Fundamentals of Magnetism

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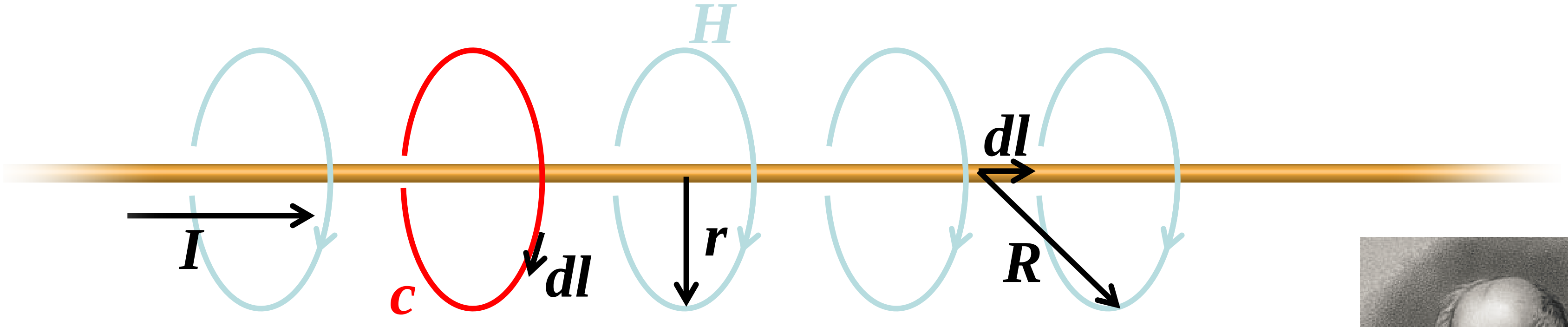
Part I: $\boldsymbol{H, M, B, \chi, \mu}$
Part II: $\boldsymbol{M(H)}$

Magnetic Field from Current in a Wire



Ampere's Circuital Law:

$$\oint_{c} \vec{H}\cdot d\vec{l} = I$$

Biot-Savart Law:

$$d\vec{H} = \frac{I d\vec{l} \times \hat{R}}{4\pi\left|\vec{R}\right|^{2}}$$

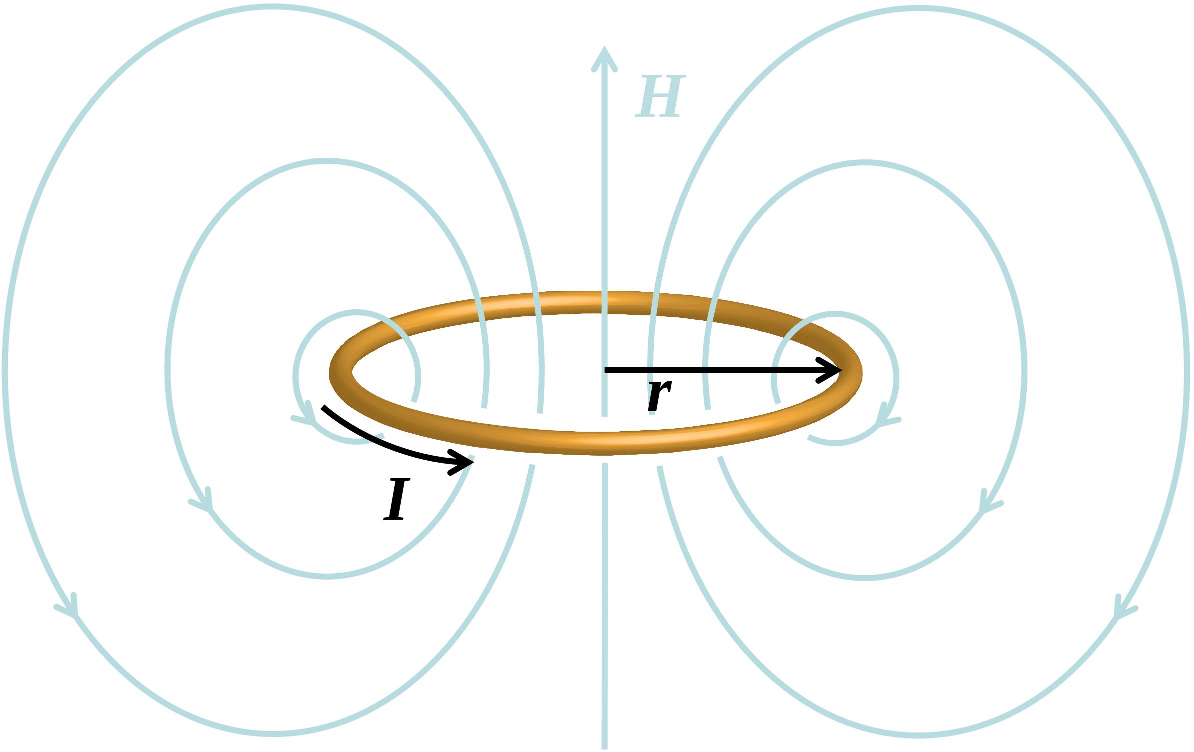
$$H = \frac{I}{2\pi r}$$ [A/m]

André-Marie Ampère
(1775–1836)

Jean-Baptiste Biot
(1774 –1862)

Félix Savart
(1791 – 1841)

Magnetic Field from Current Loop



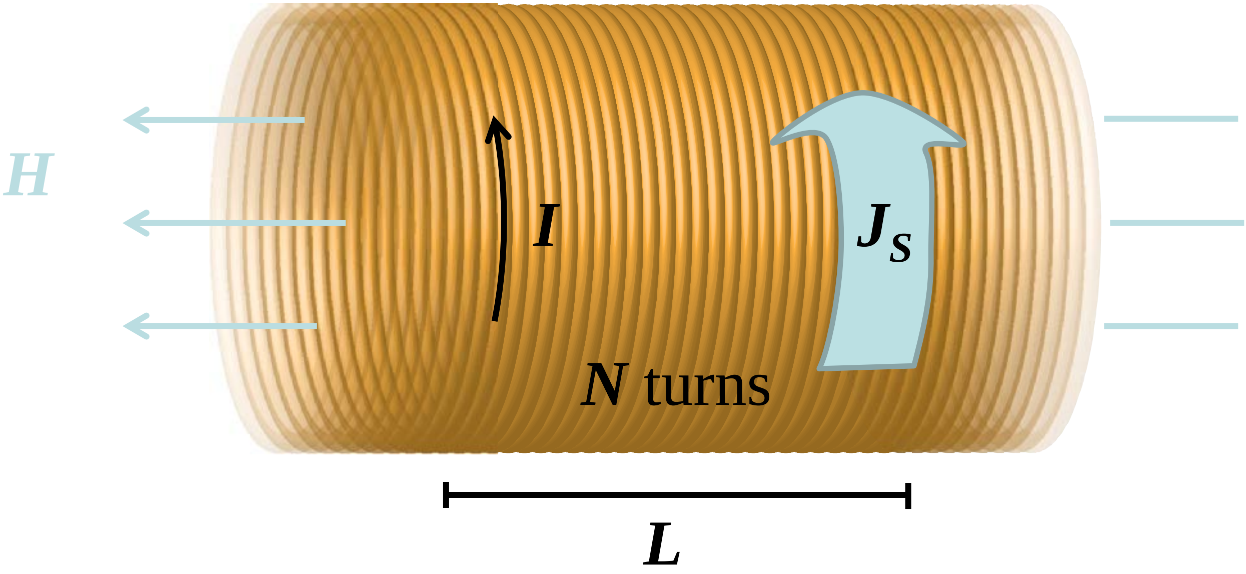
Use Biot-Savart Law:

$$d\vec{H} = \frac{I d\vec{l} \times \hat{R}}{4\pi \left|\vec{R}\right|^2}$$

In the center:

$$H = \frac{I}{2r}$$ [A/m]

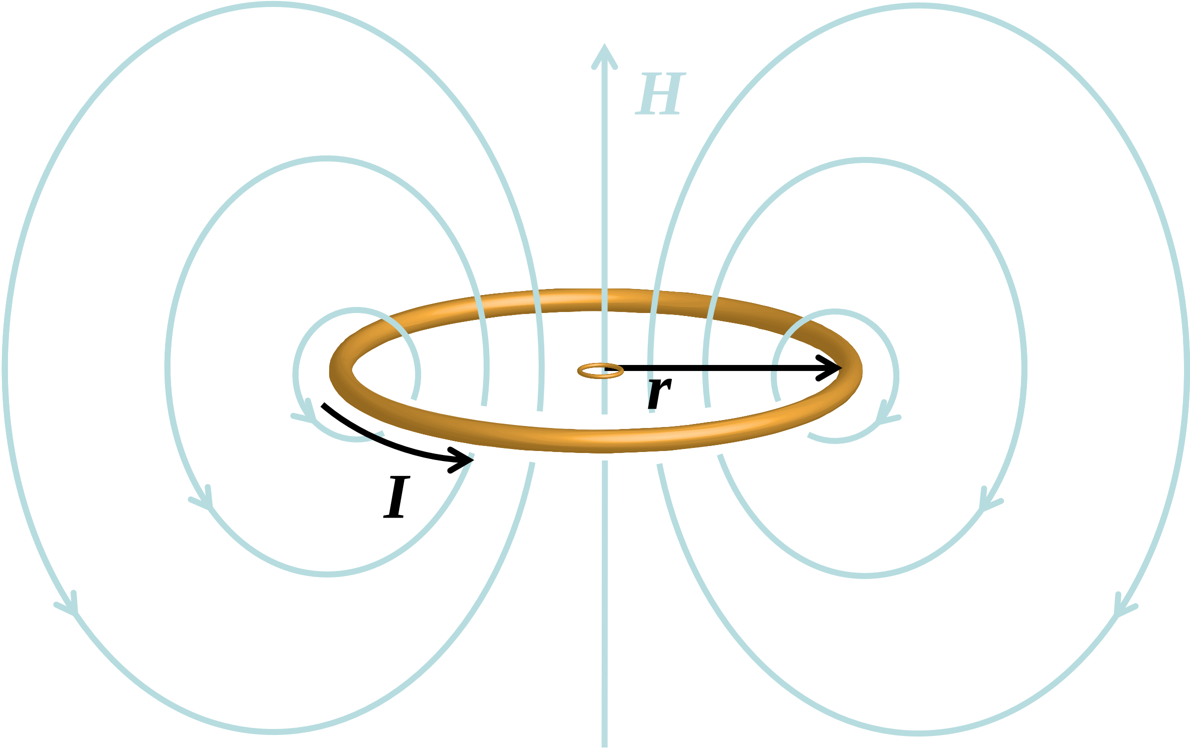
Magnetic Field in a Long Solenoid



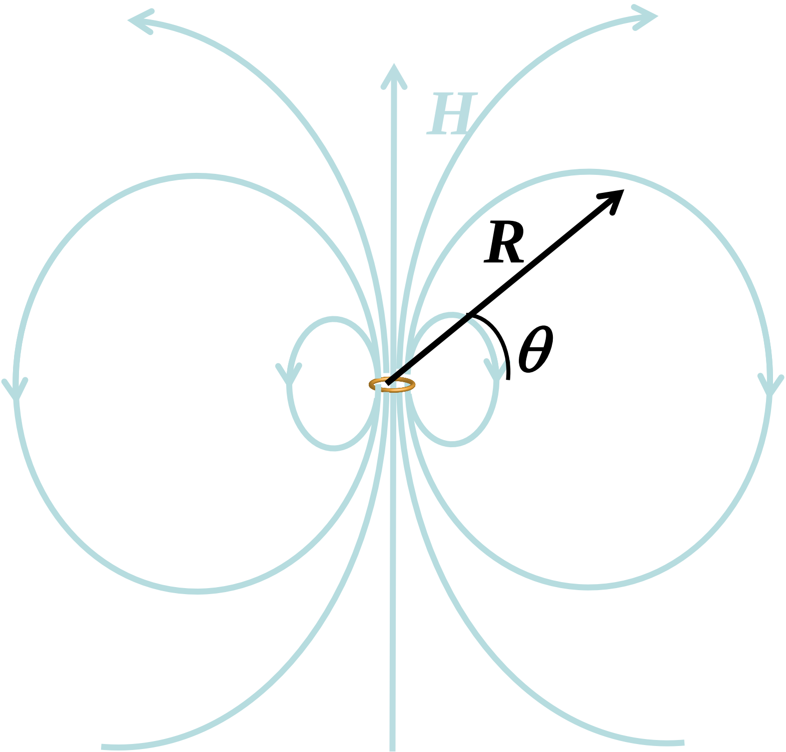
Inside, field is uniform with:

$$H = \frac{NI}{L} \text{ [A/m]} \qquad H = J_S \text{ [A/m]}$$

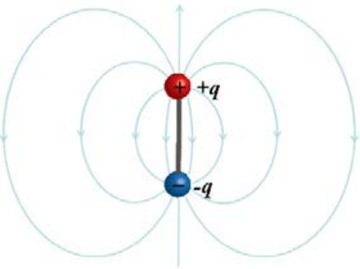
Magnetic Field from “Small” Loop (Dipole)



Magnetic Field from Small Loop (Dipole)



Compare to
Electric dipole:

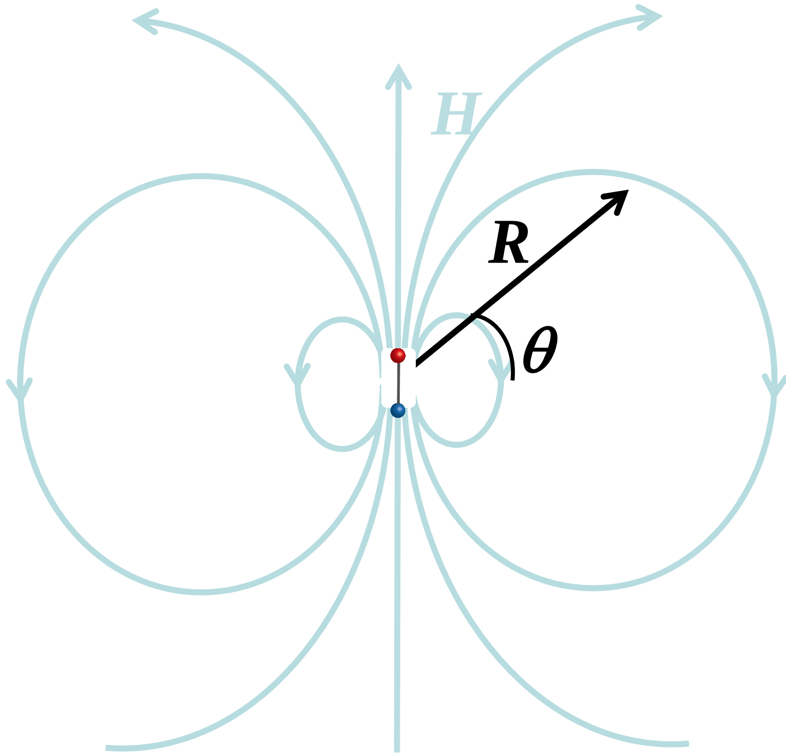


For ***R*** >> ***r***

$$H = \frac{\boxed{IA}}{4\pi R^3}\left[2\hat{\boldsymbol{r}}\cos(\theta) + \hat{\boldsymbol{\theta}}\sin(\theta)\right] \quad \text{[A/m]}$$

