $E(\pm\infty) = -3m^2$. Thus we recognize that the role of the self-coupling term of the ϕ field is just to increase the stability of its fundamental symmetry breaking constant solution.

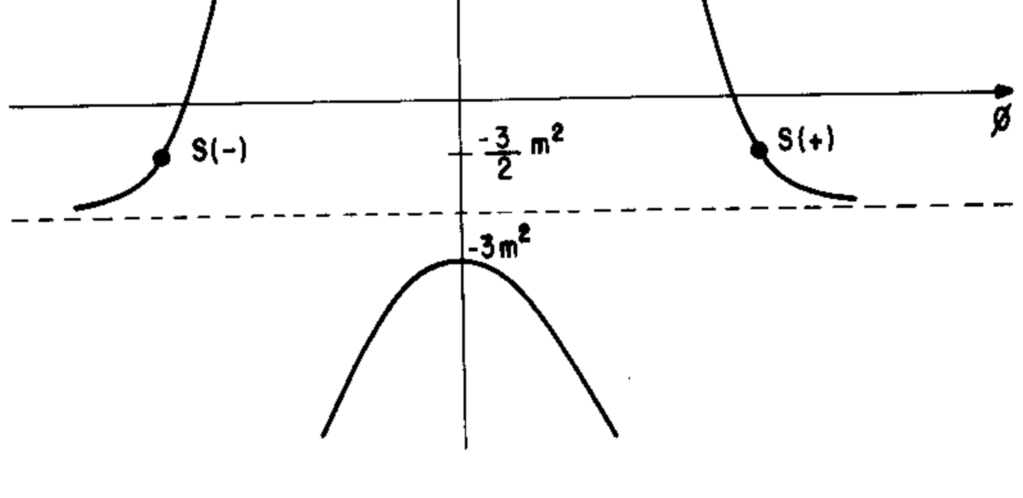

FIGURE 5.2 - Case in which $\sigma = 0$. $S_{(+)}$ and $S_{(-)}$ are De Sitter Universes.

III.2 - REPULSIVE GRAVITY OF MASSLESS PARTICLES INDUCED BY THE SCALAR FIELD

In this section we present a consequence of the existence of the symmetry breaking mechanism on the properties of the gravitational field. From figure (5.1) we can conclude that in the ϕ_0-states the scalar field does not contribute to the energy which is responsible for curving the space-time. This is due to the fact that $E(\phi_0) = 0$. Let us see the consequences of this in case there are other sources of energy present.

In this case, equation (17) has an additional term of energy:

$$(1 - \frac{1}{6}\phi^2)\,(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}) = - E_{\mu\nu} - T_{\mu\nu} \tag{19}$$

in which $E_{\mu\nu}$ is given by the right hand side of (17) and $T_{\mu\nu}$ represents the energy momentum tensor of the rest of the matter. In the fundamental state equation (19) takes the form

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = - \frac{3\, m^2}{3\, m^2 - 2\,\Lambda}\, T_{\mu\nu} \tag{20}$$

Remark that in order that the value (24) of ϕ_0 remain valid in case $T_{\mu\nu} \neq 0$, equation (4) must hold and this imposes that the energy-momentum tensor must be trace-free ($T \equiv T_{\mu\nu} g^{\mu\nu} = 0$).

