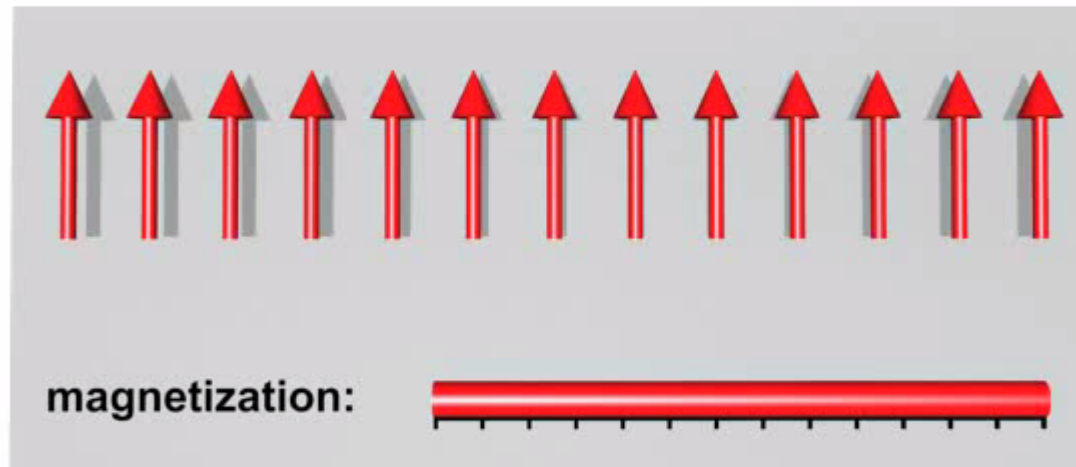


Magnetization Dynamics I: Fundamentals of spin dynamics and Brillouin light scattering

Burkard Hillebrands

Fachbereich Physik and Landesforschungszentrum OPTIMAS,
Technische Universität Kaiserslautern, Germany

Ferromagnetic spin chain: magnon





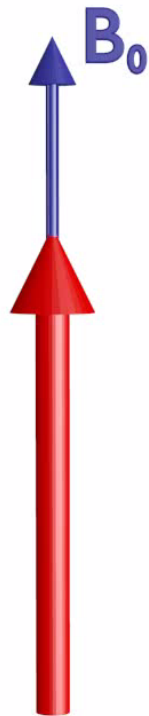
- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves



Basics: Spin Waves

- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

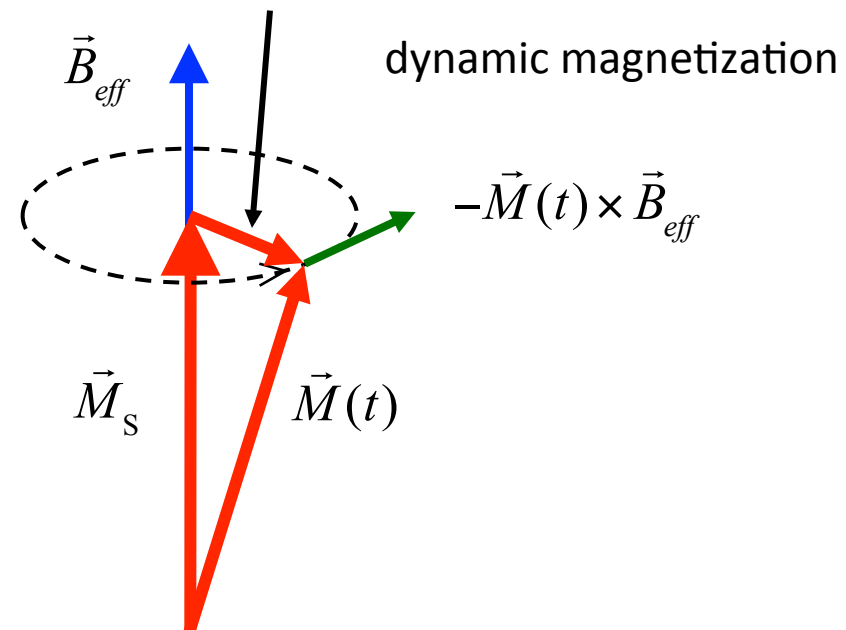
Spin wave: collective motion of magnetic moments



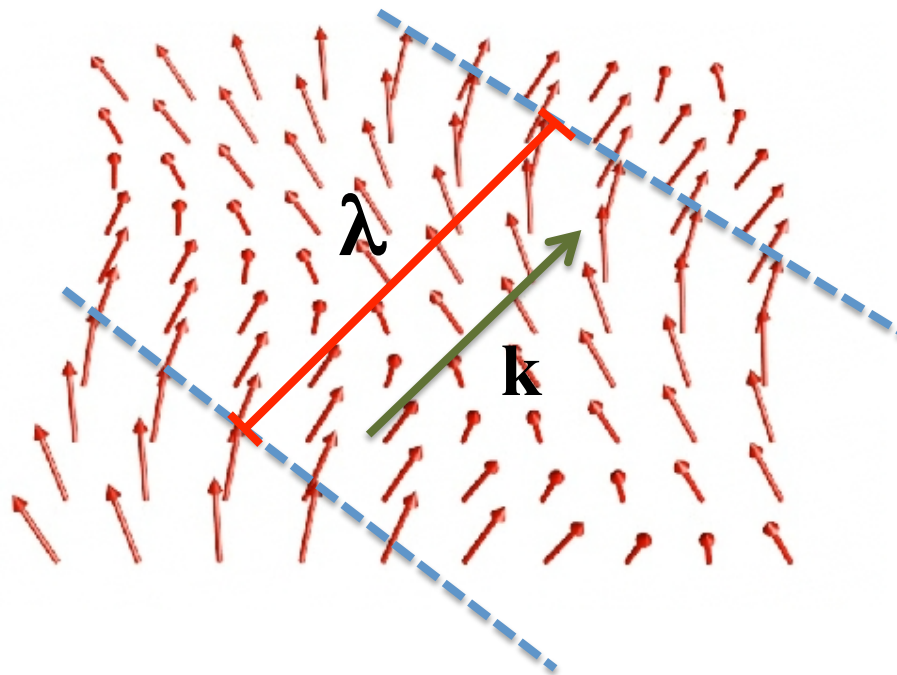
Landau-Lifshitz torque equation

$$\frac{1}{|\gamma|} \frac{d\vec{M}(t)}{dt} = -\vec{M}(t) \times \vec{B}_{eff}(t)$$

$$\vec{m}(\vec{r}, t) = \vec{m}_0(\vec{r}) \times e^{i(\vec{k}\vec{r} - \omega t)}$$



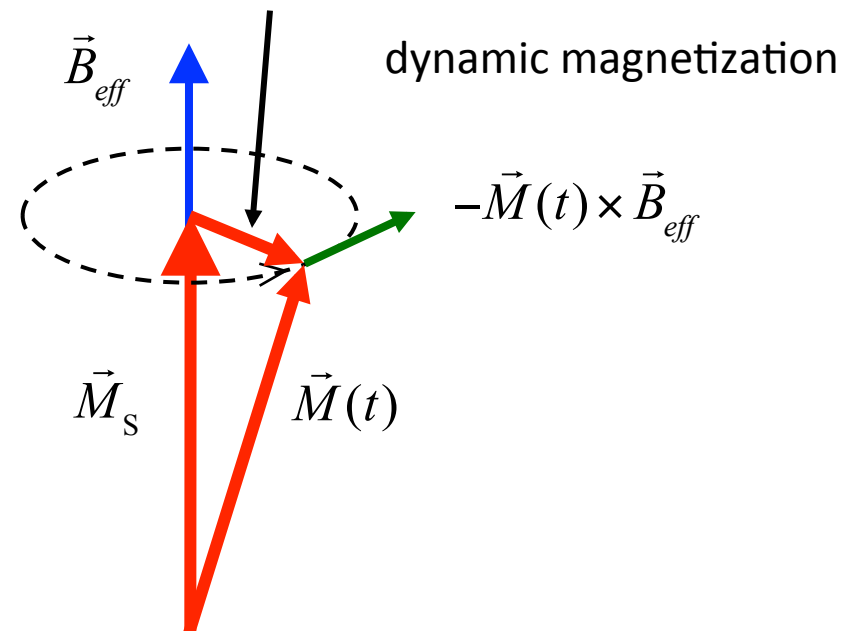
Spin wave: collective motion of magnetic moments



Landau-Lifshitz torque equation

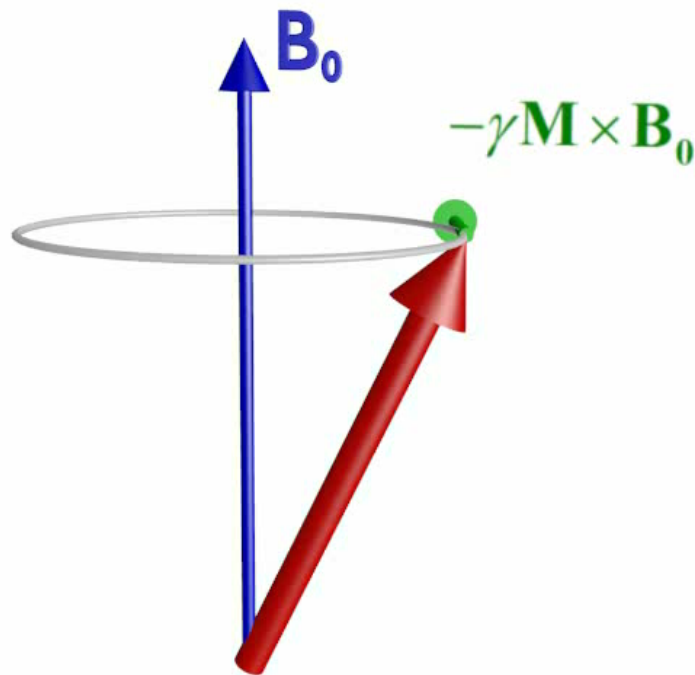
$$\frac{1}{|\gamma|} \frac{d\vec{M}(t)}{dt} = -\vec{M}(t) \times \vec{B}_{eff}(t)$$

$$\vec{m}(\vec{r}, t) = \vec{m}_0(\vec{r}) \times e^{i(\vec{k}\vec{r} - \omega t)}$$

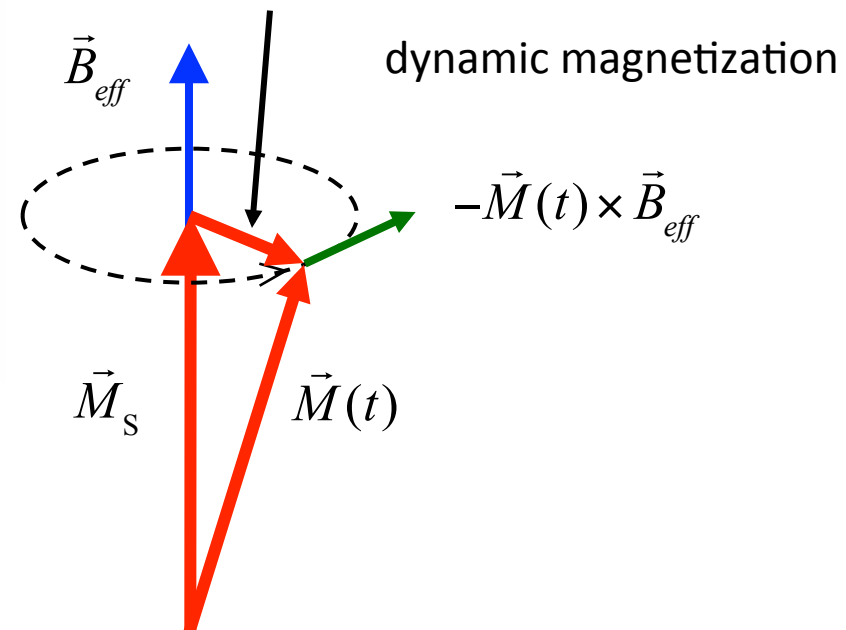


Landau-Lifshitz torque equation

$$\frac{1}{|\gamma|} \frac{d\vec{M}(t)}{dt} = -\vec{M}(t) \times \vec{B}_{eff}(t) + \frac{\alpha}{M_s} \vec{M}(t) \times \frac{d\vec{M}(t)}{dt}$$