Magnetic (dipole) moment: $\vec{m} = I\vec{A}$ [Am^2]

Magnetic Field of a Dipole



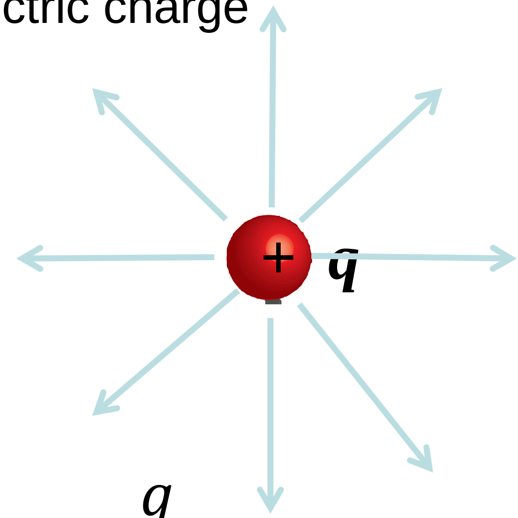
For ***R*** >> ***r***

$$H = \frac{q_m d}{4\pi R^3}\left[2\hat{r}\cos(\theta) + \hat{\theta}\sin(\theta)\right] \quad \text{[A/m]}$$

Magnetic (dipole) moment: $\vec{m} = q_m\vec{d}$ [Am^2]

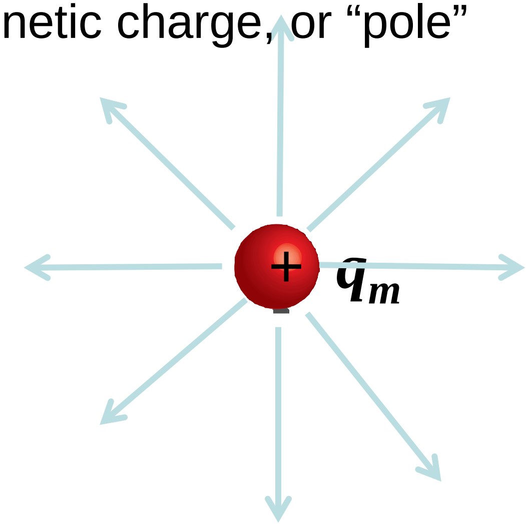
Magnetic Poles and Dipoles

Electric charge



$$D = \frac{q}{4\pi R^2}$$

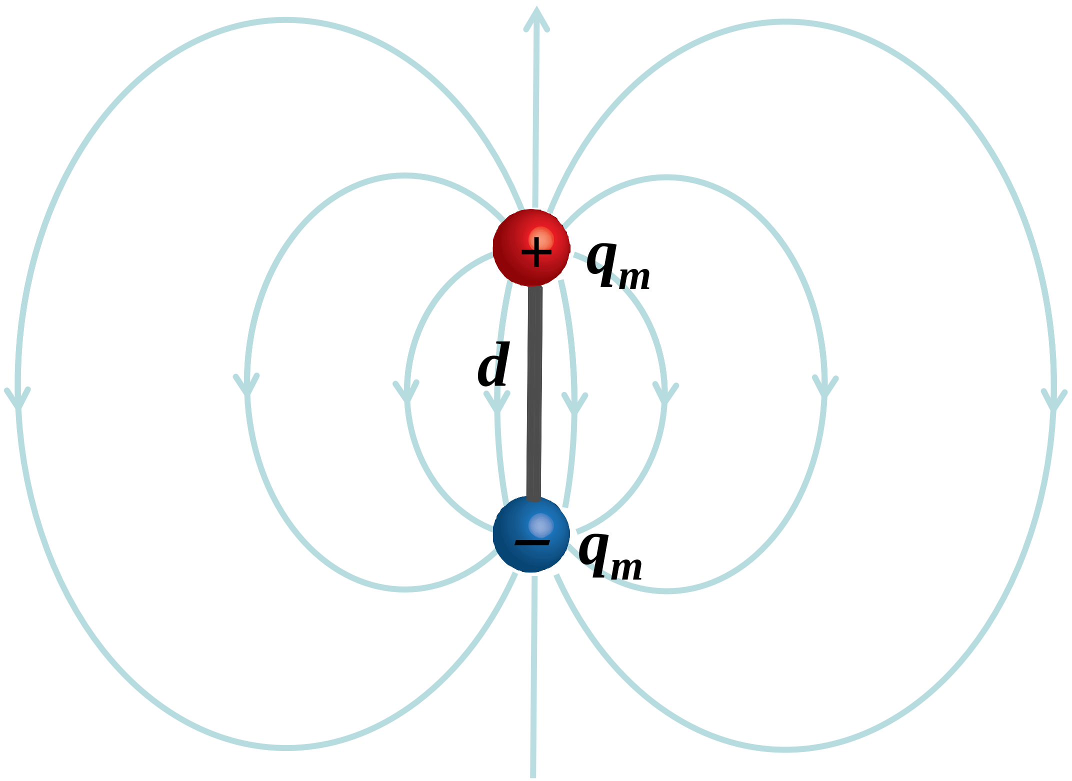
Magnetic charge, or “pole”



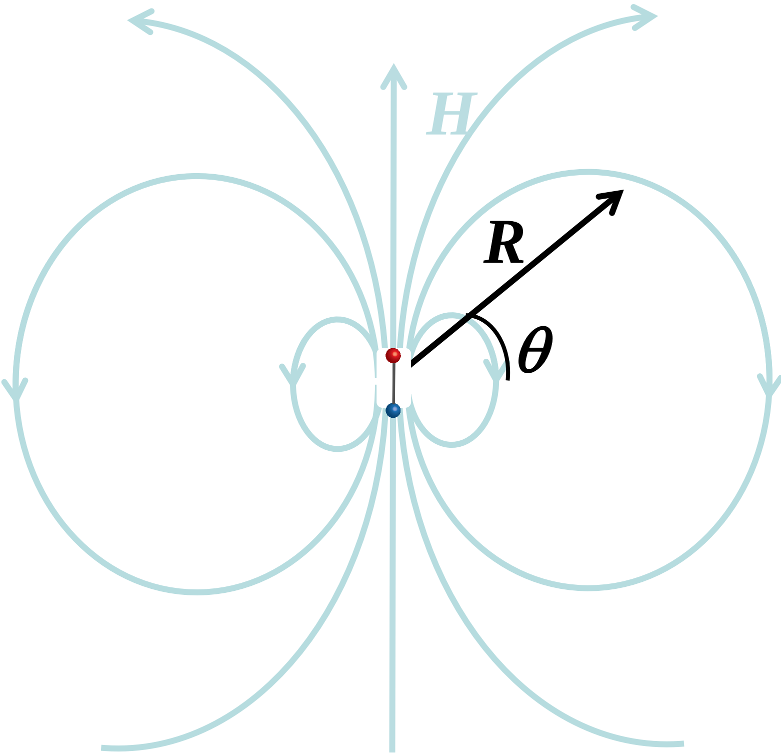
Magnetic field from a magnetic “monopole”

$$H = \frac{q_m}{4\pi R^2}$$ [A/m]

Magnetic Field of a Dipole



Magnetic Field of a Dipole



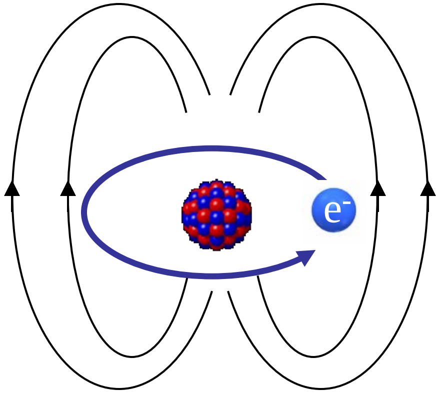
q_m [A·m]

For ***R*** >> ***r***

$$H = \frac{q_m d}{4\pi R^3}\left[2\hat{r}\cos(\theta) + \hat{\theta}\sin(\theta)\right] \quad \text{[A/m]}$$

Magnetic (dipole) moment: $\vec{m} = q_m \vec{d}$ [Am^2]

Orbital Magnetic Moment



- Electrons orbiting a nucleus are like a circulating current producing a magnetic field.

- For an electron with charge $\boldsymbol{q_e}$ orbiting at a radius $\boldsymbol{R}$ with frequency f, the Orbital Magnetic Moment is

$$m = IA = -q_e f \pi R^2 \qquad [\mathrm{Am}^2]$$

- It also has an Orbital Angular Momentum

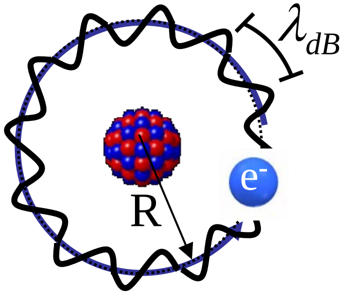
$$L = m_e v R = m_e 2\pi R f R$$

- Note:

$$\frac{m}{L} = \frac{-q_e}{2m_e} = gyromagnetic\ ratio$$

Orbital Moment is Quantized

Bohr model of the atom



- DeBroglie wavelength is:

$$\lambda_{dB} = \frac{h}{m_e v}$$

- Bohr Model: orbit must be integer number of wavelengths

$$2\pi R = N\lambda_{dB} = N\frac{h}{m_e v} = N\frac{h}{m_e 2\pi R f}$$

- Thus the orbital magnetic moment is quantized:

$$m = -q_e f\,\pi R^2 = N\frac{h}{2\pi}\frac{q_e}{2m_e} = N\frac{\hbar q_e}{2m_e}$$

- Magnetic moment restricted to multiples of

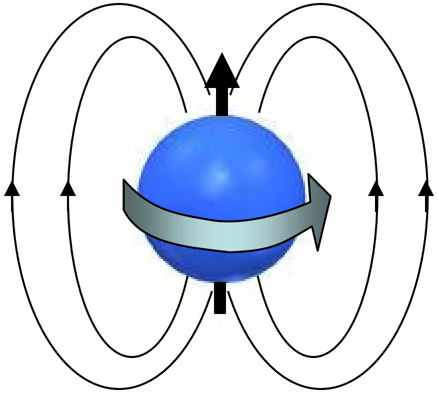
Bohr Magneton

$$\mu_B = \frac{\hbar q_e}{2m_e} = 9.2742\times 10^{-24}[Am^2]$$

Niels Bohr (1885-1962)

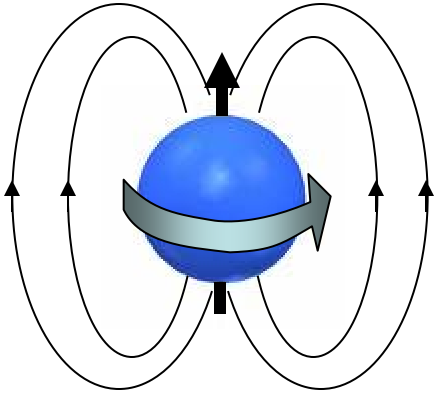
Louis de Broglie (1892-1987)

Spin



- Spin is a property of subatomic particles (just like charge or mass.)
- A particle with spin has a magnetic dipole moment and angular momentum.
- Spin may be thought of ***conceptually*** as arising from a spinning sphere of charge. (However, note that neutrons also have spin but no charge!)

Pauli and Bohr contemplate the “spin” of a tippy-top



Electron Spin

- *When measured in a particular direction*, the measured angular momentum of an electron is

$$L_z = \pm \frac{\hbar}{2}$$

- *When measured in a particular direction*, the measured magnetic moment of an electron is

$$m_z = s_z \frac{\hbar q_e}{m_e} = \pm \mu_B$$

- We say “spin up” and “spin down”

- Note: $\frac{m}{L} = \frac{-q_e}{m_e}$

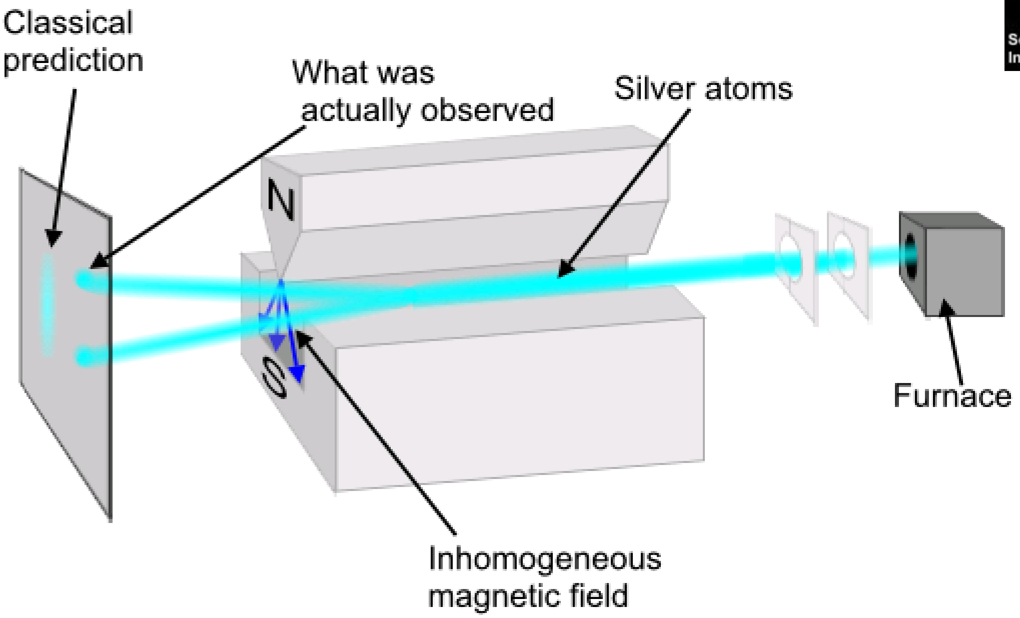
Compare:

Spin	Orbital
$m_z = \pm \mu_B$	$m_z = N \mu_B$
$\frac{m}{L} = \frac{-q_e}{m_e}$	$\frac{m}{L} = \frac{-q_e}{2m_e}$
$g = 2$	$g = 1$

$$\frac{m}{L} = g \frac{-q_e}{2m_e}$$

Stern-Gerlach Experiment - 1922

Demonstrated that magnetic moment is quantized with $\pm\mu_B$

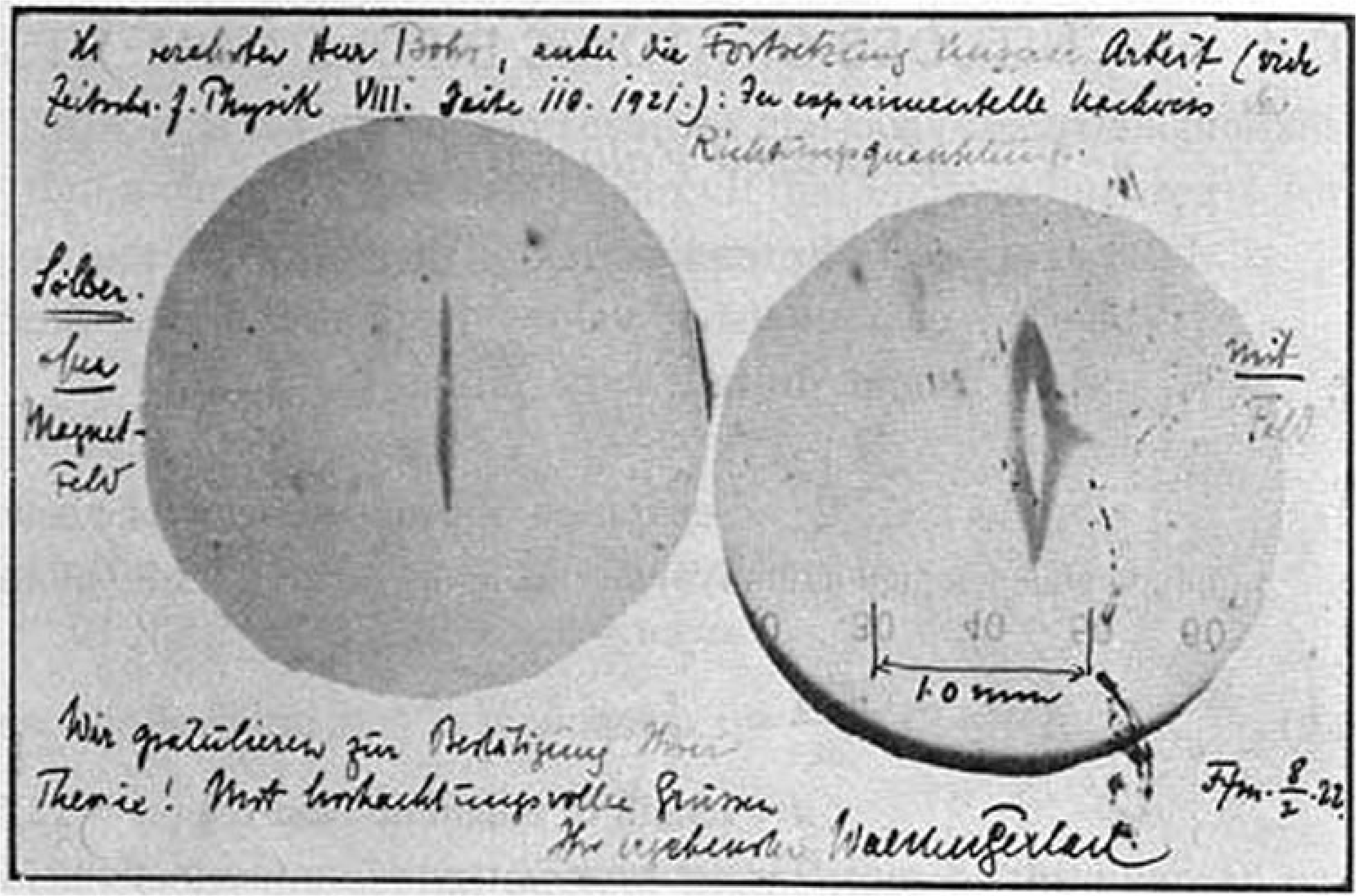


Otto Stern (1888-1969)

Walter Gerlach (1889-1979)

Walther Gerlach, Otto Stern (1922). "Das magnetische Moment des Silberatoms". *Zeitschrift für Physik A Hadrons and Nuclei* **9**.

Stern-Gerlach Experiment



Hochverehrter Herr Bohr, anbei die Fortsetzung unserer Arbeit (vide Zeitschr. f. Physik VIII. Seite 110. 1921): Der experimentelle Nachweis der Richtungsquantelung.

Silber. Ohne Magnet-Feld

Mit Feld

Wir gratulieren zur Bestätigung Ihrer Theorie! Mit hochachtungsvollen Grüssen Ihr ergebenster Walther Gerlach

Ffm. $\frac{8}{2}$.22.

Gerlach's postcard, dated 8 February 1922, to Niels Bohr. It shows a photograph of the beam splitting, with the message, in translation: "Attached [is] the experimental proof of directional quantization. We congratulate [you] on the confirmation of your theory." (Physics Today December 2003)

Quantum Numbers of Electron Orbitals

- Electrons bound to a nucleus move in orbits identified by quantum numbers (from solution of Schrödinger's equation):

n	1…	principal quantum number, identifies the "shell"
ℓ	0..n-1	angular momentum q. #, type of orbital, e.g. s,p,d,f
m_ℓ	$-\ell..\ell$	magnetic quantum number
m_s	+½ or -½	spin quantum number

For example, the electron orbiting the hydrogen nucleus (in the ground state) has:

$n=1,\ \ell=0,\ m_\ell=0,\ m_s=+½$

In spectroscopic notation: $1s^1$

<u>Spectroscopic notation</u>

$\ell=0$	s
$\ell=1$	p
$\ell=2$	d
$\ell=3$	f

n ℓ # of electrons in orbital

$1s^2 2s^2$

Spin and Orbital Magnetic Moment

- Total orbital magnetic moment (sum over all electron orbitals)

$$m_{tot\_orbital} = \mu_B \sum m_\ell$$

- Total spin magnetic moment

$$m_{tot\_spin} = 2\mu_B \sum m_s$$

full orbitals: no net moment

E.g. Iron:

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$$

How to fill remaining d orbitals (ℓ=2) 6 electrons for 10 spots:

	$m_\ell=-2$	$m_\ell=-1$	$m_\ell=0$	$m_\ell=+1$	$m_\ell=+2$
$m_s=+½$	↑	↑	↑	↑	↑
$m_s=-½$					↓

$$m_{tot\_orbital} = 2\mu_B \qquad m_{tot\_spin} = 4\mu_B$$

Hund's rules:

Maximize Σm_s

Then maximize Σm_ℓ

PERIODIC TABLE

Atomic Properties of the Elements

NIST

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Physics Laboratory
physics.nist.gov

Standard Reference Data Group
www.nist.gov/srd

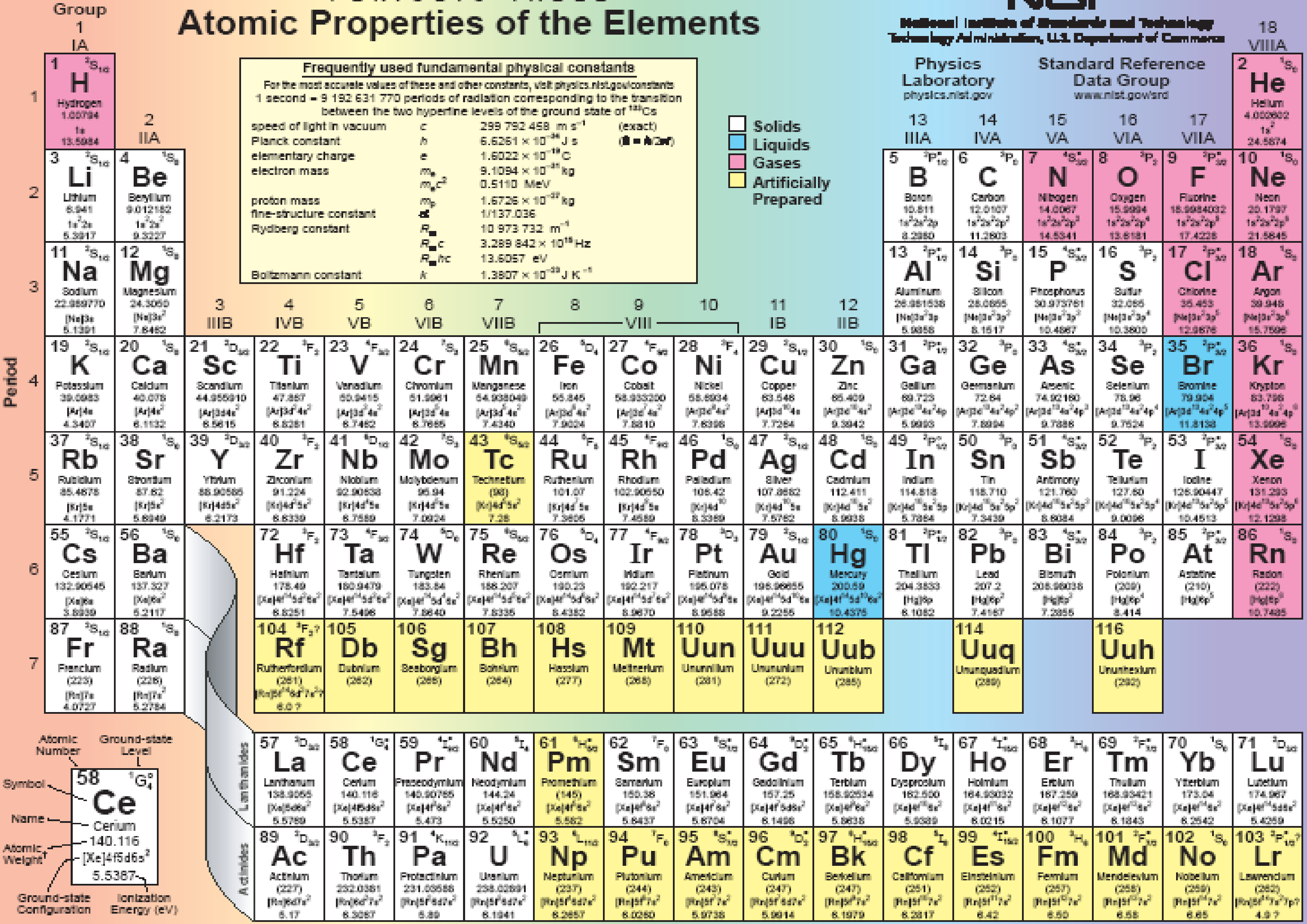
Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.6261×10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.6022×10^{-19} C	
electron mass	m_e	9.1094×10^{-31} kg	
	$m_e c^2$	0.5110 MeV	
proton mass	m_p	1.6726×10^{-27} kg	
fine-structure constant	α	1/137.036	
Rydberg constant	R_∞	10 973 732 m^{-1}	
	$R_\infty c$	$3.289\,842 \times 10^{15}$ Hz	
	$R_\infty hc$	13.6057 eV	
Boltzmann constant	k	1.3807×10^{-23} J K^{-1}	