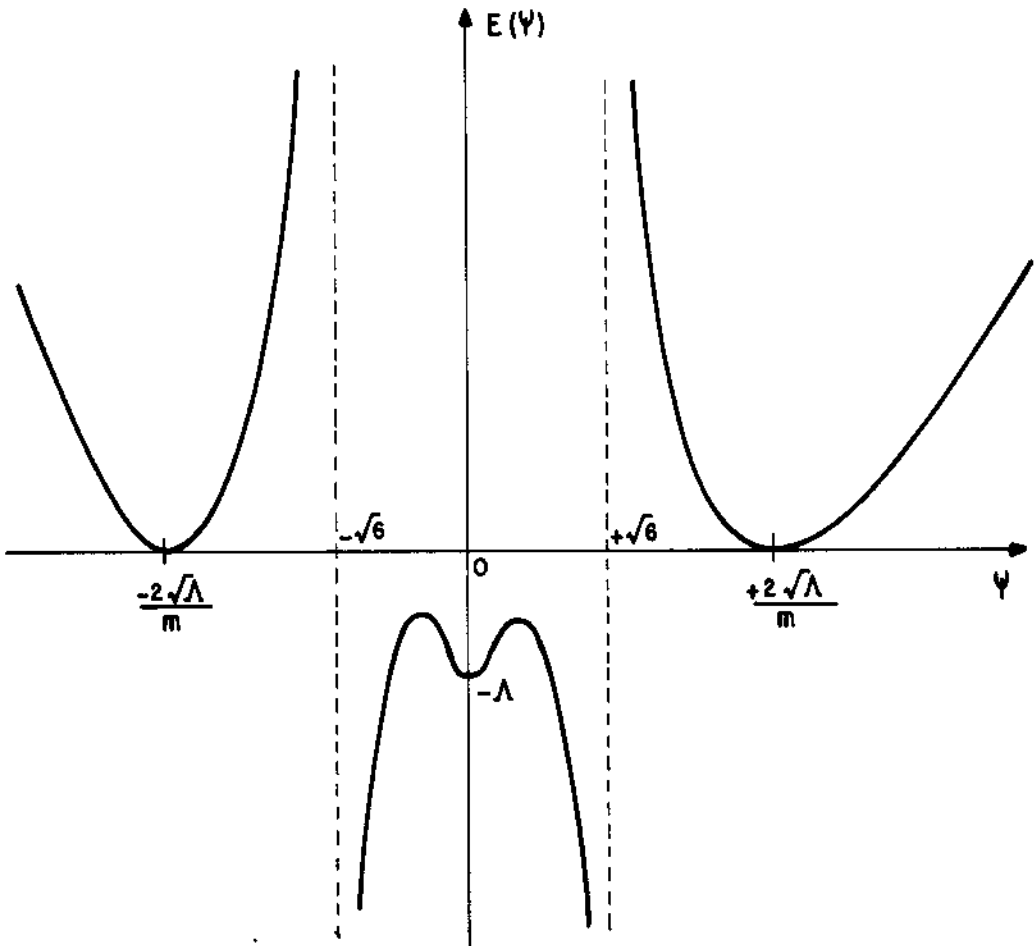

FIGURE 5. - Case in which $2\Lambda > 3m^2 > \Lambda > \frac{3}{2} m^2$. (See the text).

From equation (20) we obtain that the net consequence of the existence of the scalar field in the fundamental state ϕ_0 is just to renormalize the gravitational constant K (which we take as unity in the system we are using) to an effective constant

$$K_{REN} = \frac{3\,m^2}{3m^2 - 2\,\Lambda}\;K \tag{21}$$

The sign of the value of the renormalized gravitational constant depends crucially on the mass of ϕ and on the cosmological constant Λ.

Thus, if the mass of the scalar field is such that $3m^2 - 2\,\Lambda < 0$, then K_{REN} becomes negative. We then conclude that the gravitational field created by radiation is repulsive in the fundamental state ϕ_0 of the scalar field.

Let us point out that this mechanism does not change any property of photons and neutrinos , but only creates amedium in which the gravitational field generated by them becomes repulsive.

The application of the above mechanism to our Universe could lead us to an alternative explanation of its actual expanding era. Indeed, the dominant energy at early times comes from massless particles (this is a direct consequence of the conservation law, which states that for a perfect fluid with equation of state relating the pressure p to the density of energy ρ by $p = \lambda\rho$, in a Friedmann Universe, we have $\rho = \rho_0\, A^{-3(1+\lambda)}$, see section 2).

If the scalar field has a very small mass $m^2 < \frac{2}{3}\,\Lambda$ (that is $m \lesssim 10^{-34}$ Mev) and if the system exists in the fundamental state ϕ_0 then the conditions are fulfilled in order to the above result of cosmic repulsion be applied.

This is a matter for further investigations.

REFERENCES

1 - J.Plebansky - Lectures on non-linear Electrodynamics - Nordita (1968).

2 - M.Novello - Excitations of the gravitational field - Rev.Bras.de Física, vol. 8, 442 (1978).

3 - M.Novello and J.Duarte - On Dual Properties of Weyl Tensor - GRG, vol.12,871 (1980).