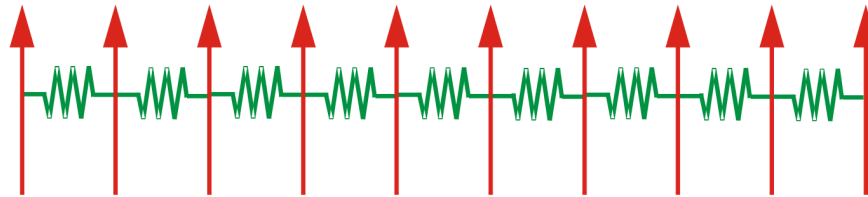


$$\vec{m}(\vec{r}, t) = \vec{m}_0(\vec{r}) \times e^{i(\vec{k}\vec{r} - \omega t)}$$

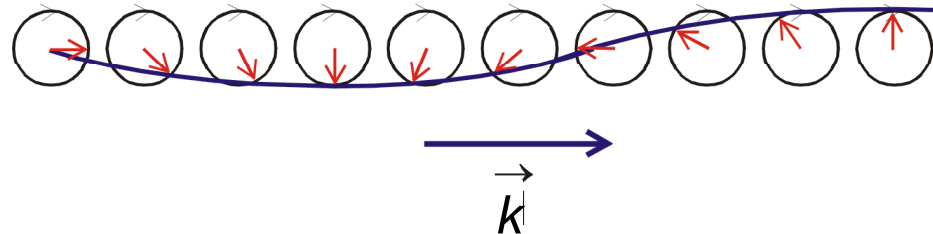


Two types of energy contributions

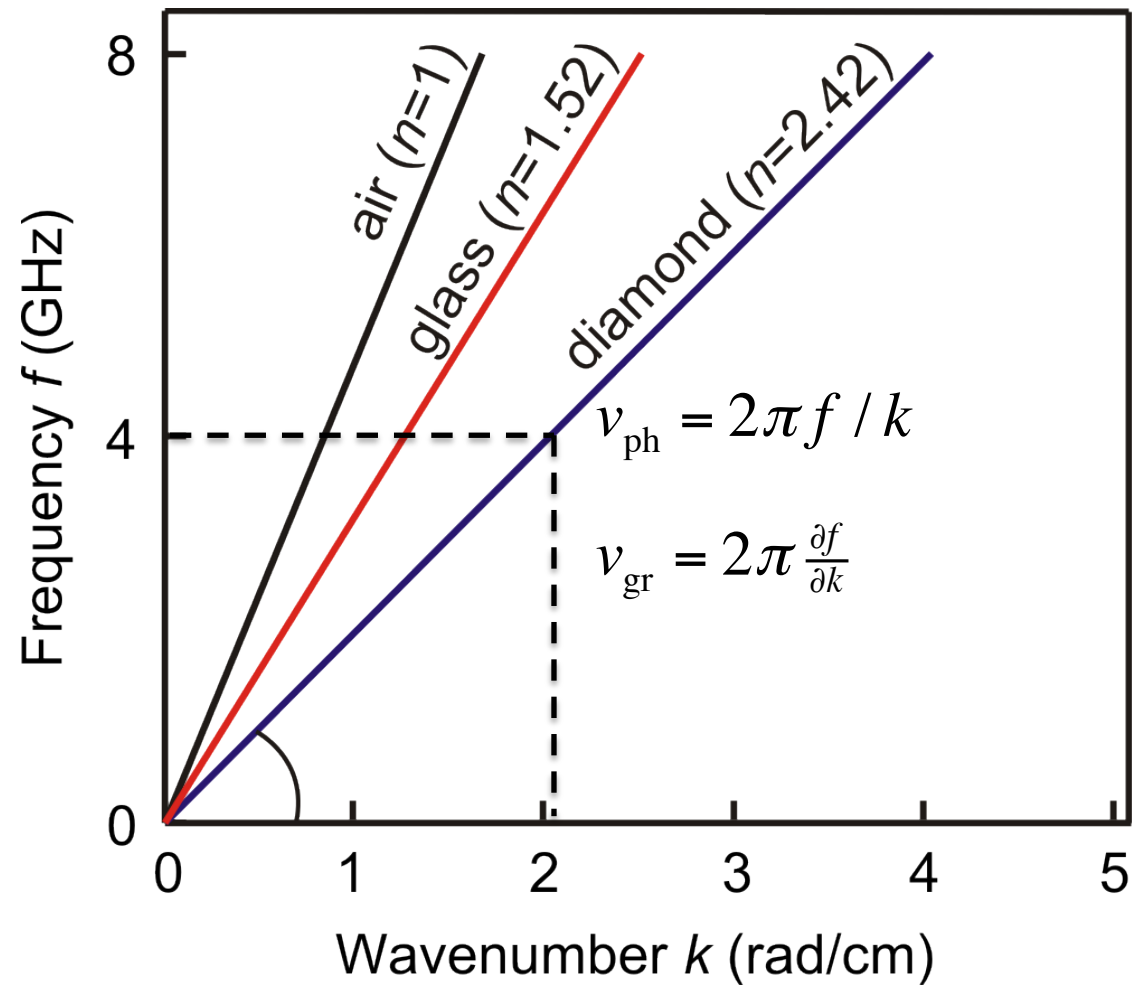
- exchange energy:
generated by twist of neighbored spins



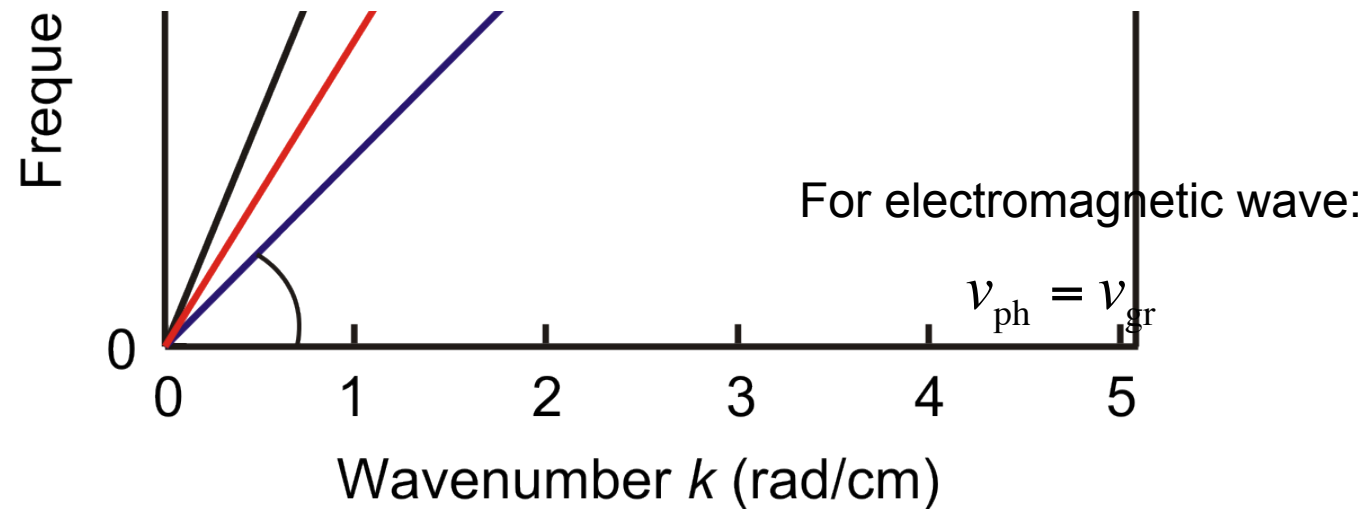
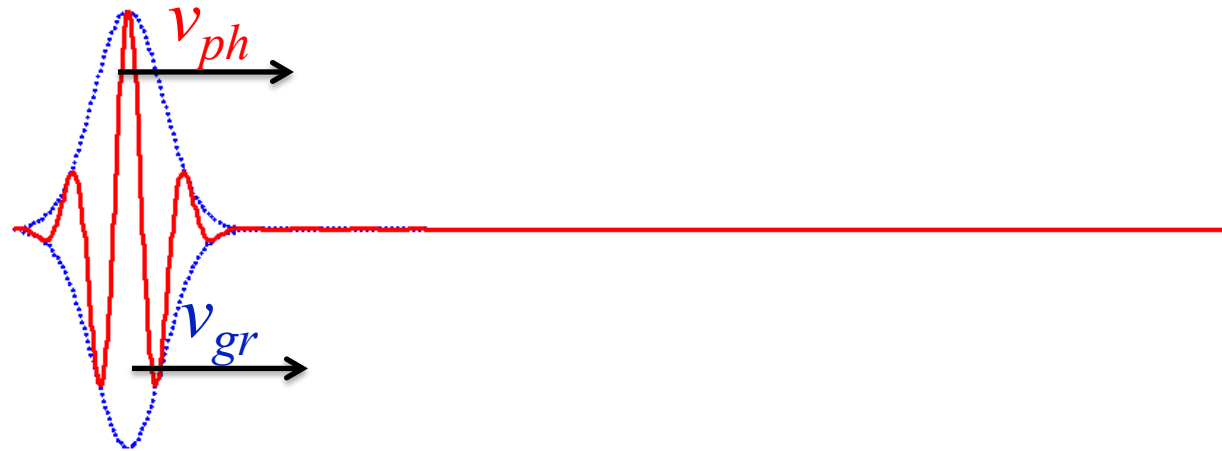
- dipolar energy:
generated by magnetic poles in long-wavelength spin waves