- ⬜ Solids
- 🟦 Liquids
- 🟥 Gases
- 🟨 Artificially Prepared



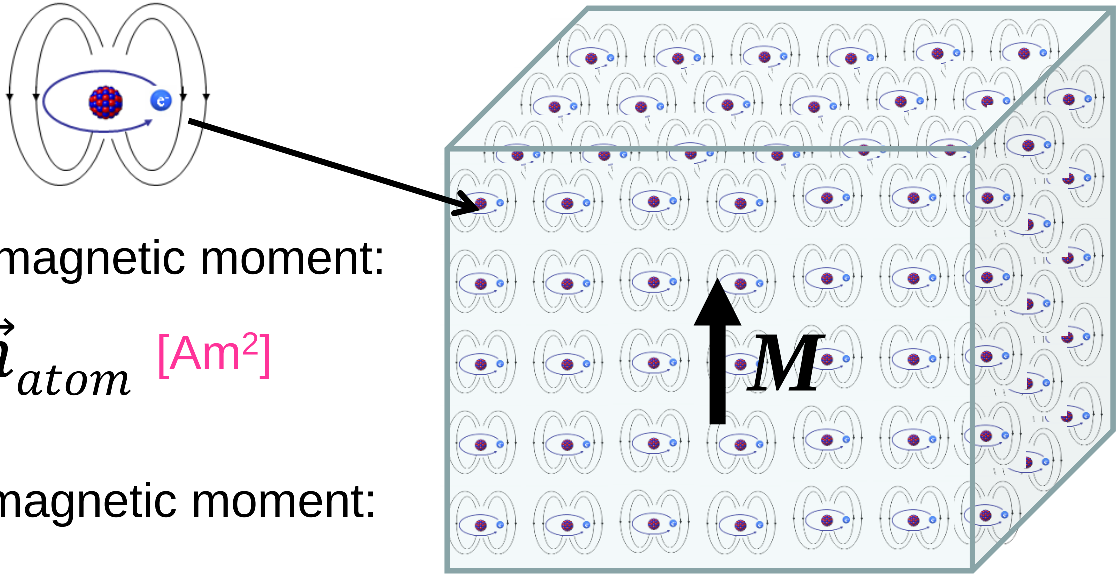
Period	Group	Atomic Number	Symbol	Name	Ground-state Level	Atomic Weight†	Ground-state Configuration	Ionization Energy (eV)
1	1 IA	1	H	Hydrogen	$^2S_{1/2}$	1.00794	$1s$	13.5984
1	18 VIIIA	2	He	Helium	$^1S_0$	4.002602	$1s^2$	24.5874
2	1 IA	3	Li	Lithium	$^2S_{1/2}$	6.941	$1s^2 2s$	5.3917
2	2 IIA	4	Be	Beryllium	$^1S_0$	9.012182	$1s^2 2s^2$	9.3227
2	13 IIIA	5	B	Boron	$^2P^\circ_{1/2}$	10.811	$1s^2 2s^2 2p$	8.2980
2	14 IVA	6	C	Carbon	$^3P_0$	12.0107	$1s^2 2s^2 2p^2$	11.2603
2	15 VA	7	N	Nitrogen	$^4S^\circ_{3/2}$	14.0067	$1s^2 2s^2 2p^3$	14.5341
2	16 VIA	8	O	Oxygen	$^3P_2$	15.9994	$1s^2 2s^2 2p^4$	13.6181
2	17 VIIA	9	F	Fluorine	$^2P^\circ_{3/2}$	18.9984032	$1s^2 2s^2 2p^5$	17.4228
2	18 VIIIA	10	Ne	Neon	$^1S_0$	20.1797	$1s^2 2s^2 2p^6$	21.5645
3	1 IA	11	Na	Sodium	$^2S_{1/2}$	22.989770	[Ne]$3s$	5.1391
3	2 IIA	12	Mg	Magnesium	$^1S_0$	24.3050	[Ne]$3s^2$	7.6462
3	13 IIIA	13	Al	Aluminum	$^2P^\circ_{1/2}$	26.981538	[Ne]$3s^2 3p$	5.9858
3	14 IVA	14	Si	Silicon	$^3P_0$	28.0855	[Ne]$3s^2 3p^2$	8.1517
3	15 VA	15	P	Phosphorus	$^4S^\circ_{3/2}$	30.973761	[Ne]$3s^2 3p^3$	10.4867
3	16 VIA	16	S	Sulfur	$^3P_2$	32.065	[Ne]$3s^2 3p^4$	10.3600
3	17 VIIA	17	Cl	Chlorine	$^2P^\circ_{3/2}$	35.453	[Ne]$3s^2 3p^5$	12.9676
3	18 VIIIA	18	Ar	Argon	$^1S_0$	39.948	[Ne]$3s^2 3p^6$	15.7596
4	1 IA	19	K	Potassium	$^2S_{1/2}$	39.0983	[Ar]$4s$	4.3407
4	2 IIA	20	Ca	Calcium	$^1S_0$	40.078	[Ar]$4s^2$	6.1132
4	3 IIIB	21	Sc	Scandium	$^2D_{3/2}$	44.955910	[Ar]$3d4s^2$	6.5615
4	4 IVB	22	Ti	Titanium	$^3F_2$	47.867	[Ar]$3d^2 4s^2$	6.8281
4	5 VB	23	V	Vanadium	$^4F_{3/2}$	50.9415	[Ar]$3d^3 4s^2$	6.7462
4	6 VIB	24	Cr	Chromium	$^7S_3$	51.9961	[Ar]$3d^5 4s$	6.7665
4	7 VIIB	25	Mn	Manganese	$^6S_{5/2}$	54.938049	[Ar]$3d^5 4s^2$	7.4340
4	8 VIII	26	Fe	Iron	$^5D_4$	55.845	[Ar]$3d^6 4s^2$	7.9024
4	9 VIII	27	Co	Cobalt	$^4F_{9/2}$	58.933200	[Ar]$3d^7 4s^2$	7.8810
4	10 VIII	28	Ni	Nickel	$^3F_4$	58.6934	[Ar]$3d^8 4s^2$	7.6398
4	11 IB	29	Cu	Copper	$^2S_{1/2}$	63.546	[Ar]$3d^{10} 4s$	7.7264
4	12 IIB	30	Zn	Zinc	$^1S_0$	65.409	[Ar]$3d^{10} 4s^2$	9.3942
4	13 IIIA	31	Ga	Gallium	$^2P^\circ_{1/2}$	69.723	[Ar]$3d^{10} 4s^2 4p$	5.9993
4	14 IVA	32	Ge	Germanium	$^3P_0$	72.64	[Ar]$3d^{10} 4s^2 4p^2$	7.8994
4	15 VA	33	As	Arsenic	$^4S^\circ_{3/2}$	74.92160	[Ar]$3d^{10} 4s^2 4p^3$	9.7886
4	16 VIA	34	Se	Selenium	$^3P_2$	78.96	[Ar]$3d^{10} 4s^2 4p^4$	9.7524
4	17 VIIA	35	Br	Bromine	$^2P^\circ_{3/2}$	79.904	[Ar]$3d^{10} 4s^2 4p^5$	11.8138
4	18 VIIIA	36	Kr	Krypton	$^1S_0$	83.798	[Ar]$3d^{10} 4s^2 4p^6$	13.9996
5	1 IA	37	Rb	Rubidium	$^2S_{1/2}$	85.4678	[Kr]$5s$	4.1771
5	2 IIA	38	Sr	Strontium	$^1S_0$	87.62	[Kr]$5s^2$	5.6949
5	3 IIIB	39	Y	Yttrium	$^2D_{3/2}$	88.90585	[Kr]$4d5s^2$	6.2173
5	4 IVB	40	Zr	Zirconium	$^3F_2$	91.224	[Kr]$4d^2 5s^2$	6.6339
5	5 VB	41	Nb	Niobium	$^6D_{1/2}$	92.90638	[Kr]$4d^4 5s$	6.7589
5	6 VIB	42	Mo	Molybdenum	$^7S_3$	95.94	[Kr]$4d^5 5s$	7.0924
5	7 VIIB	43	Tc	Technetium	$^6S_{5/2}$	(98)	[Kr]$4d^5 5s^2$	7.28
5	8 VIII	44	Ru	Ruthenium	$^5F_5$	101.07	[Kr]$4d^7 5s$	7.3605
5	9 VIII	45	Rh	Rhodium	$^4F_{9/2}$	102.90550	[Kr]$4d^8 5s$	7.4589
5	10 VIII	46	Pd	Palladium	$^1S_0$	106.42	[Kr]$4d^{10}$	8.3369
5	11 IB	47	Ag	Silver	$^2S_{1/2}$	107.8682	[Kr]$4d^{10} 5s$	7.5762
5	12 IIB	48	Cd	Cadmium	$^1S_0$	112.411	[Kr]$4d^{10} 5s^2$	8.9938
5	13 IIIA	49	In	Indium	$^2P^\circ_{1/2}$	114.818	[Kr]$4d^{10} 5s^2 5p$	5.7864
5	14 IVA	50	Sn	Tin	$^3P_0$	118.710	[Kr]$4d^{10} 5s^2 5p^2$	7.3439
5	15 VA	51	Sb	Antimony	$^4S^\circ_{3/2}$	121.760	[Kr]$4d^{10} 5s^2 5p^3$	8.6084
5	16 VIA	52	Te	Tellurium	$^3P_2$	127.60	[Kr]$4d^{10} 5s^2 5p^4$	9.0096
5	17 VIIA	53	I	Iodine	$^2P^\circ_{3/2}$	126.90447	[Kr]$4d^{10} 5s^2 5p^5$	10.4513
5	18 VIIIA	54	Xe	Xenon	$^1S_0$	131.293	[Kr]$4d^{10} 5s^2 5p^6$	12.1298
6	1 IA	55	Cs	Cesium	$^2S_{1/2}$	132.90545	[Xe]$6s$	3.8939
6	2 IIA	56	Ba	Barium	$^1S_0$	137.327	[Xe]$6s^2$	5.2117
6	4 IVB	72	Hf	Hafnium	$^3F_2$	178.49	[Xe]$4f^{14} 5d^2 6s^2$	6.8251
6	5 VB	73	Ta	Tantalum	$^4F_{3/2}$	180.9479	[Xe]$4f^{14} 5d^3 6s^2$	7.5496
6	6 VIB	74	W	Tungsten	$^5D_0$	183.84	[Xe]$4f^{14} 5d^4 6s^2$	7.8640
6	7 VIIB	75	Re	Rhenium	$^6S_{5/2}$	186.207	[Xe]$4f^{14} 5d^5 6s^2$	7.8335
6	8 VIII	76	Os	Osmium	$^5D_4$	190.23	[Xe]$4f^{14} 5d^6 6s^2$	8.4382
6	9 VIII	77	Ir	Iridium	$^4F_{9/2}$	192.217	[Xe]$4f^{14} 5d^7 6s^2$	8.9670
6	10 VIII	78	Pt	Platinum	$^3D_3$	195.078	[Xe]$4f^{14} 5d^9 6s$	8.9588
6	11 IB	79	Au	Gold	$^2S_{1/2}$	196.96655	[Xe]$4f^{14} 5d^{10} 6s$	9.2255
6	12 IIB	80	Hg	Mercury	$^1S_0$	200.59	[Xe]$4f^{14} 5d^{10} 6s^2$	10.4375
6	13 IIIA	81	Tl	Thallium	$^2P^\circ_{1/2}$	204.3833	[Hg]$6p$	6.1082
6	14 IVA	82	Pb	Lead	$^3P_0$	207.2	[Hg]$6p^2$	7.4167
6	15 VA	83	Bi	Bismuth	$^4S^\circ_{3/2}$	208.98038	[Hg]$6p^3$	7.2855
6	16 VIA	84	Po	Polonium	$^3P_2$	(209)	[Hg]$6p^4$	8.414
6	17 VIIA	85	At	Astatine	$^2P^\circ_{3/2}$	(210)	[Hg]$6p^5$	
6	18 VIIIA	86	Rn	Radon	$^1S_0$	(222)	[Hg]$6p^6$	10.7485
7	1 IA	87	Fr	Francium	$^2S_{1/2}$	(223)	[Rn]$7s$	4.0727
7	2 IIA	88	Ra	Radium	$^1S_0$	(226)	[Rn]$7s^2$	5.2784
7	4 IVB	104	Rf	Rutherfordium	$^3F_2$?	(261)	[Rn]$5f^{14} 6d^2 7s^2$?	6.0 ?
7	5 VB	105	Db	Dubnium		(262)		
7	6 VIB	106	Sg	Seaborgium		(266)		
7	7 VIIB	107	Bh	Bohrium		(264)		
7	8 VIII	108	Hs	Hassium		(277)		
7	9 VIII	109	Mt	Meitnerium		(268)		
7	10 VIII	110	Uun	Ununnilium		(281)		
7	11 IB	111	Uuu	Unununium		(272)		
7	12 IIB	112	Uub	Ununbium		(285)		
7	14 IVA	114	Uuq	Ununquadium		(289)		
7	16 VIA	116	Uuh	Ununhexium		(292)		
Lanthanides		57	La	Lanthanum	$^2D_{3/2}$	138.9055	[Xe]$5d6s^2$	5.5769
Lanthanides		58	Ce	Cerium	$^1G^\circ_4$	140.116	[Xe]$4f5d6s^2$	5.5387
Lanthanides		59	Pr	Praseodymium	$^4I^\circ_{9/2}$	140.90765	[Xe]$4f^3 6s^2$	5.473
Lanthanides		60	Nd	Neodymium	$^5I_4$	144.24	[Xe]$4f^4 6s^2$	5.5250
Lanthanides		61	Pm	Promethium	$^6H^\circ_{5/2}$	(145)	[Xe]$4f^5 6s^2$	5.582
Lanthanides		62	Sm	Samarium	$^7F_0$	150.36	[Xe]$4f^6 6s^2$	5.6437
Lanthanides		63	Eu	Europium	$^8S^\circ_{7/2}$	151.964	[Xe]$4f^7 6s^2$	5.6704
Lanthanides		64	Gd	Gadolinium	$^9D^\circ_2$	157.25	[Xe]$4f^7 5d6s^2$	6.1498
Lanthanides		65	Tb	Terbium	$^6H^\circ_{15/2}$	158.92534	[Xe]$4f^9 6s^2$	5.8638
Lanthanides		66	Dy	Dysprosium	$^5I_8$	162.500	[Xe]$4f^{10} 6s^2$	5.9389
Lanthanides		67	Ho	Holmium	$^4I^\circ_{15/2}$	164.93032	[Xe]$4f^{11} 6s^2$	6.0215
Lanthanides		68	Er	Erbium	$^3H_6$	167.259	[Xe]$4f^{12} 6s^2$	6.1077
Lanthanides		69	Tm	Thulium	$^2F^\circ_{7/2}$	168.93421	[Xe]$4f^{13} 6s^2$	6.1843
Lanthanides		70	Yb	Ytterbium	$^1S_0$	173.04	[Xe]$4f^{14} 6s^2$	6.2542
Lanthanides		71	Lu	Lutetium	$^2D_{3/2}$	174.967	[Xe]$4f^{14} 5d6s^2$	5.4259
Actinides		89	Ac	Actinium	$^2D_{3/2}$	(227)	[Rn]$6d7s^2$	5.17
Actinides		90	Th	Thorium	$^3F_2$	232.0381	[Rn]$6d^2 7s^2$	6.3067
Actinides		91	Pa	Protactinium	$^4K_{11/2}$	231.03588	[Rn]$5f^2 6d7s^2$	5.89
Actinides		92	U	Uranium	$^5L^\circ_6$	238.02891	[Rn]$5f^3 6d7s^2$	6.1941
Actinides		93	Np	Neptunium	$^6L_{11/2}$	(237)	[Rn]$5f^4 6d7s^2$	6.2657
Actinides		94	Pu	Plutonium	$^7F_0$	(244)	[Rn]$5f^6 7s^2$	6.0260
Actinides		95	Am	Americium	$^8S^\circ_{7/2}$	(243)	[Rn]$5f^7 7s^2$	5.9738
Actinides		96	Cm	Curium	$^9D^\circ_2$	(247)	[Rn]$5f^7 6d7s^2$	5.9914
Actinides		97	Bk	Berkelium	$^6H^\circ_{15/2}$	(247)	[Rn]$5f^9 7s^2$	6.1979
Actinides		98	Cf	Californium	$^5I_8$	(251)	[Rn]$5f^{10} 7s^2$	6.2817
Actinides		99	Es	Einsteinium	$^4I^\circ_{15/2}$	(252)	[Rn]$5f^{11} 7s^2$	6.42
Actinides		100	Fm	Fermium	$^3H_6$	(257)	[Rn]$5f^{12} 7s^2$	6.50
Actinides		101	Md	Mendelevium	$^2F^\circ_{7/2}$	(258)	[Rn]$5f^{13} 7s^2$	6.58
Actinides		102	No	Nobelium	$^1S_0$	(259)	[Rn]$5f^{14} 7s^2$	6.65
Actinides		103	Lr	Lawrencium	$^2P^\circ_{1/2}$?	(262)	[Rn]$5f^{14} 7s^2 7p$?	4.9 ?

†Based upon ^{12}C. () indicates the mass number of the most stable isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2003)

Magnetization, $\vec{M}$



Atomic magnetic moment:

$$\vec{m}_{atom}$$ [Am²]

What is H here?

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom}$$ [Am²]

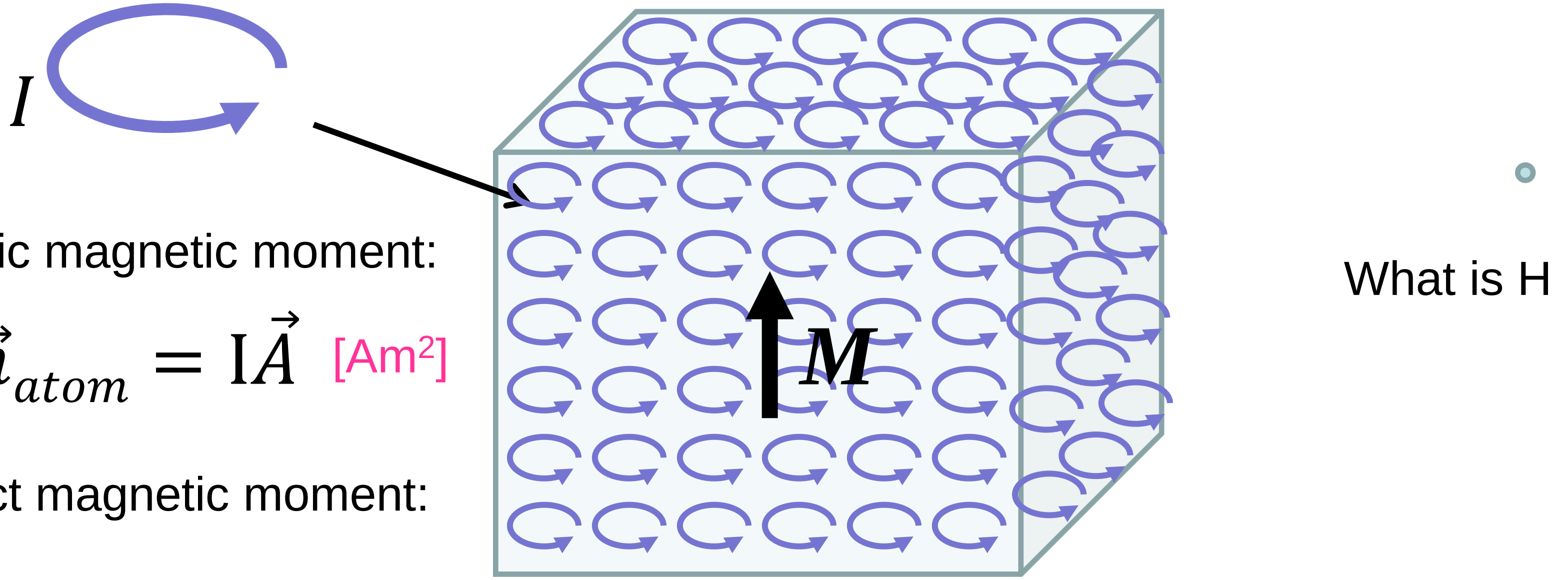
Atomic density:

$$d = \frac{N}{vol}$$ [1/m³]

Magnetic moment per unit volume:

$$\vec{M} \stackrel{\text{def}}{=} \frac{\vec{m}_{obj.}}{vol} = \vec{m}_{atom} d$$ [A/m]

Equivalent Loop Currents



What is H here?