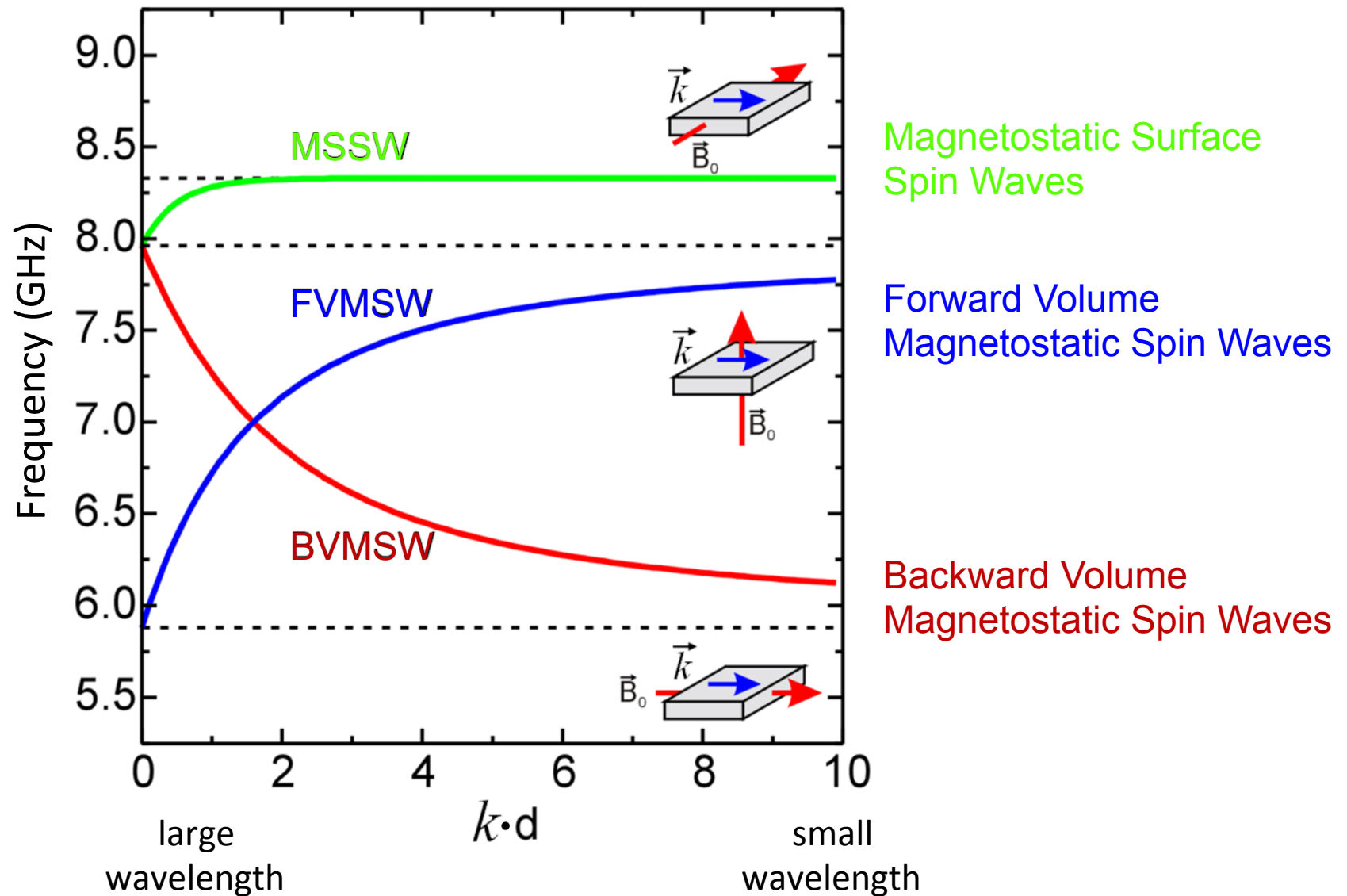
Dispersion of electromagnetic wave

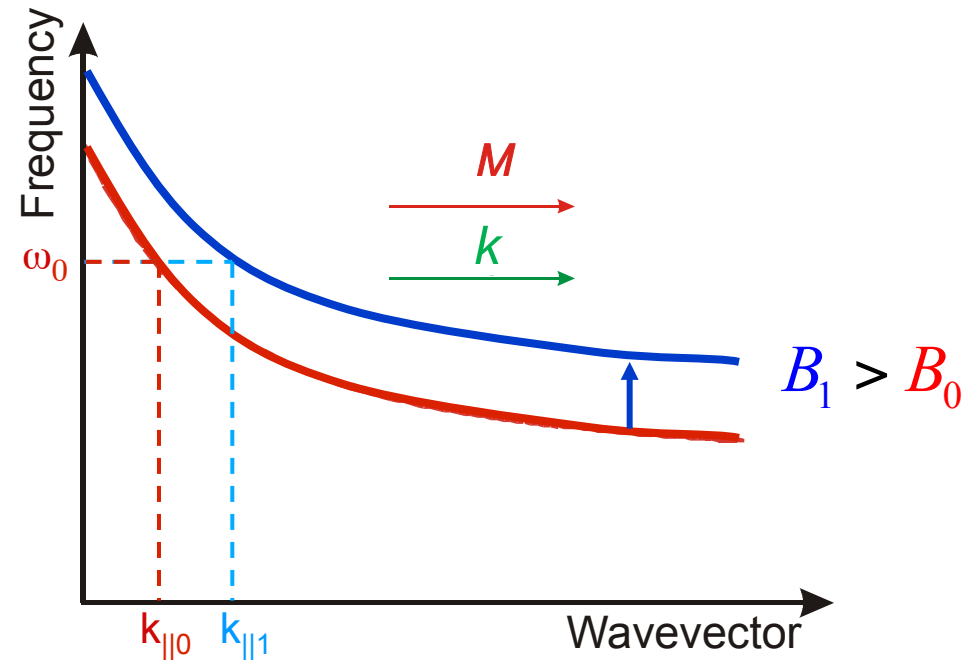
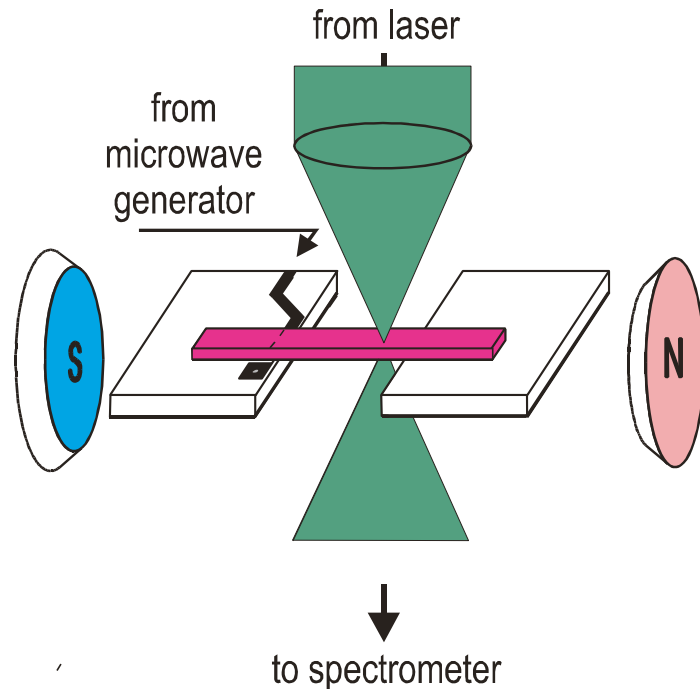


Dispersion of electromagnetic wave



Dipolar spin waves



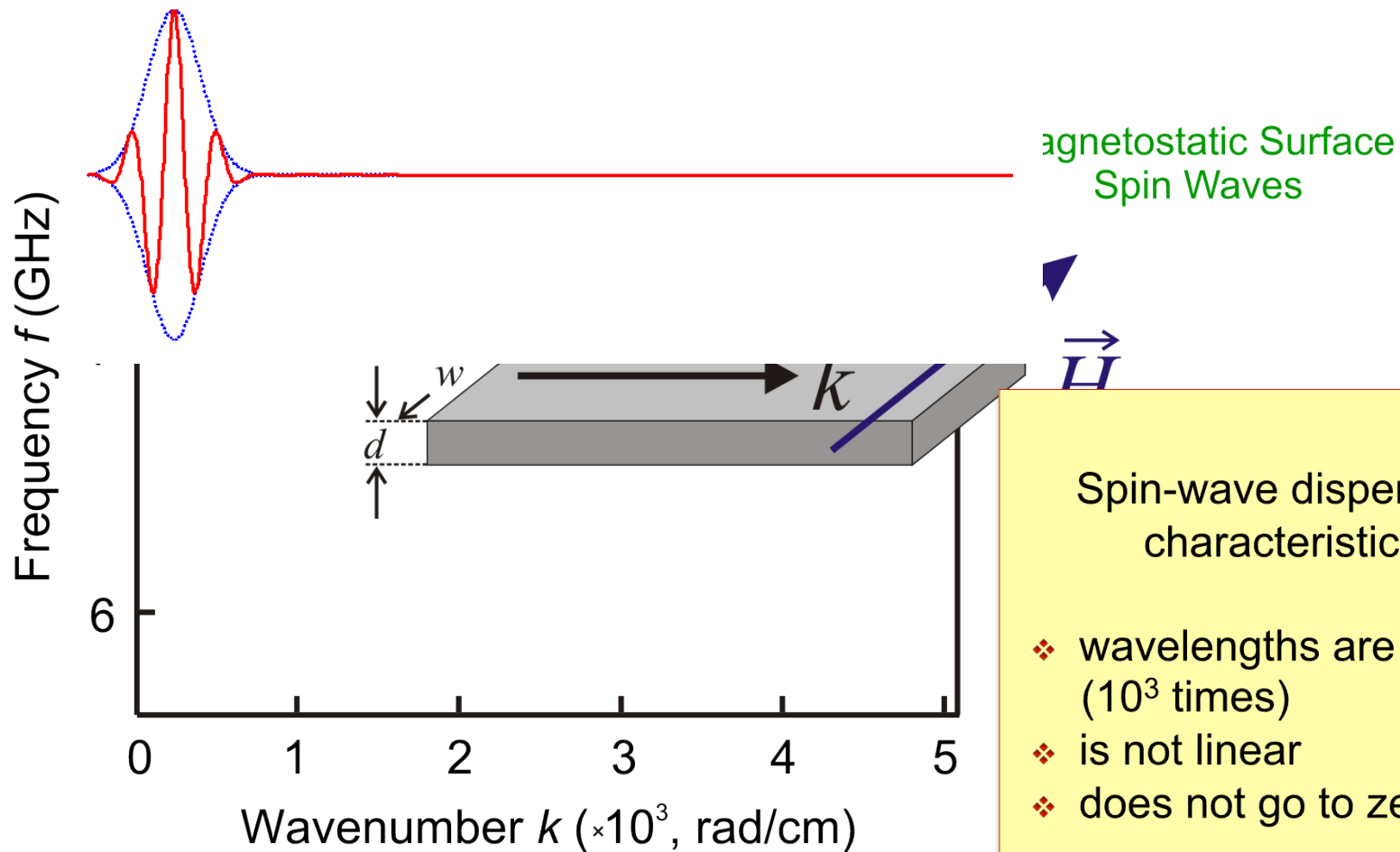


Wavevector k :

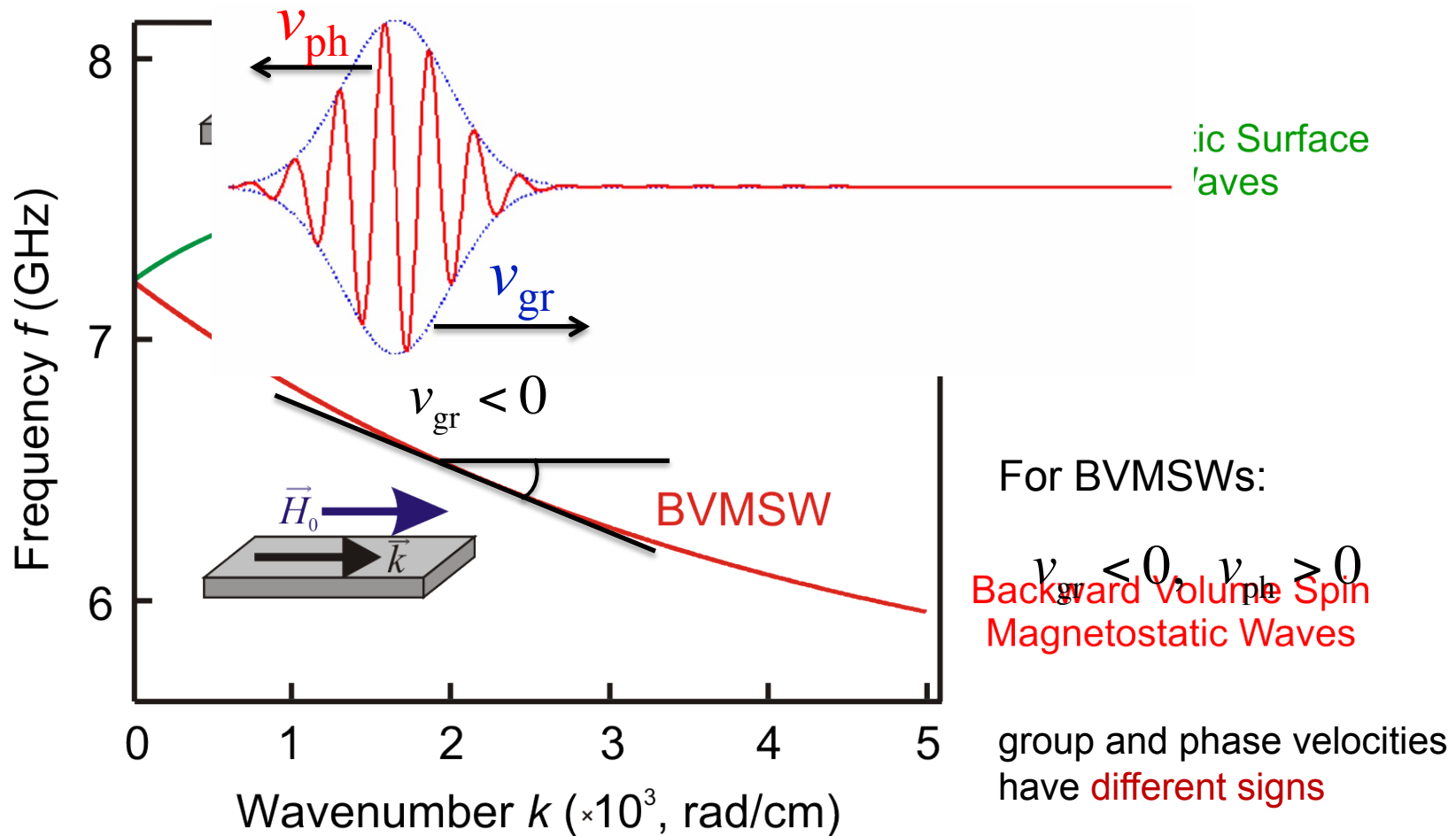
k_{parallel} defined by input frequency and dispersion