Atomic magnetic moment:

$$\vec{m}_{atom} = \mathrm{I}\vec{A}$$ [Am²]

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom}$$ [Am²]

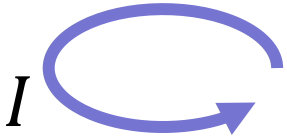
Atomic density:

$$d = \frac{N}{vol}$$ [1/m³]

Magnetic moment per unit volume:

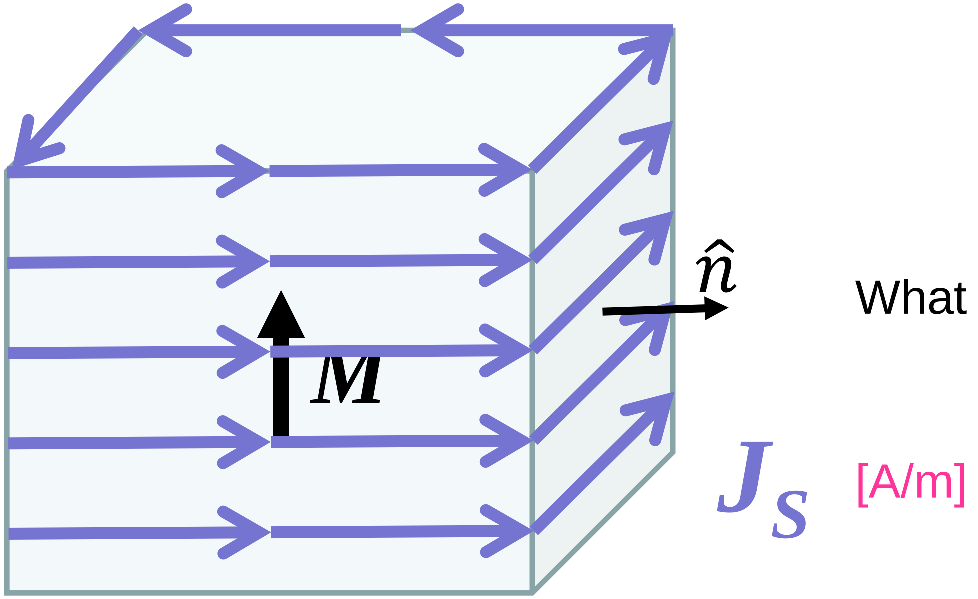
$$\vec{M} \overset{\text{def}}{=} \frac{\vec{m}_{obj.}}{vol} = \vec{m}_{atom} d$$ [A/m]

Equivalent Surface Current



Atomic magnetic moment:

$$\vec{m}_{atom} = \mathrm{I}\vec{A} \quad \text{[Am}^2\text{]}$$



What is H here?

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \quad \text{[Am}^2\text{]}$$

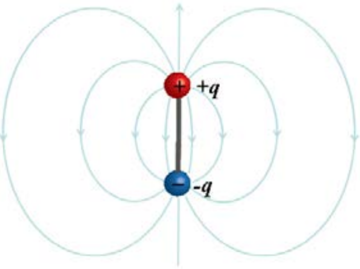
Atomic density:

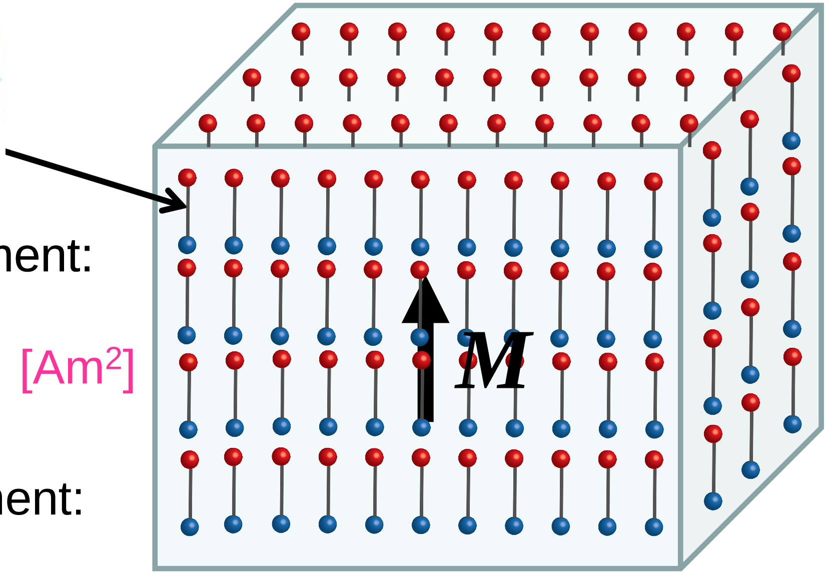
$$d = \frac{N}{vol} \quad \text{[1/m}^3\text{]}$$

Equivalent surface current:

$$\vec{J}_S = \vec{M} \times \hat{n} \quad \text{[A/m]}$$

Magnetic Pole Model





What is H here?

Atomic magnetic moment:

$$\vec{m}_{atom} = q_m \vec{d} \quad [\mathrm{Am^2}]$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \quad [\mathrm{Am^2}]$$

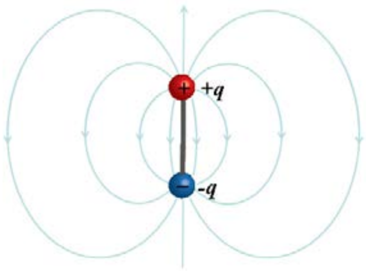
Atomic density:

$$d = \frac{N}{vol} \quad [1/\mathrm{m^3}]$$

Magnetic moment per unit volume:

$$\vec{M} \overset{\text{def}}{=} \frac{\vec{m}}{vol} \quad [\mathrm{A/m}]$$

Equivalent Surface Pole Density



Atomic magnetic moment:

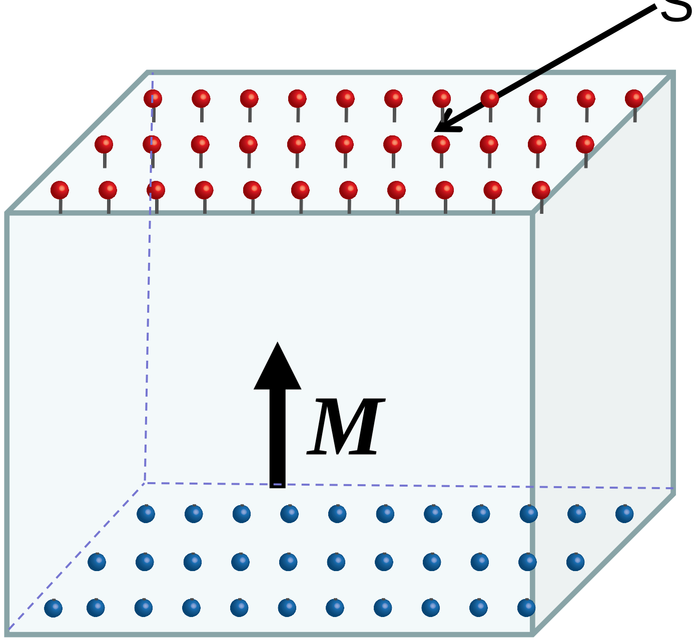
$$\vec{m}_{atom} = q_m \vec{d} \quad [\mathrm{Am}^2]$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \quad [\mathrm{Am}^2]$$

Atomic density:

$$d = \frac{N}{vol} \quad [1/\mathrm{m}^3]$$



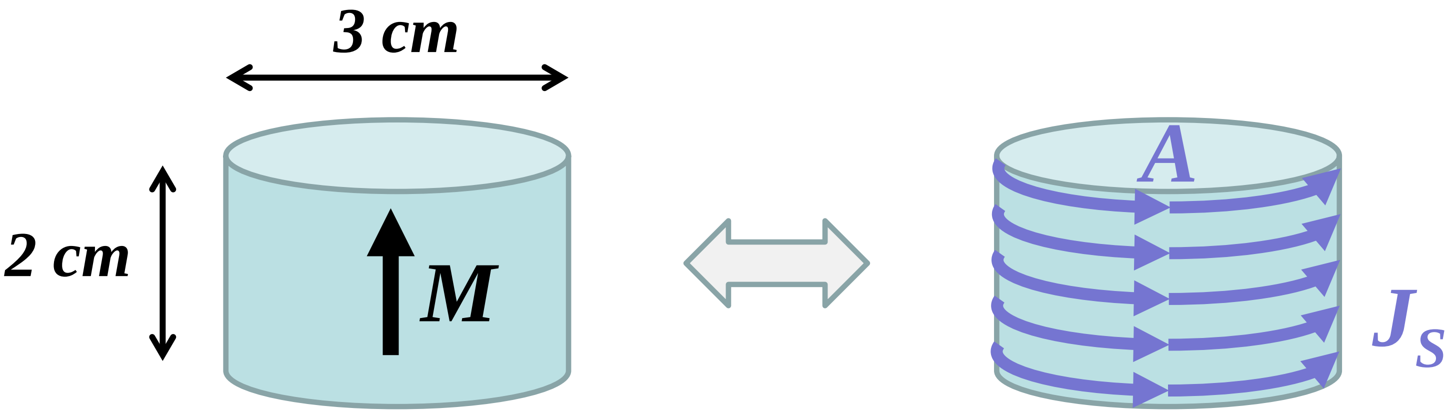
Surface charge density:

$$\rho_{ms} = \frac{q_m}{area} \quad \frac{[\mathrm{Am}]}{[\mathrm{m}^2]}$$

Magnetic pole density:

$$\rho_{ms} = \vec{M} \cdot \hat{n} \quad [\mathrm{A/m}]$$

Example: NdFeB Cylinder



$M = 10^6$ [A/m]

$vol \simeq 14 \cdot 10^{-6}$ [m^3]

$m = 14$ [Am^2]

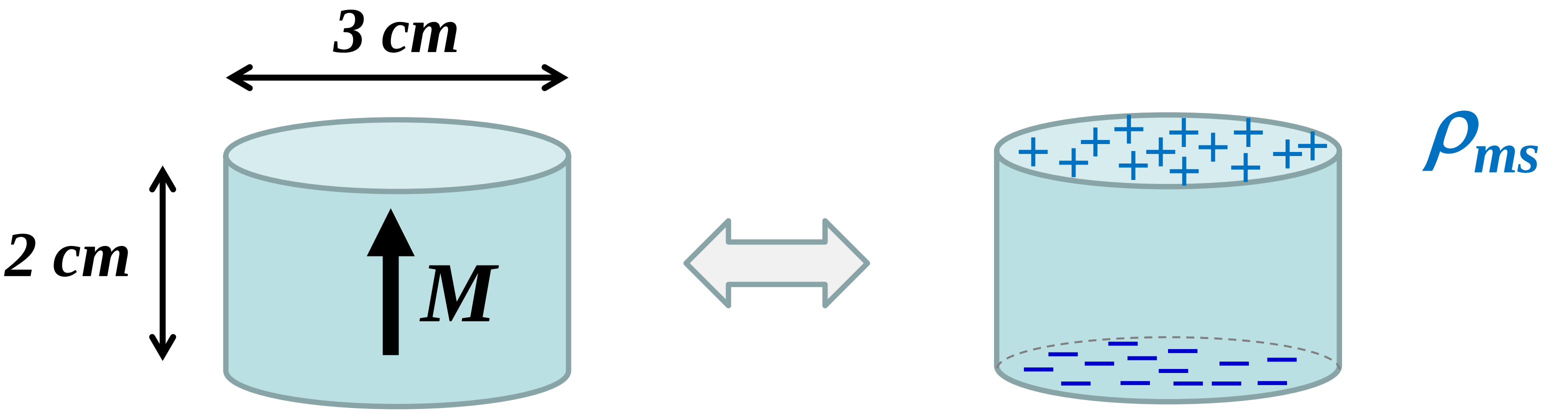
$J_S = 10^6$ [A/m]

$I = 20000$ [A]

$A \simeq 7 \cdot 10^{-4}$ [m^2]

$m = IA = 14$ [Am^2]

Example: NdFeB Cylinder



$M = 10^6$ [A/m]

$vol \simeq 14 \cdot 10^{-6}$ [m^3]

$m = 14$ [Am^2]

$\rho_{ms} = 10^6$ [A/m]

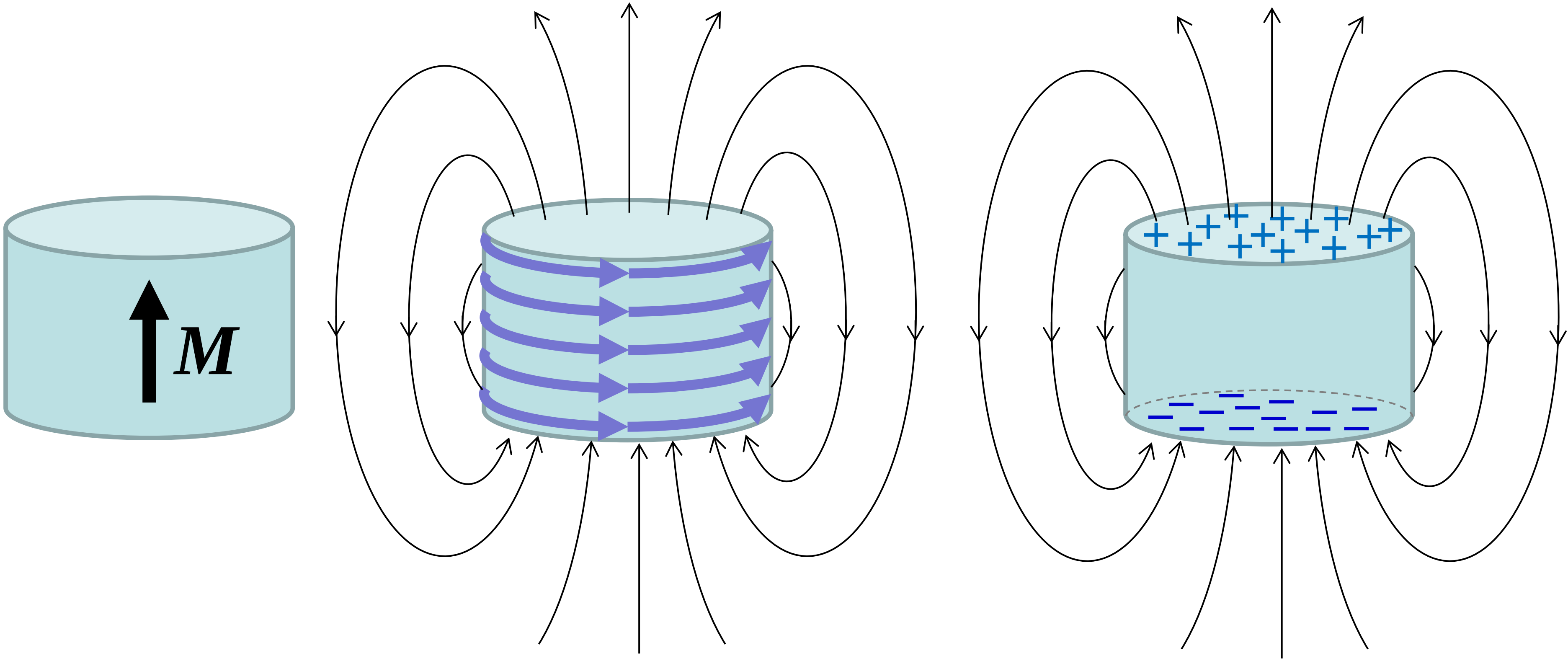
$A \simeq 7 \cdot 10^{-4}$ [m^2]

$q_m = 700$ [Am]

$d = 2 \cdot 10^{-2}$ [m]

$m = q_m d = 14$ [Am^2]

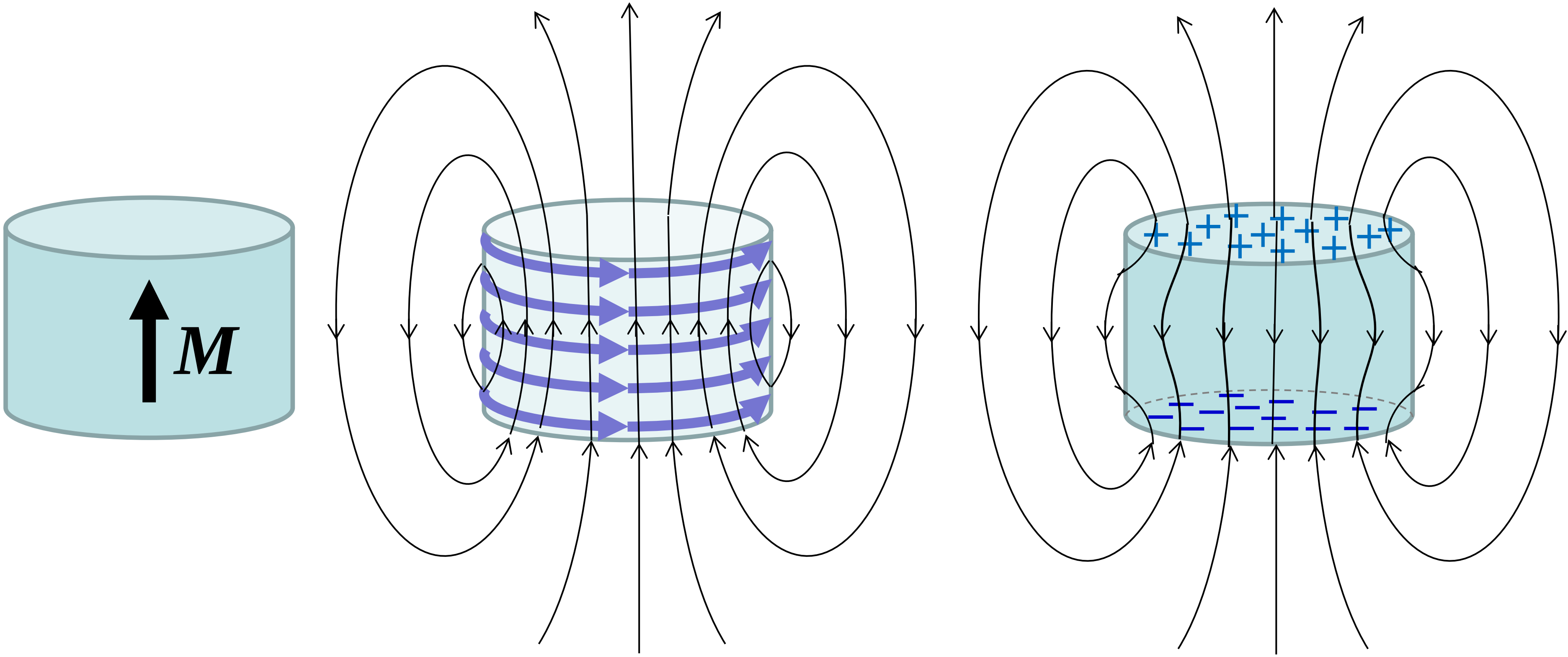
Example: NdFeB Cylinder



Result is the <u>same</u> <u>external</u> to magnetic material.

Hint: we are “far” away from the dipoles!!!

Example: NdFeB Cylinder



Result is <u>different</u> <u>inside</u> the magnetic material.

The Constitutive Relation

$$\boxed{\overrightarrow{\boldsymbol{B}} = \boldsymbol{\mu_0}(\overrightarrow{\boldsymbol{H}} + \overrightarrow{\boldsymbol{M}})}$$

$\boldsymbol{B}$ = Magnetic flux density [Tesla]

$\boldsymbol{H}$ = Magnetic field, “Magnetizing force” [A/m]

$\boldsymbol{M}$ = Magnetization [A/m]

$\boldsymbol{\mu_0}$ = Magnetic constant, “Permeability of free space” [Tesla-m/A] [Henry/m] [N/A^2]

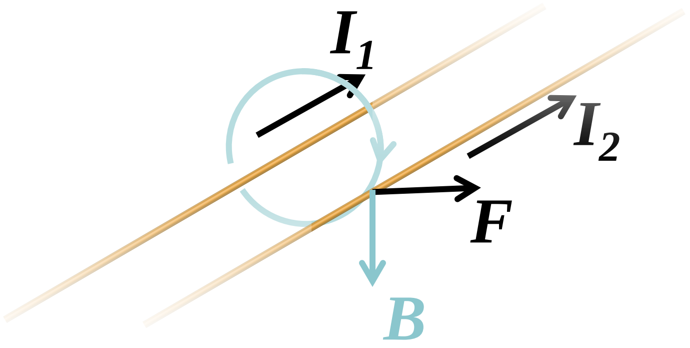
Note, in vacuum, $\boldsymbol{M}$ is zero:

$$\overrightarrow{\boldsymbol{B}} = \boldsymbol{\mu_0}\overrightarrow{\boldsymbol{H}}$$

What is μ_o ?

μ_0 comes from the SI definition of the Ampere:

> The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.



$$\boldsymbol{B} = \boldsymbol{\mu_0 H} = \frac{\boldsymbol{\mu_0 I_1}}{\boldsymbol{2\pi r}}$$

$$\vec{\boldsymbol{F}} = \boldsymbol{I_2}\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

Maxwell's Equations (Magnetostatics)

Differential form

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

Integral form

$$\oiint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{H} \cdot d\vec{l} = I$$

With:

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

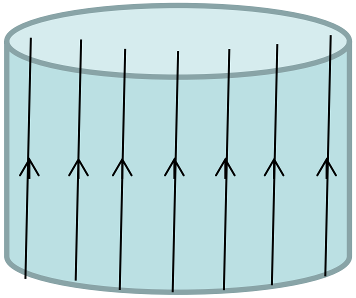
$$\nabla \cdot \vec{B} = \mu_0(\nabla \cdot \vec{H} + \nabla \cdot \vec{M})$$

$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M} = \rho_m$$

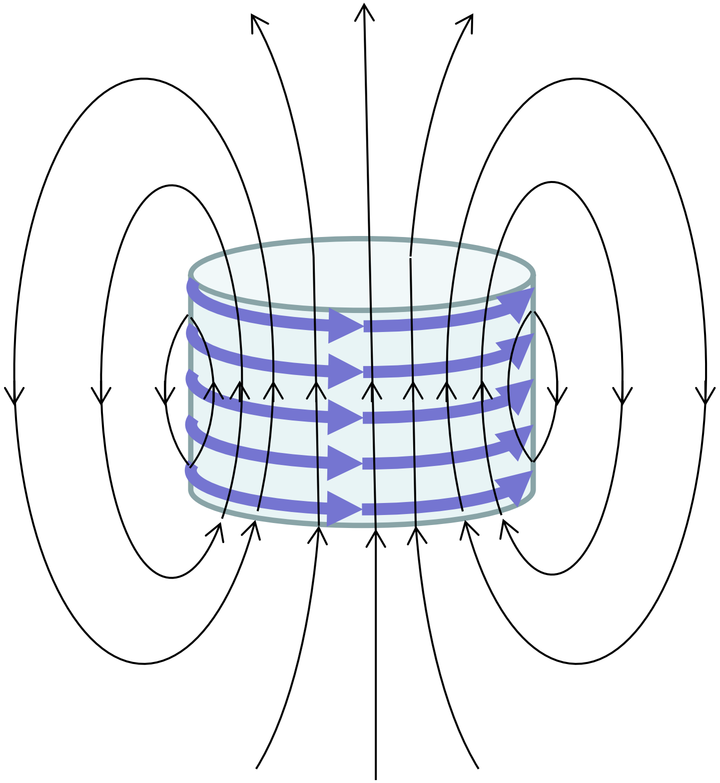
$[A/m^2]$

Magnetic charge density
i.e. magnetic charge/volume

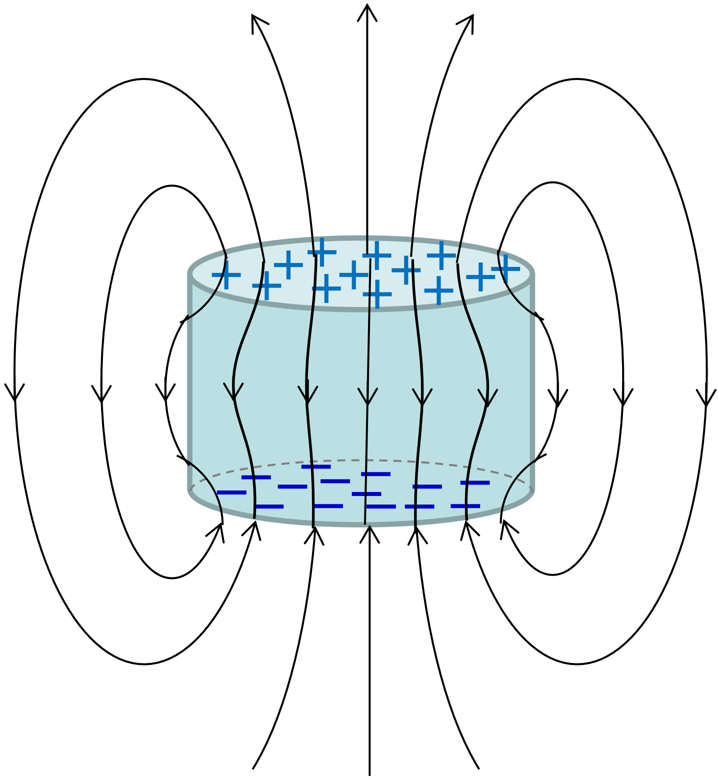
Example: NdFeB Cylinder



M



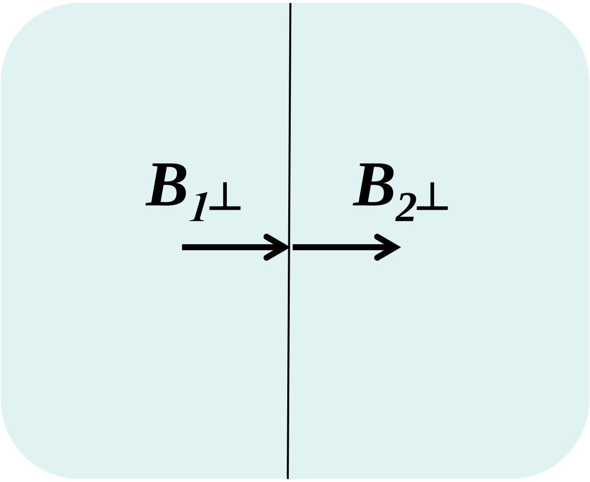
B



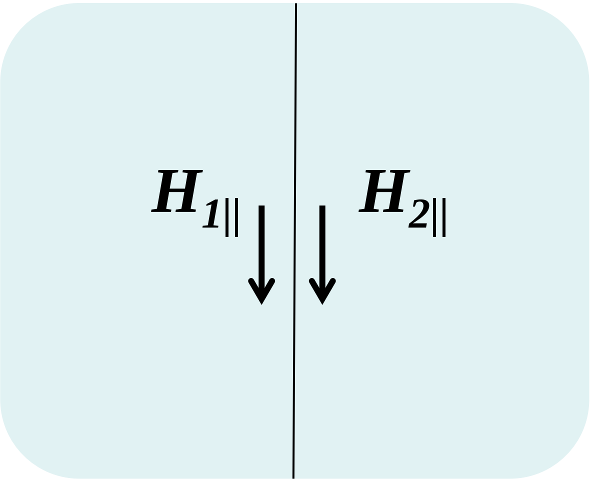
H

$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}(\vec{\boldsymbol{H}} + \vec{\boldsymbol{M}})$$

Boundary Conditions

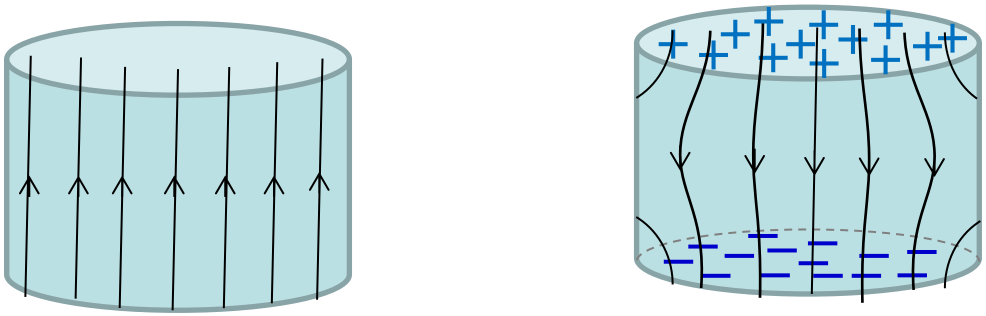


The perpendicular component of B is continuous across a boundary



The transverse component of H is continuous across a boundary (unless there is a <u>true</u> current on the boundary)