Dispersion shifted vertically by change in magnetic field

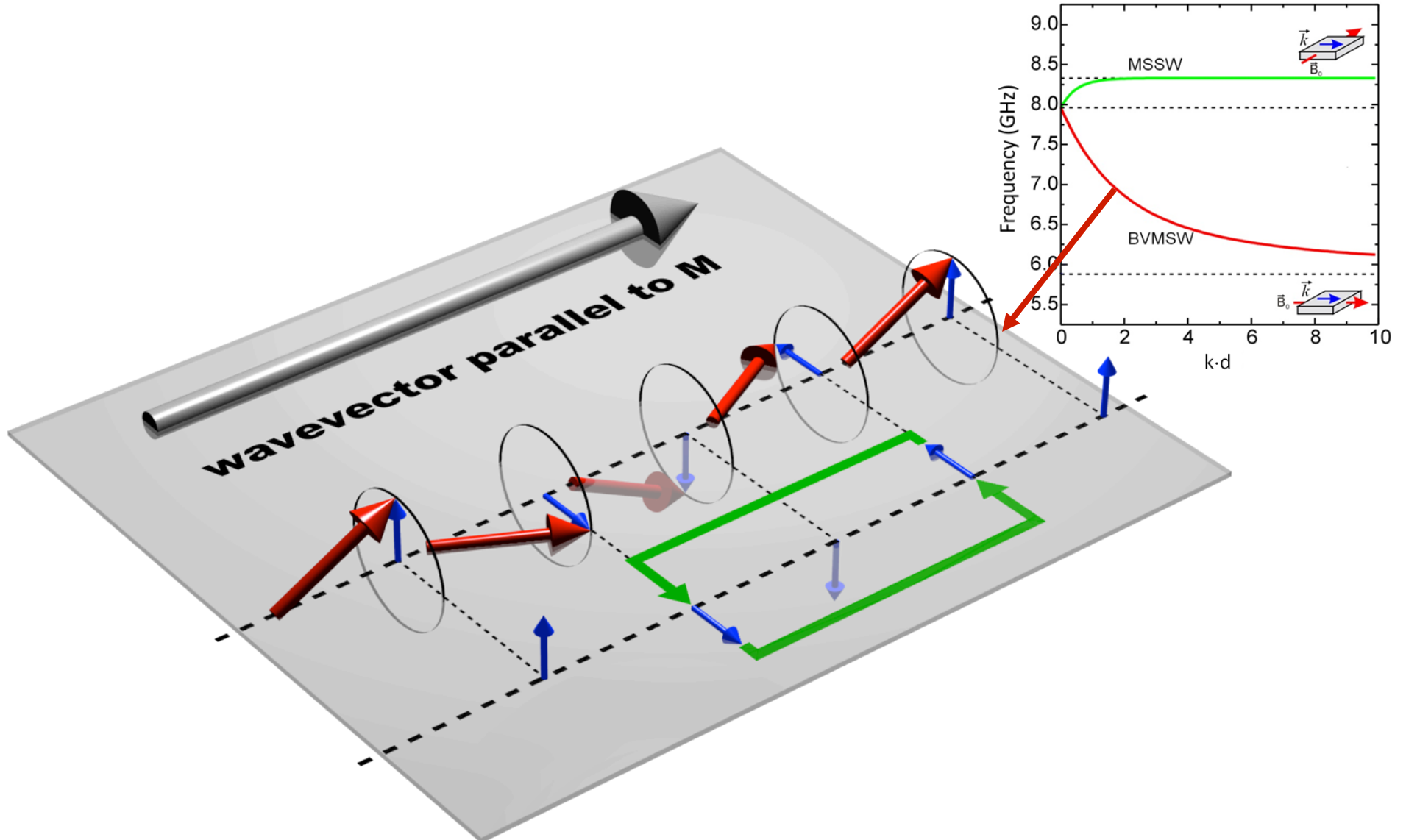
Dispersion curves for spin waves



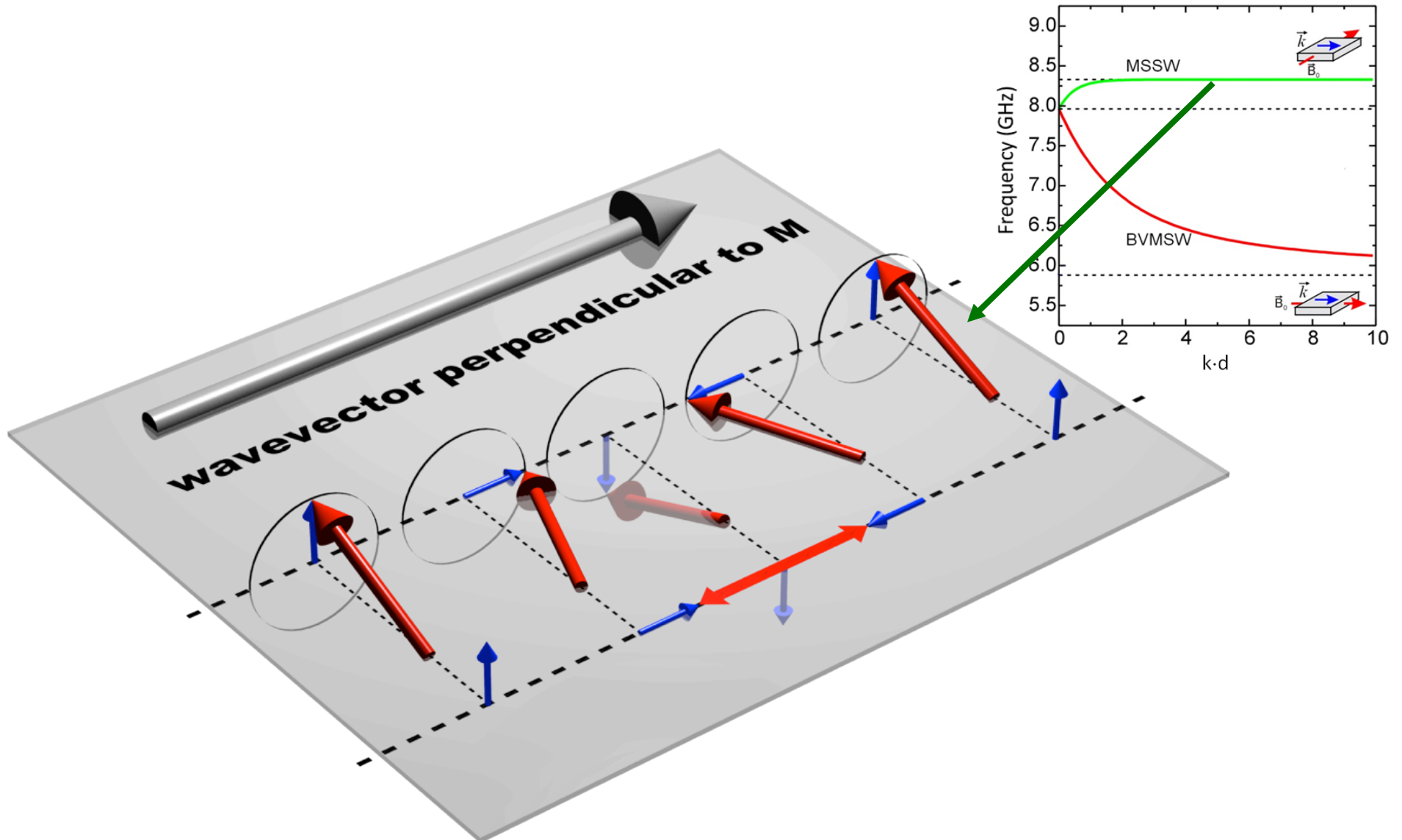
Dispersion curves for spin waves



Backward volume magnetostatic spin wave

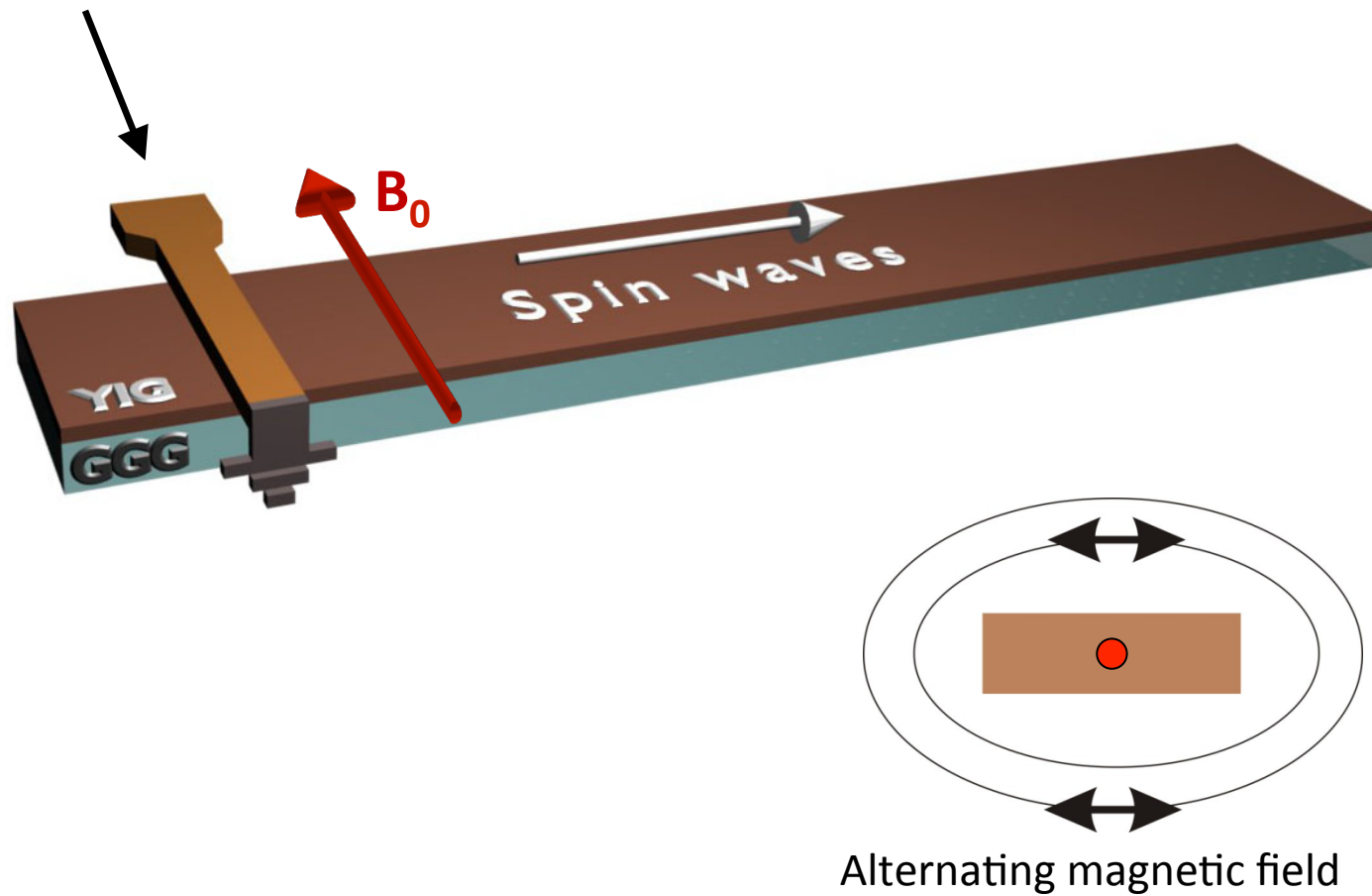


Magnetostatic surface spin wave

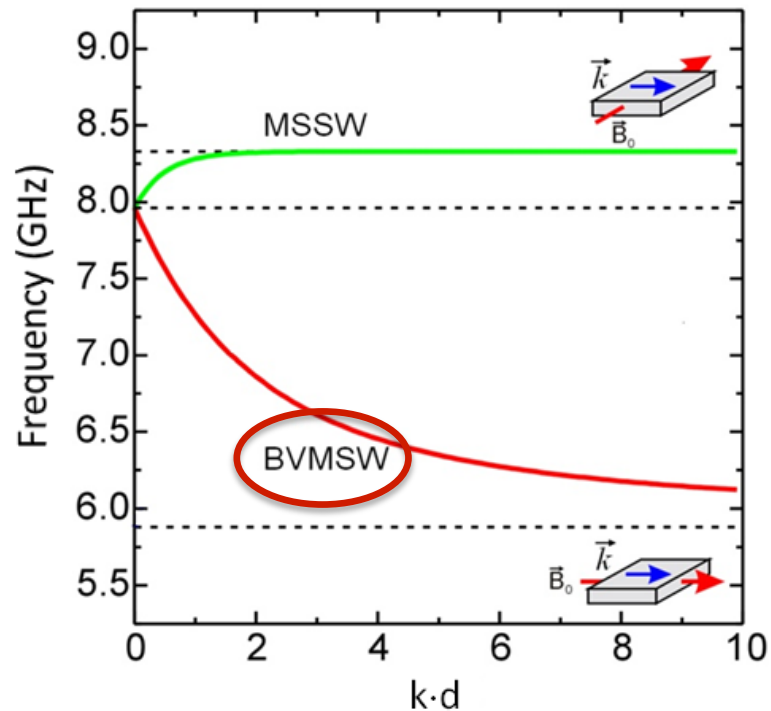


Excitation of dipolar spin waves

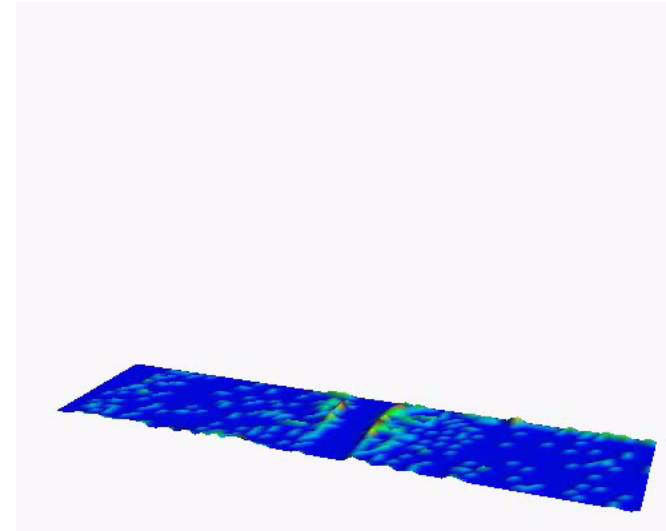
Input microwave signal