Demagnetizing Fields



$\mathbf{M}$

$\mathbf{H_d}$

$$\boldsymbol{H_d} \propto \boldsymbol{M}$$

$$\boldsymbol{H_d} = -\boldsymbol{N}\boldsymbol{M}$$

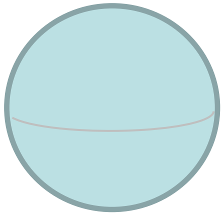
Demagnetizing Factor

Demagnetizing Factors: Special Cases

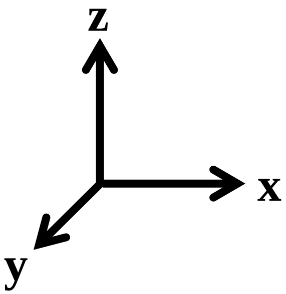
$$\boldsymbol{H_d = -NM}$$

$$\boldsymbol{N_x + N_y + N_z = 1}$$

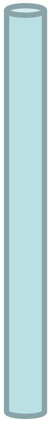
Sphere



$$\boldsymbol{N_x = N_y = N_z = \frac{1}{3}}$$



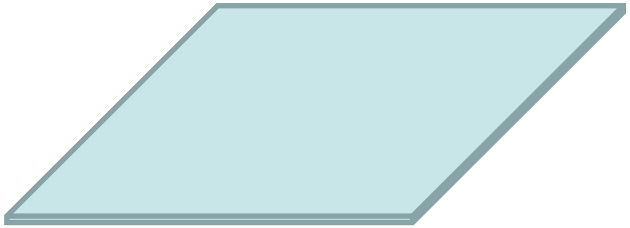
Long Rod



$$\boldsymbol{N_x = N_y = \frac{1}{2}}$$

$$\boldsymbol{N_z = 0}$$

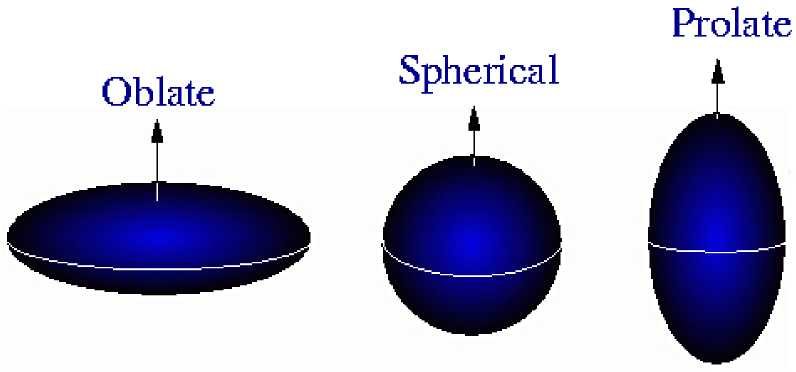
Thin Sheet



$$\boldsymbol{N_x = N_y = 0}$$

$$\boldsymbol{N_z = 1}$$

Demagnetizing Factors



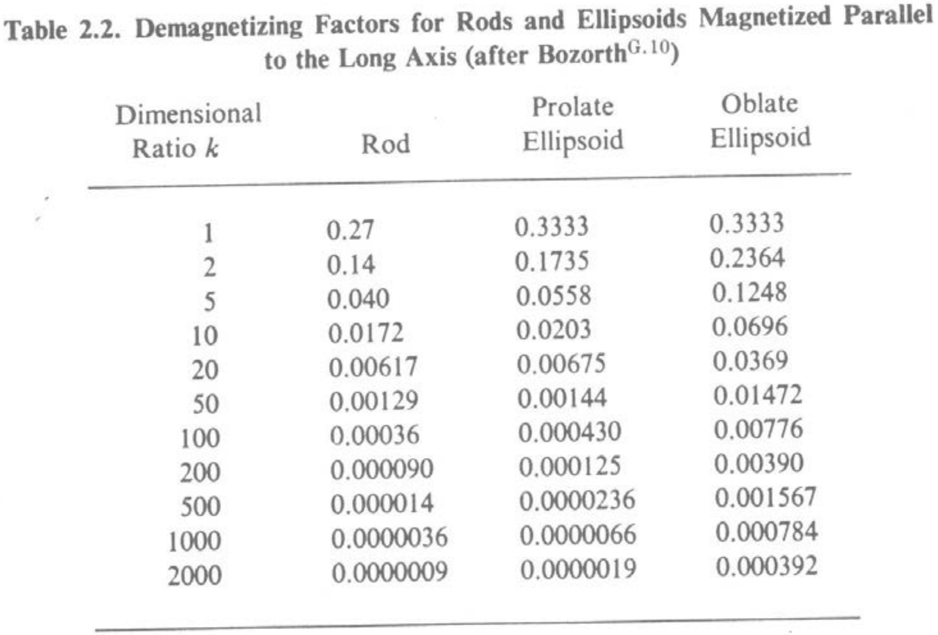


Table 2.2. Demagnetizing Factors for Rods and Ellipsoids Magnetized Parallel to the Long Axis (after Bozorth$^{G.10}$)

Dimensional Ratio *k*	Rod	Prolate Ellipsoid	Oblate Ellipsoid
1	0.27	0.3333	0.3333
2	0.14	0.1735	0.2364
5	0.040	0.0558	0.1248
10	0.0172	0.0203	0.0696
20	0.00617	0.00675	0.0369
50	0.00129	0.00144	0.01472
100	0.00036	0.000430	0.00776
200	0.000090	0.000125	0.00390
500	0.000014	0.0000236	0.001567
1000	0.0000036	0.0000066	0.000784
2000	0.0000009	0.0000019	0.000392

“Demag” fields in ellipsoids of revolution are uniform.

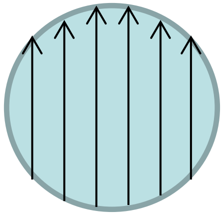
$$\boldsymbol{H_d = -NM}$$

$$\boldsymbol{N_x + N_y + N_z = 1}$$

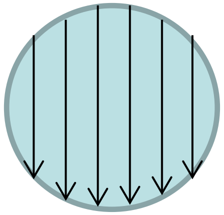
Example: Ba-Ferrite Sphere

Barrium Ferrite:

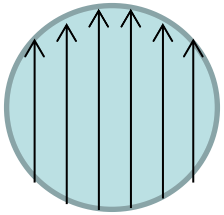
$\boldsymbol{M \sim 150}$ [kA/m]



M



H



B

$$\boldsymbol{N_x = N_y = N_z = \frac{1}{3}}$$

$$\boldsymbol{H_d = -NM}$$

$$\boldsymbol{H_d = -50}$$ [kA/m]

$$\boldsymbol{\vec{B} = \mu_0(\vec{H} + \vec{M})}$$

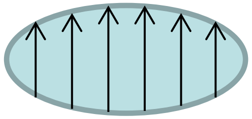
$$\boldsymbol{\vec{B} = \mu_0(100}$$ [kA/m] $)$

$$\boldsymbol{\vec{B} = 0.126}$$ [Tesla]

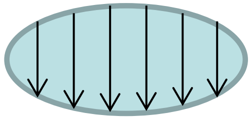
Example: Ba-Ferrite Ellipsoid

Barrium Ferrite:

$\boldsymbol{M} \sim \mathbf{150}$ [kA/m]



M

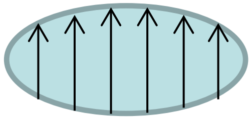


H

$$\boldsymbol{N_x = N_y = 0.45}$$

$$\boldsymbol{H_d = -NM}$$

$$\boldsymbol{H_d = -67.5} \text{ [kA/m]}$$



B

$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}(\vec{\boldsymbol{H}} + \vec{\boldsymbol{M}})$$

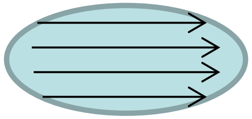
$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}(\mathbf{82.5} \text{ [kA/m] })$$

$$\vec{\boldsymbol{B}} = \mathbf{0.104} \text{ [Tesla]}$$

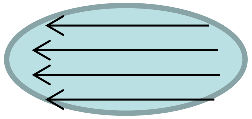
Example: Ba-Ferrite Ellipsoid

Barrium Ferrite:

$\boldsymbol{M} \sim \mathbf{150}$ [kA/m]



M

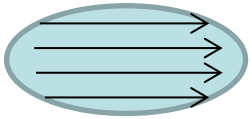


H

$$\boldsymbol{N_z = 0.1}$$

$$\boldsymbol{H_d = -NM}$$

$$\boldsymbol{H_d = -15}$$ [kA/m]



B

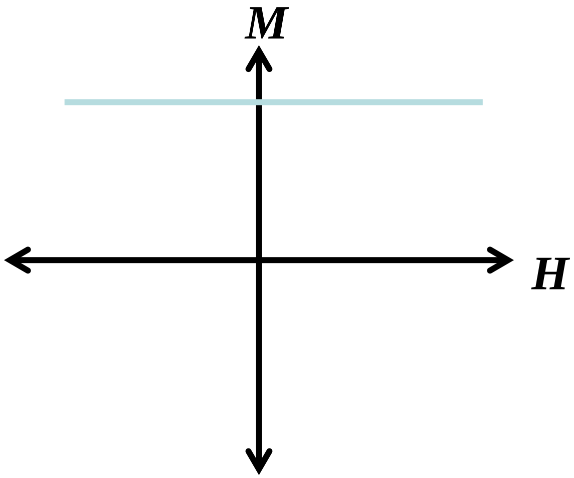
$$\boldsymbol{\vec{B} = \mu_0(\vec{H} + \vec{M})}$$

$$\boldsymbol{\vec{B} = \mu_0(135}\text{ [kA/m]}\boldsymbol{)}$$

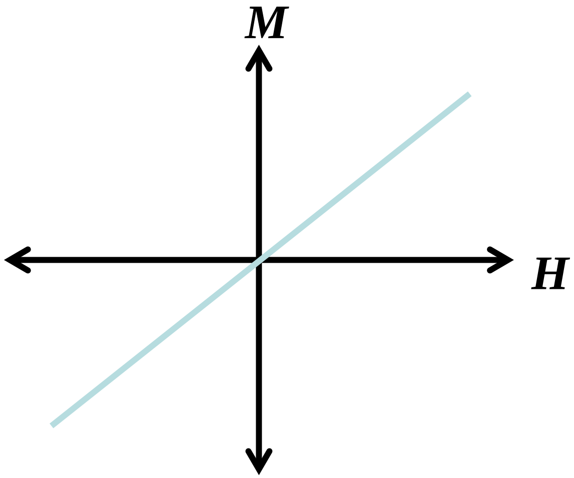
$$\boldsymbol{\vec{B} = 0.17}$$ [Tesla]

Susceptibility and Permeability

Ideal permanent magnet



Ideal, linear soft magnetic material



$$M = \chi H$$

Susceptibility [unitless]

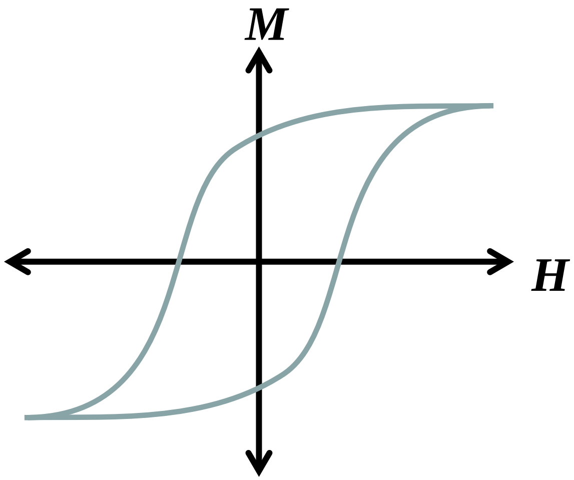
$$\vec{B} = \mu_0(\vec{H} + \chi\vec{H})$$

$$\vec{B} = \mu_0(1 + \chi)\vec{H}$$

$$\vec{B} = \mu_0\mu_r\vec{H}$$

Rel. Permeability [unitless]

Real, useful magnetic material

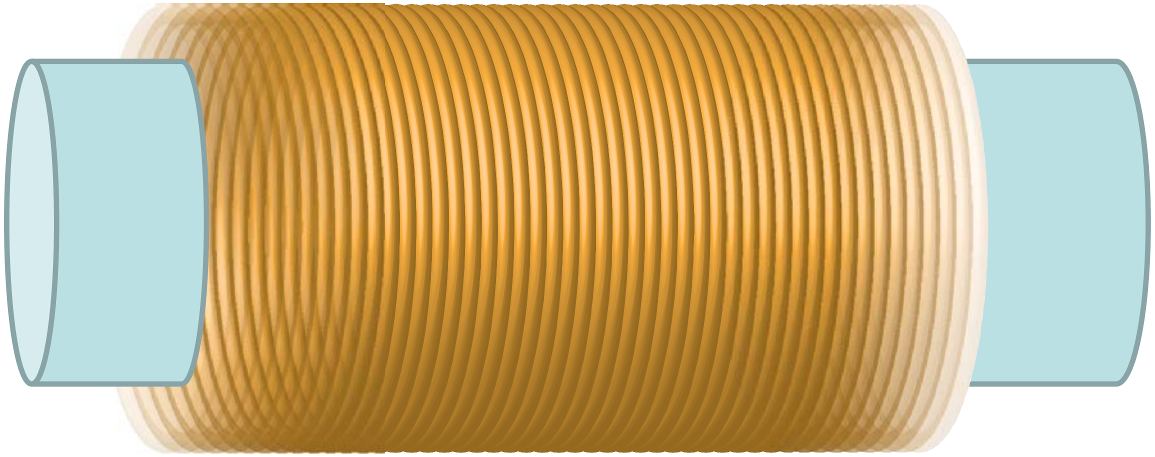


Example, Long Rod in a Long Solenoid

I =1 A

N/L = 1000/m

$\mu_r = 1000 \qquad \chi = 999$



$H = \frac{NI}{L}$ = 1 [kA/m]

$M = \chi H = \chi \frac{NI}{L} = 999$ [kA/m]

$B = \mu_0(H + M) = \mu_0$(1000kA/m)= 1.25 [Tesla]

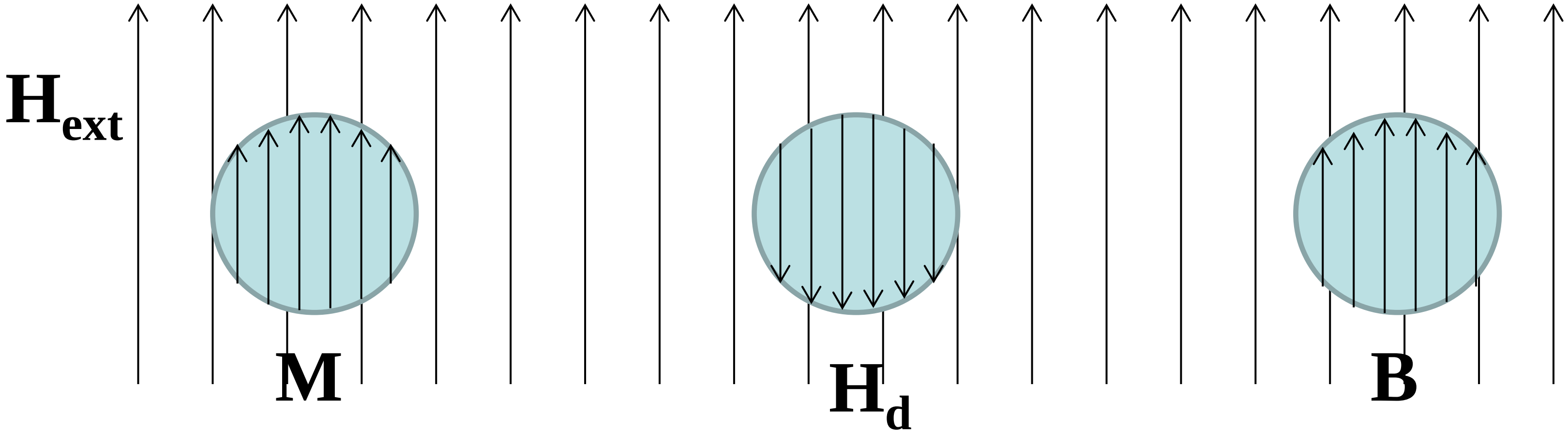
or:

$B = \mu_0 \mu_r H$= 1.25 [Tesla]

Note: without the rod:

$B = \mu_0 H$= 0.00125 [Tesla]

Example, Soft Magnetic Ball in a Field

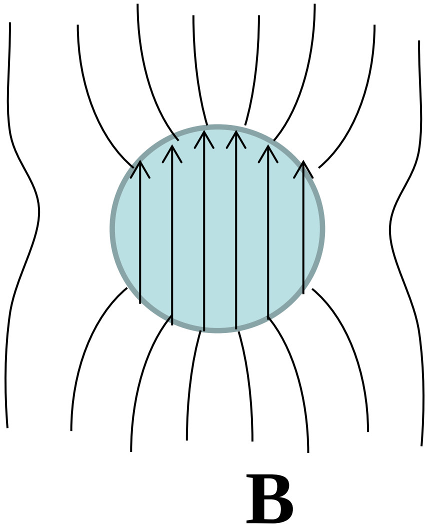


$$M = \chi H = \chi(H_{ext} + H_d) = \chi(H_{ext} - NM)$$

$$M = \frac{\chi}{1 + N\chi} H_{ext}$$

$$\boxed{\chi_{eff} = \frac{\chi}{1 + N\chi} < 3!!!}$$

Example, Soft Magnetic Ball in a Field

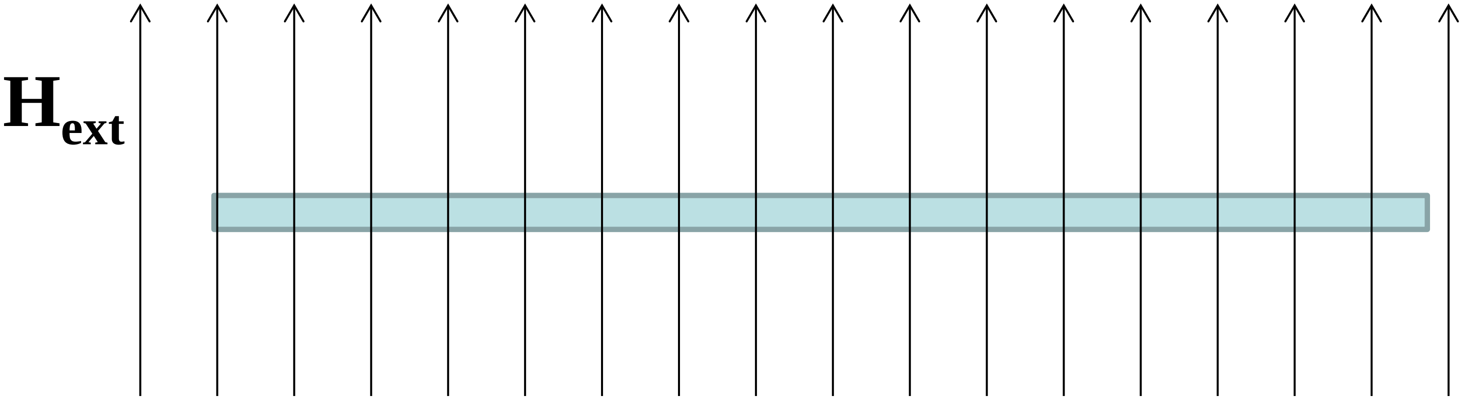


$$\boldsymbol{M} = \boldsymbol{\chi H} = \boldsymbol{\chi}(\boldsymbol{H}_{\boldsymbol{ext}} + \boldsymbol{H}_{\boldsymbol{d}}) = \boldsymbol{\chi}(\boldsymbol{H}_{\boldsymbol{ext}} - \boldsymbol{NM})$$

$$\boldsymbol{M} = \frac{\boldsymbol{\chi}}{\mathbf{1} + \boldsymbol{N\chi}}\boldsymbol{H}_{\boldsymbol{ext}}$$

$$\boxed{\boldsymbol{\chi}_{\boldsymbol{eff}} = \frac{\boldsymbol{\chi}}{\mathbf{1} + \boldsymbol{N\chi}} < \mathbf{3}!!!}$$

Example, Thin Film in a Field



$$\boldsymbol{M} = \boldsymbol{\chi} \boldsymbol{H}_{\boldsymbol{inside}} = \boldsymbol{\chi}(\boldsymbol{H}_{\boldsymbol{ext}} + \boldsymbol{H}_{\boldsymbol{d}}) = \boldsymbol{\chi}(\boldsymbol{H}_{\boldsymbol{ext}} - \boldsymbol{M})$$

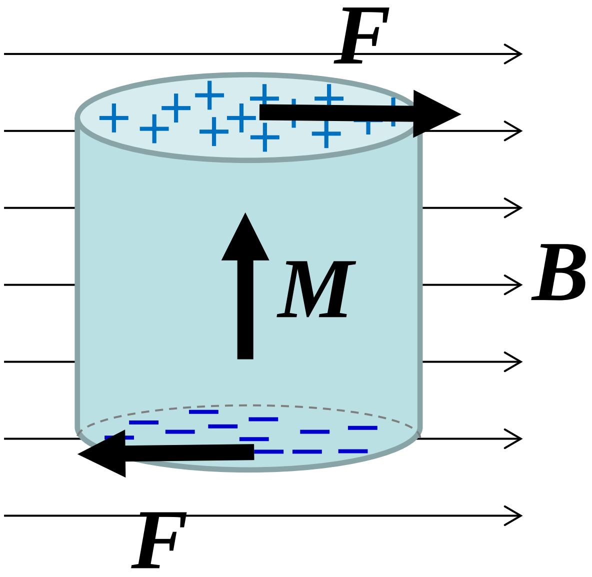
$$\boldsymbol{M} = \frac{\boldsymbol{\chi}}{\mathbf{1} + \boldsymbol{\chi}} \boldsymbol{H}_{\boldsymbol{ext}}$$

$$\boldsymbol{H}_{\boldsymbol{inside}} = \frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{\chi}} \boldsymbol{H}_{\boldsymbol{ext}}$$

$$\boldsymbol{B}_{\boldsymbol{inside}} = B_{\boldsymbol{ext}}$$

Why?

Torque and Zeeman Energy



Force:

$$\vec{F} = q_m \vec{B}$$

Torque:

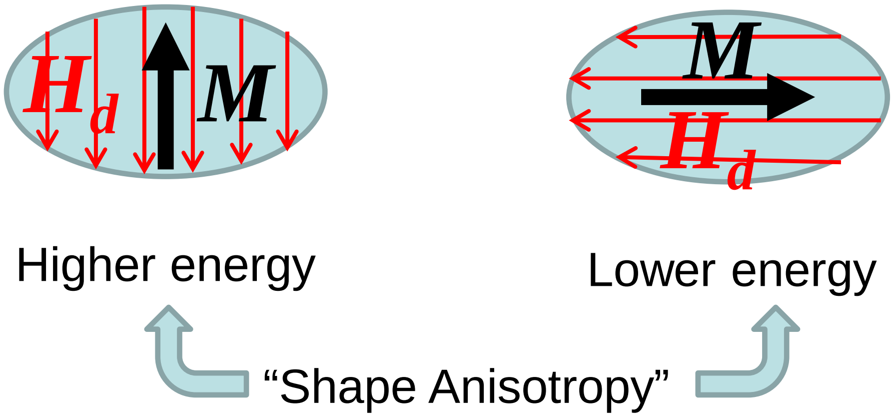
$$\vec{\mathcal{T}} = \vec{m} \times \vec{B}$$

Energy:

$$E = -\vec{m} \cdot \vec{B}$$

Self Energy and Shape Anisotropy

$$E = \frac{1}{2}\iiint_{vol} \mu_0 \vec{M}\cdot\vec{H}\,dv$$

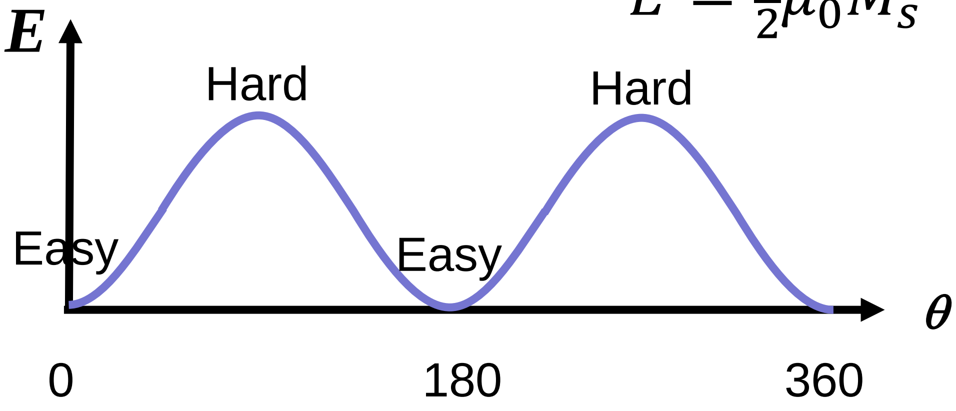


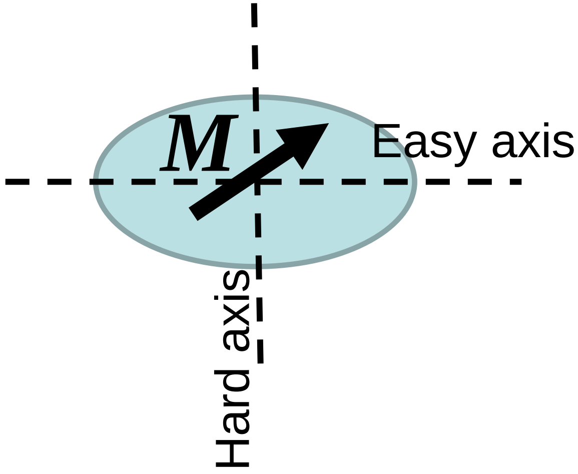
Higher energy

Lower energy

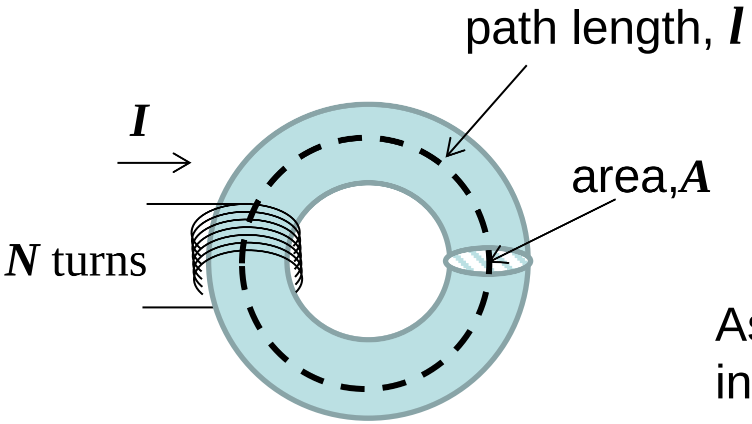
"Shape Anisotropy"

$$E = \frac{1}{2}\mu_0 {M_s}^2 (N_x - N_Y)\sin^2(\theta)$$





Magnetic Circuits



$$\oint \vec{H} \cdot d\vec{l} = NI$$

Assuming flux is uniform and contained inside permeable material:

$$B = \mu_0 \mu_r H = \frac{\phi}{A}$$

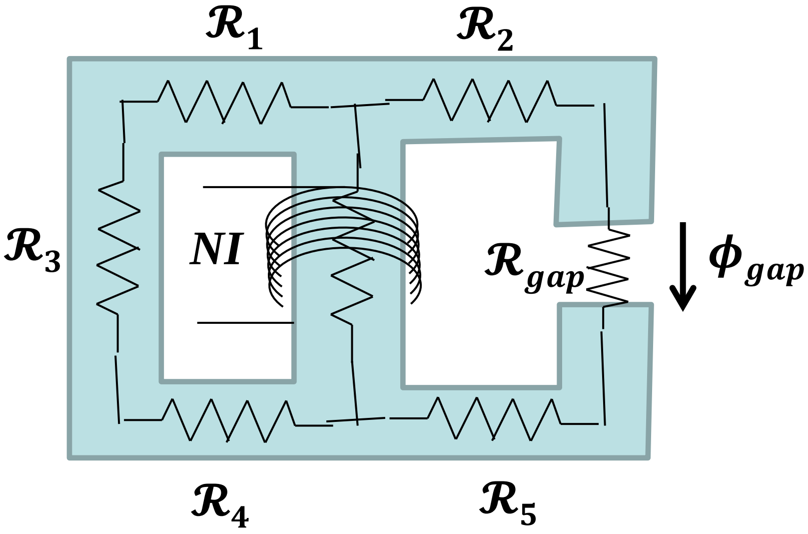
$$NI = \phi\mathcal{R}$$

Magneto-motive force, “MMF”

Flux

Reluctance, $\mathcal{R} = \frac{l}{\mu A}$

Magnetic Circuits



Solve using electric circuit theory.

Units Confusion

CGS Units

$\boldsymbol{B}$	[Gauss]
$\boldsymbol{H}$	[Oersted]
$\boldsymbol{M}$	[emu/cc]
$\boldsymbol{4\pi M}$	[Gauss]

$$\boldsymbol{B = H + 4\pi M}$$

$\boldsymbol{B \cdot H}$ [dyne/cm^3]

$$\boldsymbol{N_x + N_y + N_z = 4\pi}$$

SI Units

$\boldsymbol{B}$: [Tesla]

$\boldsymbol{H}$: [A/m]

$\boldsymbol{M}$: [A/m]

$$\boldsymbol{B = \mu_0 (H + M)}$$

$\boldsymbol{B \cdot H}$ [J/m^3]

$$\boldsymbol{N_x + N_y + N_z = 1}$$

Books on Magnetism

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M. Coey, *Magnetism and Magnetic Materials*, Cambridge University Press, 2010.

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