Backward volume magnetostatic spin waves (BVMSW)

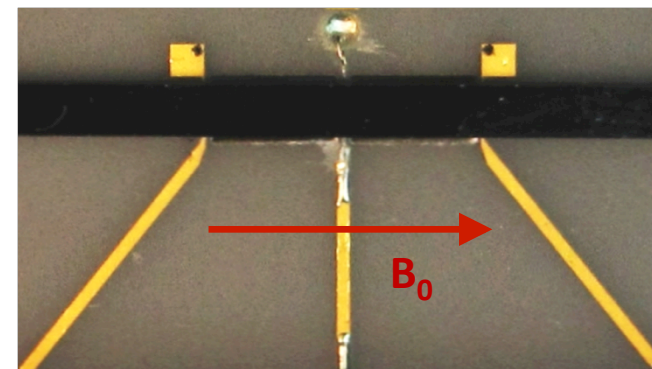
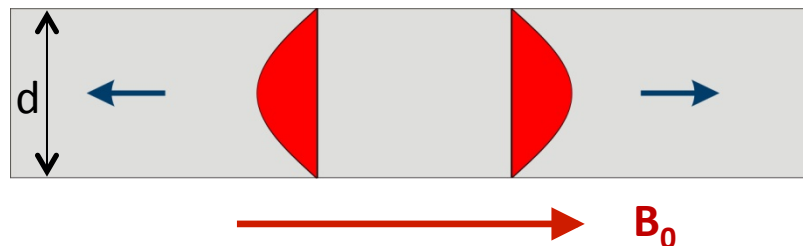


Excitation of BVMSW
measured with
Brillouin light scattering microscopy

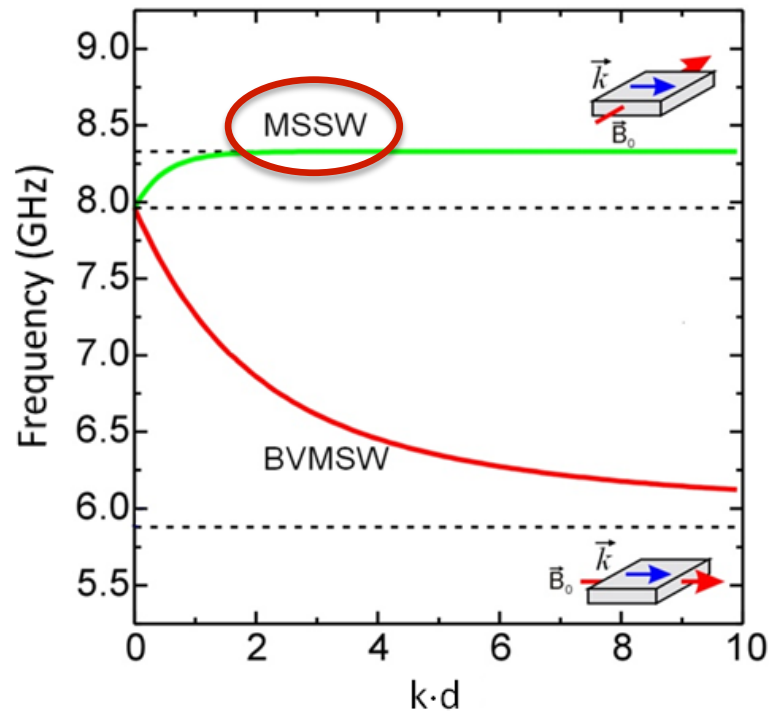


Dynamic magnetization profile

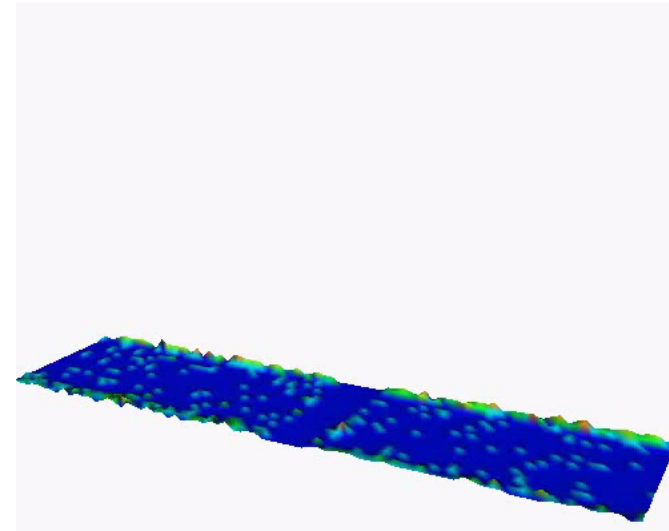
$$m_x \sim \cos(kx)$$



Magnetostatic surface spin waves (MSSW)

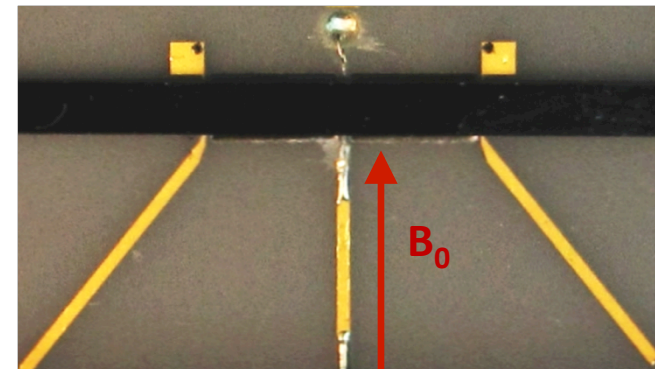
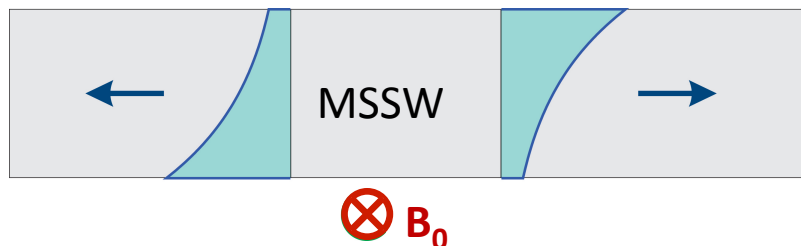


Excitation of MSSW
measured with
Brillouin light scattering microscopy

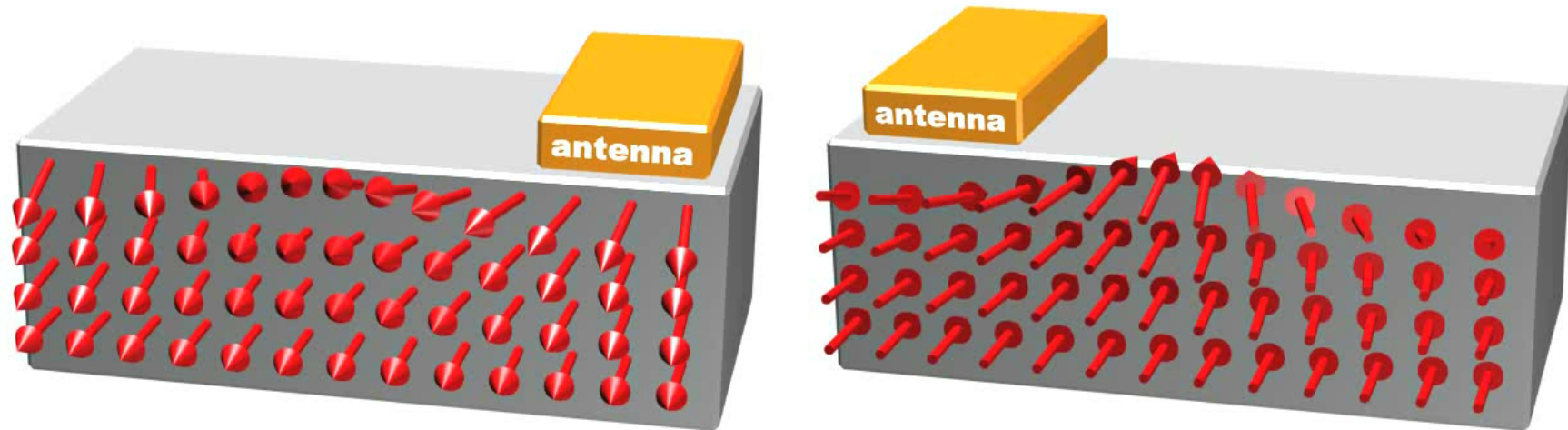


Dynamic magnetization profile

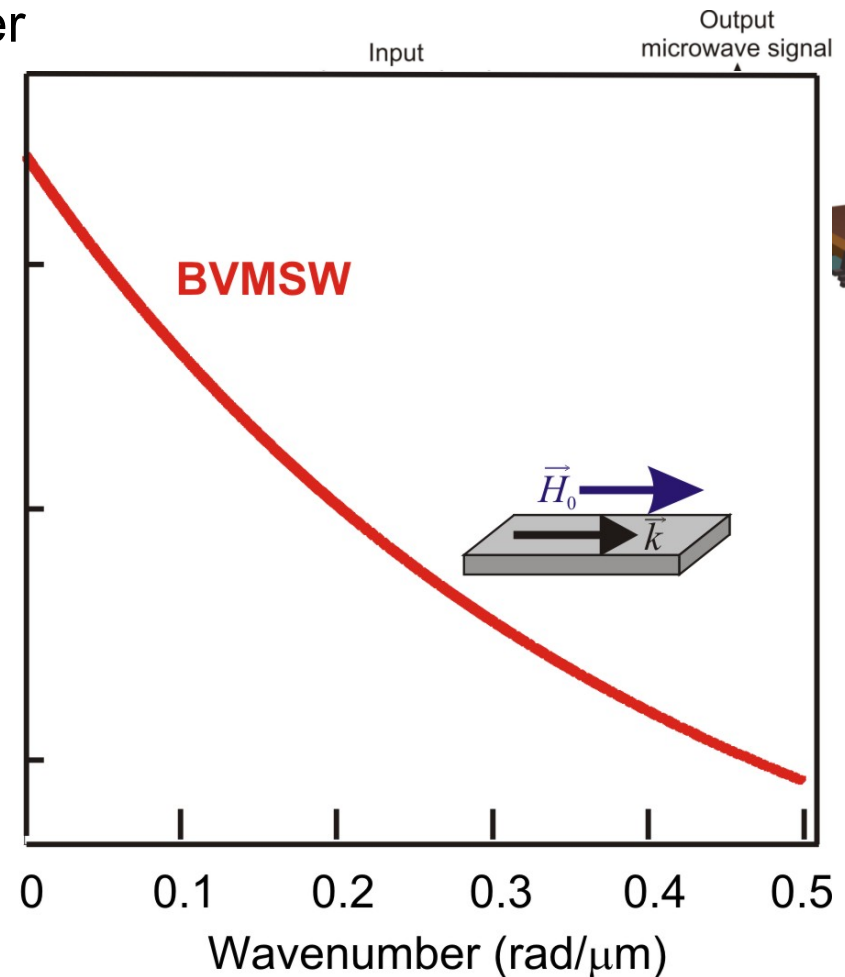
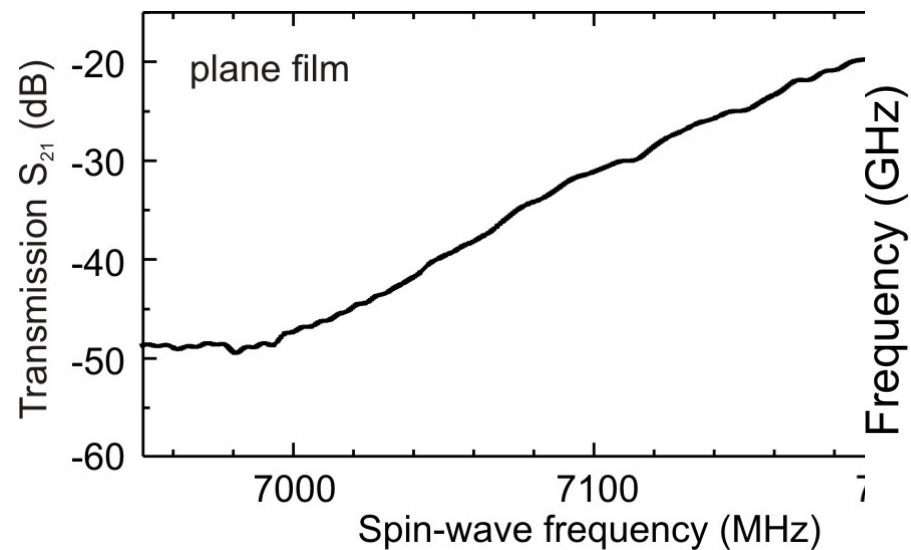
$$m_x \sim \exp(-kx)$$



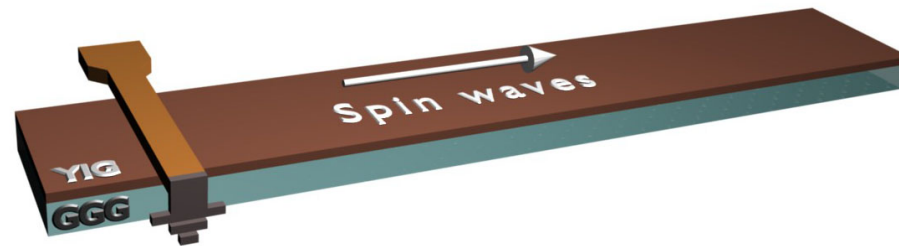
Magnetostatic surface spin wave



Dependence of the transmitted power on frequency



Chumak, et al., APL **93**, 022508 (2008)



Frequency:
$$f(k) = \gamma \sqrt{H_0 + 4\pi M_0 \frac{1 - \exp\left\{-\sqrt{(n\pi/w)^2 + k^2} d\right\}}{\sqrt{(n\pi/w)^2 + k^2} d}}$$

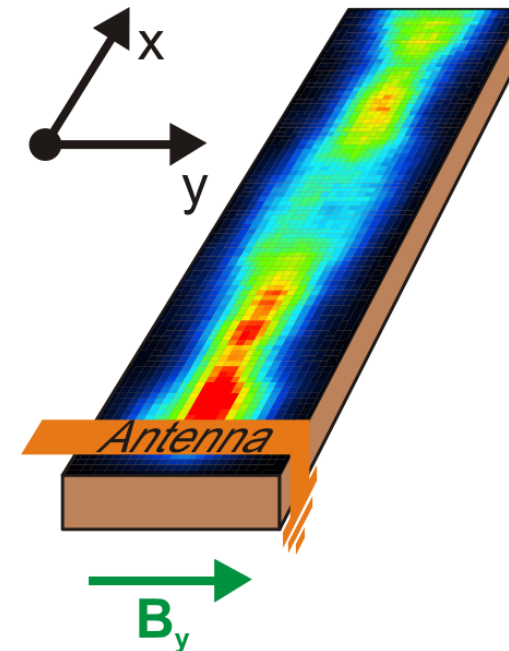
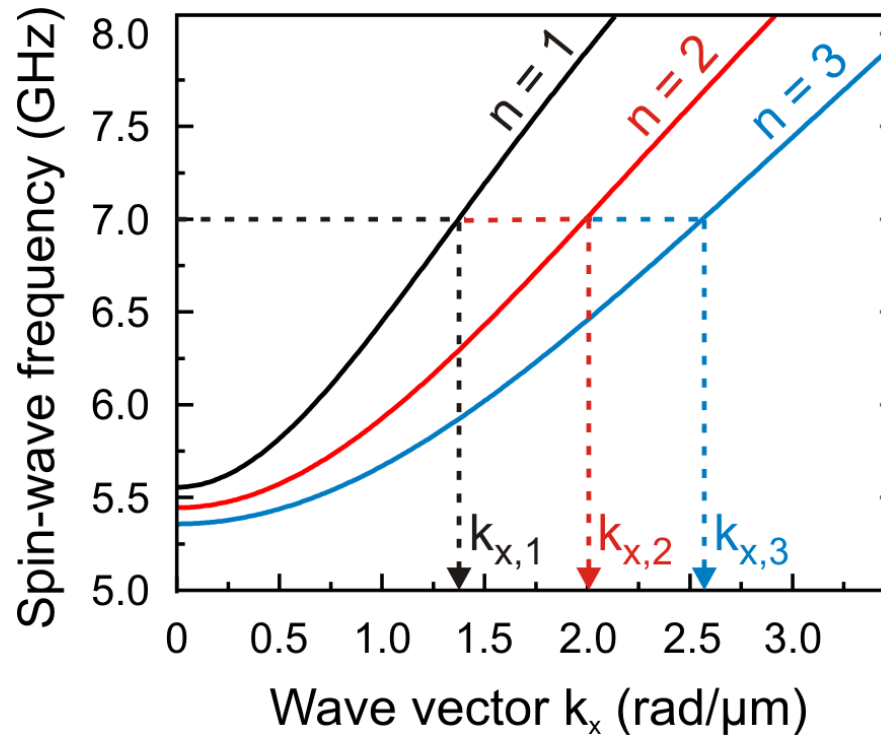
H_0 – magnetic field

M_0 – saturation magnetization

d – film thickness

w – waveguide width

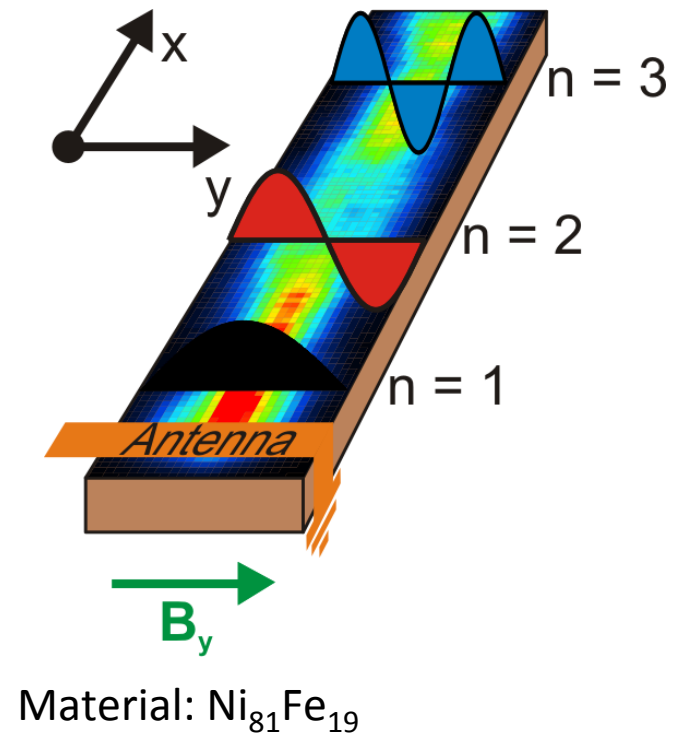
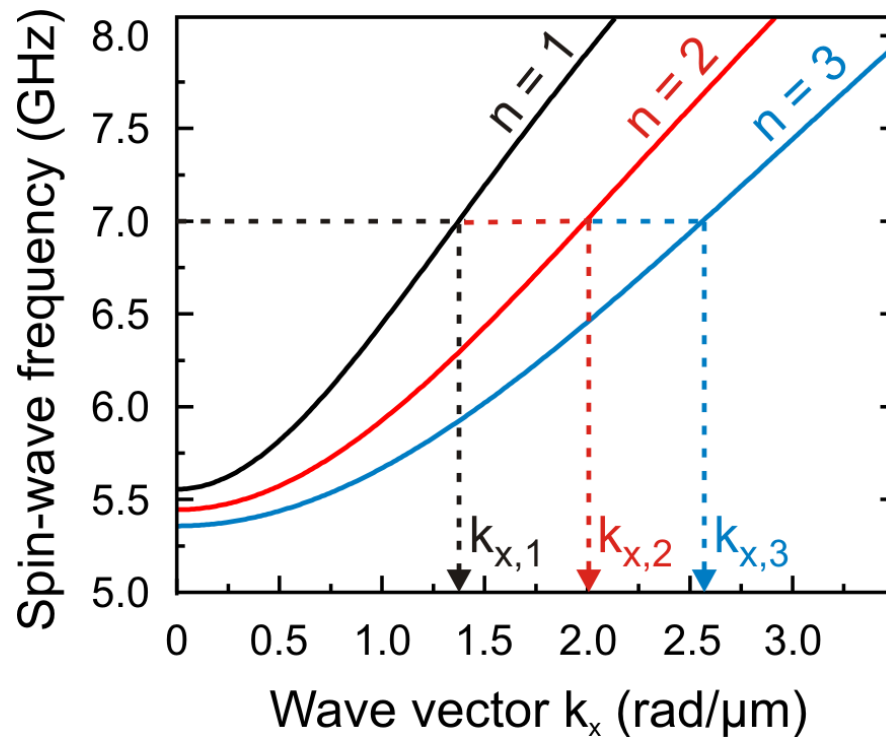
n – transverse mode order



Material: $\text{Ni}_{81}\text{Fe}_{19}$

k_x : propagating spin wave

k_y : lateral standing spin wave with mode order n

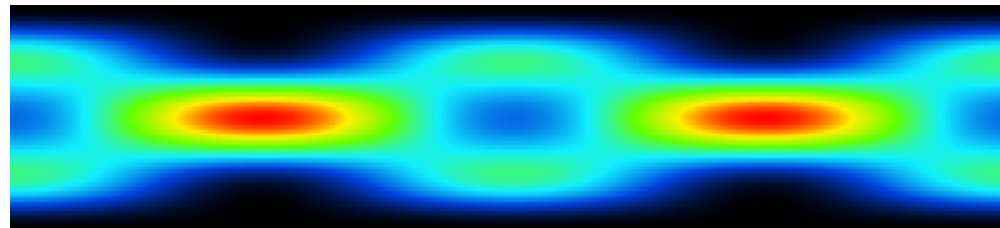


k_x : propagating spin wave

k_y : lateral standing spin wave with mode order n

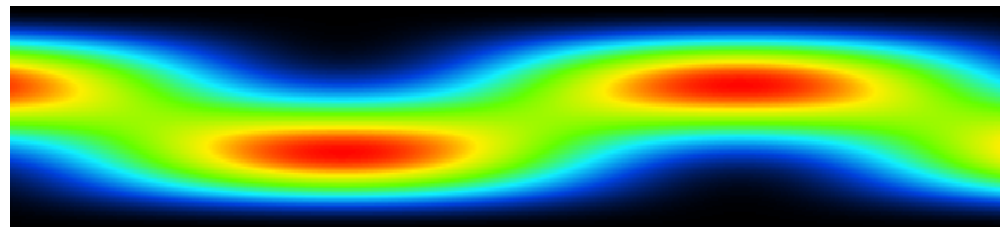
$$I(x, y) = \left| A_1 e^{-i(k_1 x)} \cos\left(n_1 \frac{\pi}{w} y\right) + A_2 e^{-i(k_2 x)} \cos\left(n_2 \frac{\pi}{w} y\right) + \dots \right|^2$$

Modes
n=1 & 3

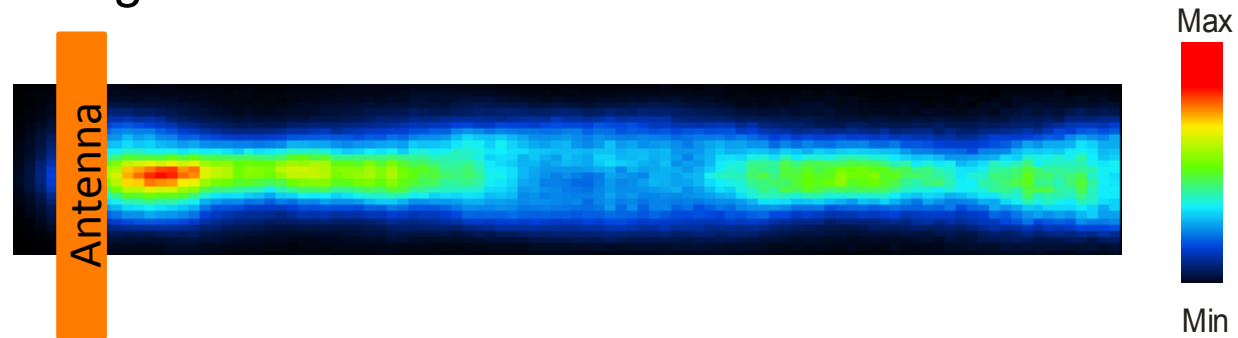


[1,2]

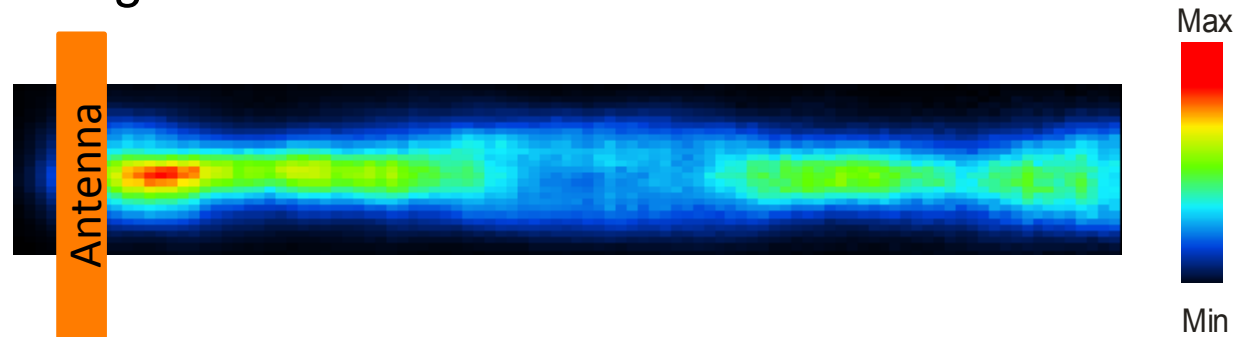
n=1 & 2



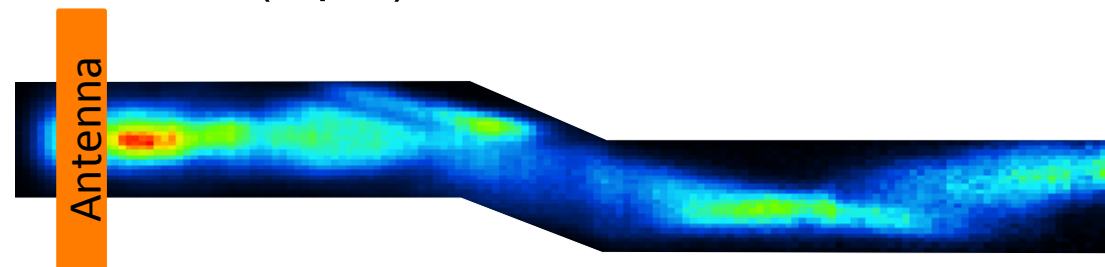
Reference waveguide



Reference waveguide

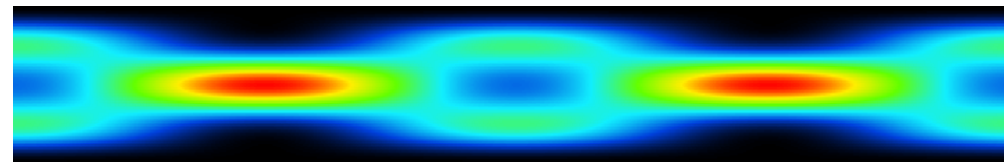
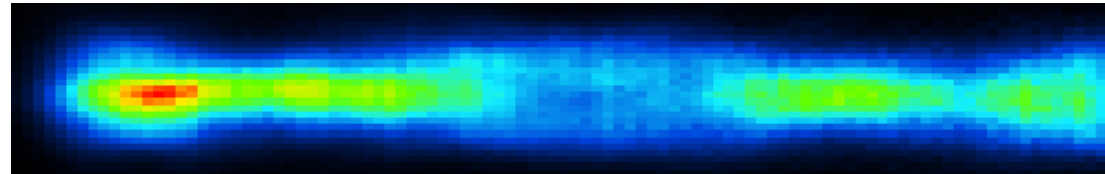


Waveguide with skew (1 μm)



- Changing interference patterns ($n=1\&3$ to $n=1\&2$)
- Edge mode: asymmetric source

n=1 & 3

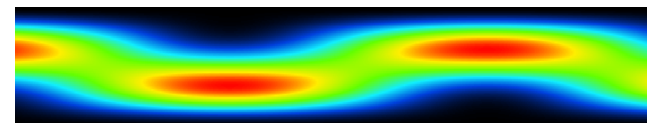
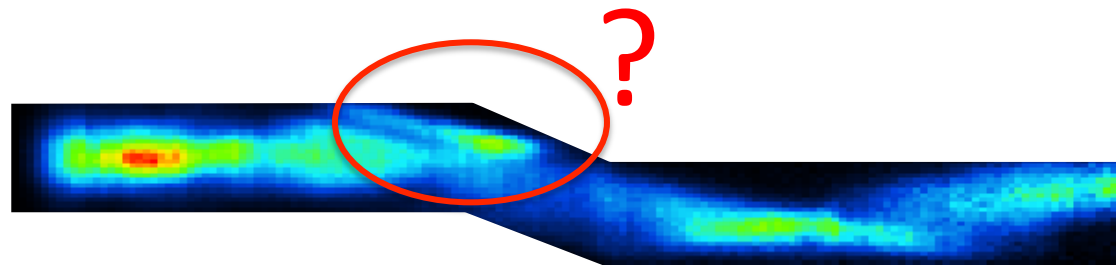


Max



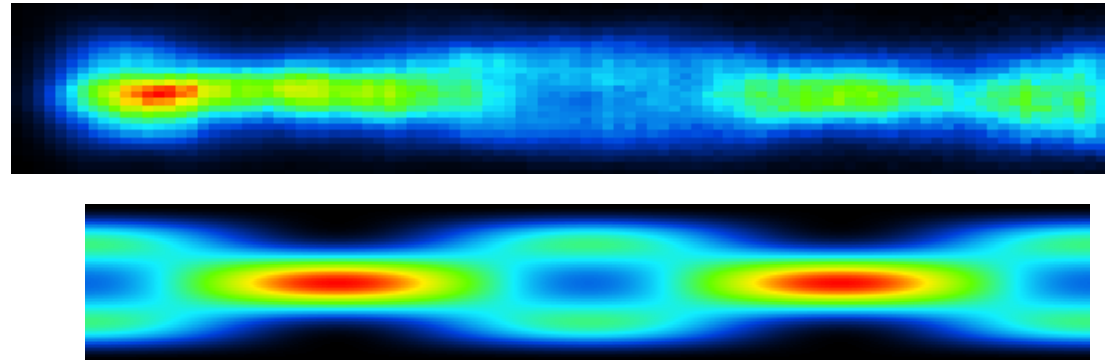
Min

n=1 & 2



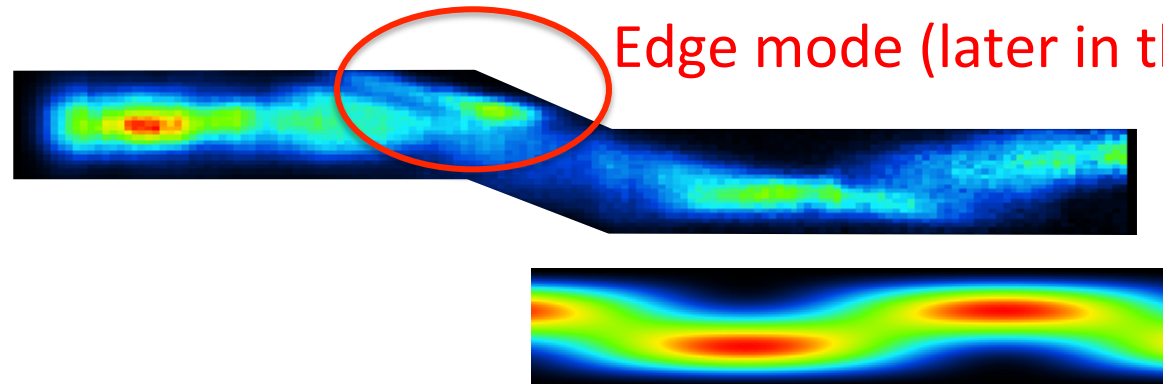
Excitation of the second width mode

n=1 & 3



Max
Min

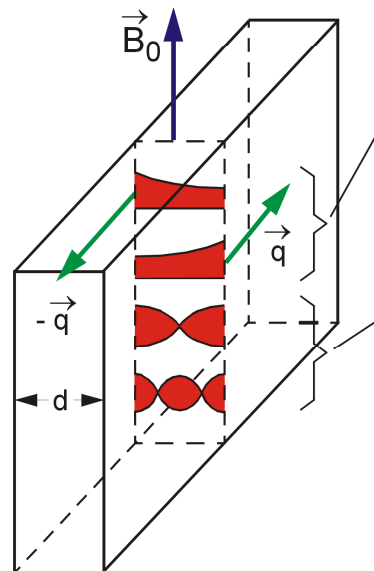
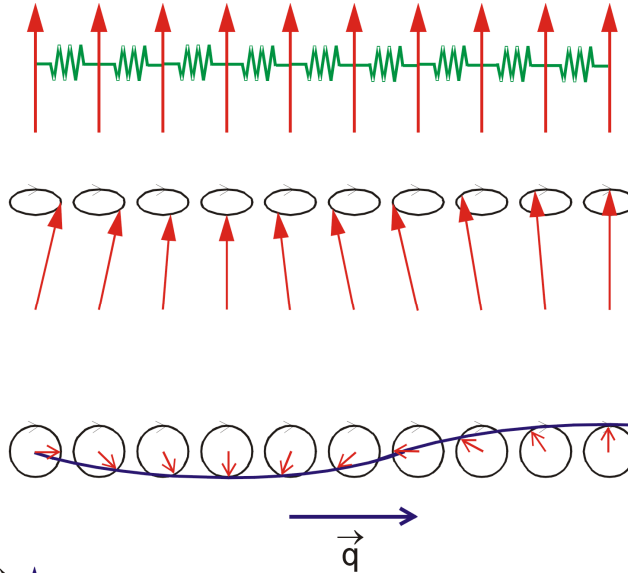
n=1 & 2



Edge mode (later in this lecture)

Excitation of the second width mode

Spin waves in a thin magnetic film



Dipolar Damon-Eshbach modes

$$\omega^2/\gamma^2 = [B_0(B_0 + J_s) + (J_s/2)^2 (1 - e^{-2qd})]$$

Standing spin waves

$$\frac{\omega}{\gamma} = \frac{2A}{M_s} \cdot q^2 = \frac{2A}{M_s} \left(\frac{n\pi}{d}\right)^2 \quad n = 1, 2, \dots$$

A: exchange constant

M_s : magnetization

small wavelength: **exchange interaction**

$$\vec{B}_{\text{exch}} = \frac{2A}{M_s^2} \nabla^2 \vec{M} = \frac{D}{M_s} \nabla^2 \vec{M}$$

must be considered.

resonance condition for wavevector component perpendicular to film:

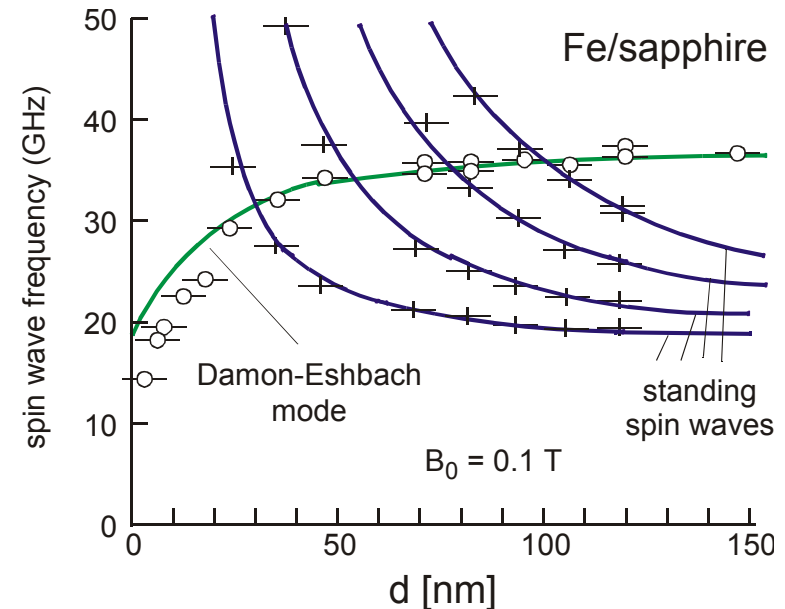
$$q_{\perp} = n \frac{\pi}{d}; \quad n = \pm 1, \pm 2, \pm 3 \dots$$

approximative calculation of exchange modes:

(outside crossing regimes with Damon-Eshbach modes)

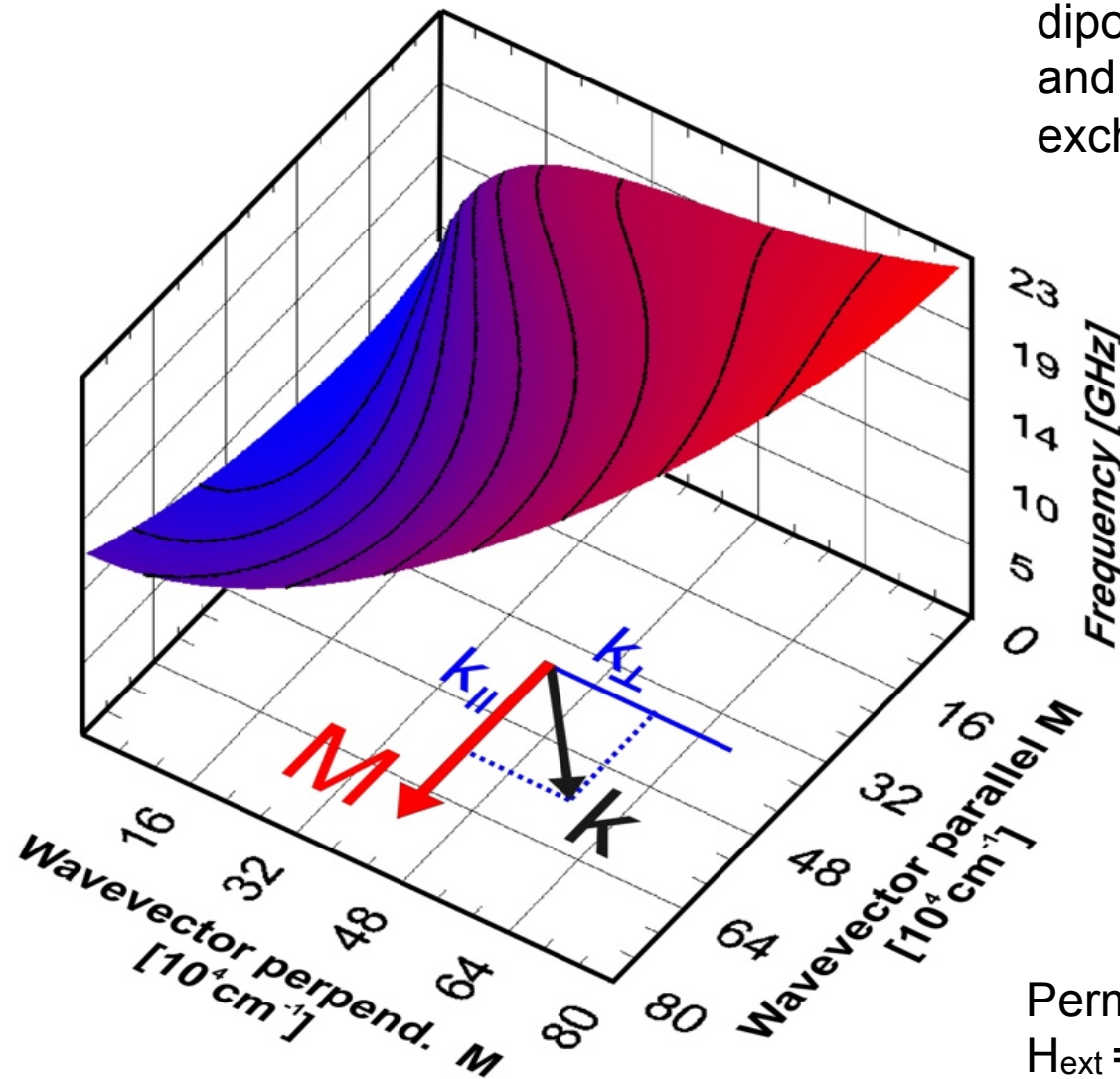
$$\left(\frac{\omega}{\gamma} \right)^2 = (B_0 + Dq^2)(B_0 + J_s + Dq^2)$$

with: $D = 2A/M_s$



P. Grünberg et al., JMMM **28**, 319 (1982)

Propagation at oblique in-plane angle



dipole-dipole interaction
 and
 exchange interaction

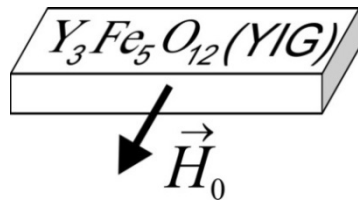
Permalloy film (15nm)
 $H_{\text{ext}} = 500 \text{ Oe}$

Dipolar and exchange spin waves

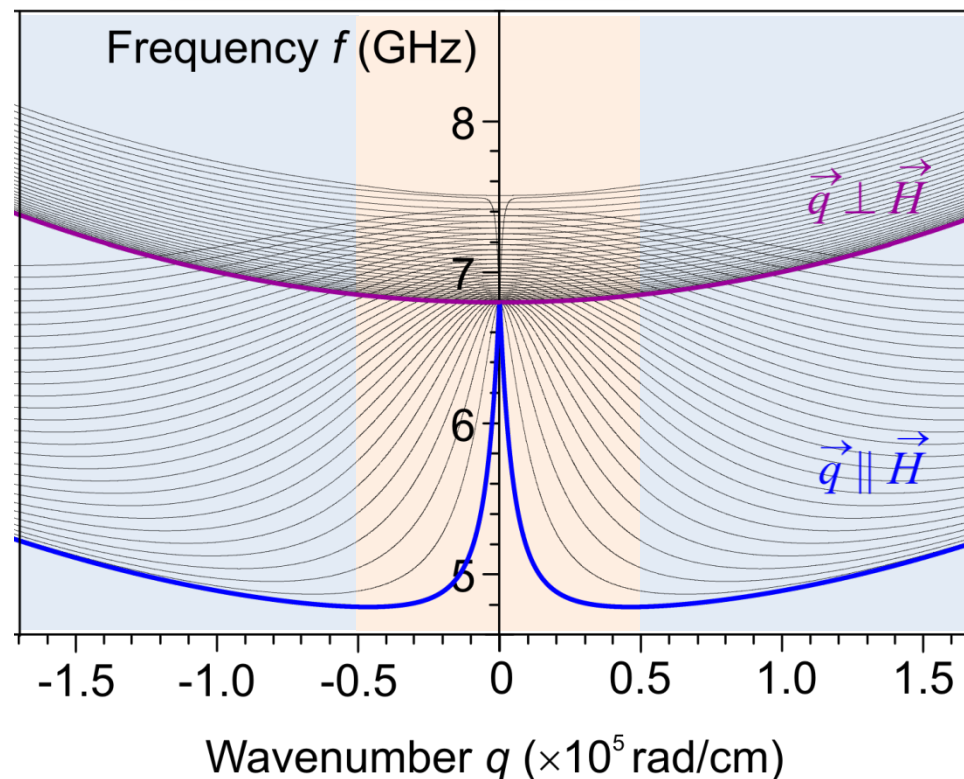
Landau-Lifshitz equation:
$$\frac{\partial \vec{M}}{\partial t} = -|\gamma| \vec{M} \times \vec{H}_{\text{eff}}$$

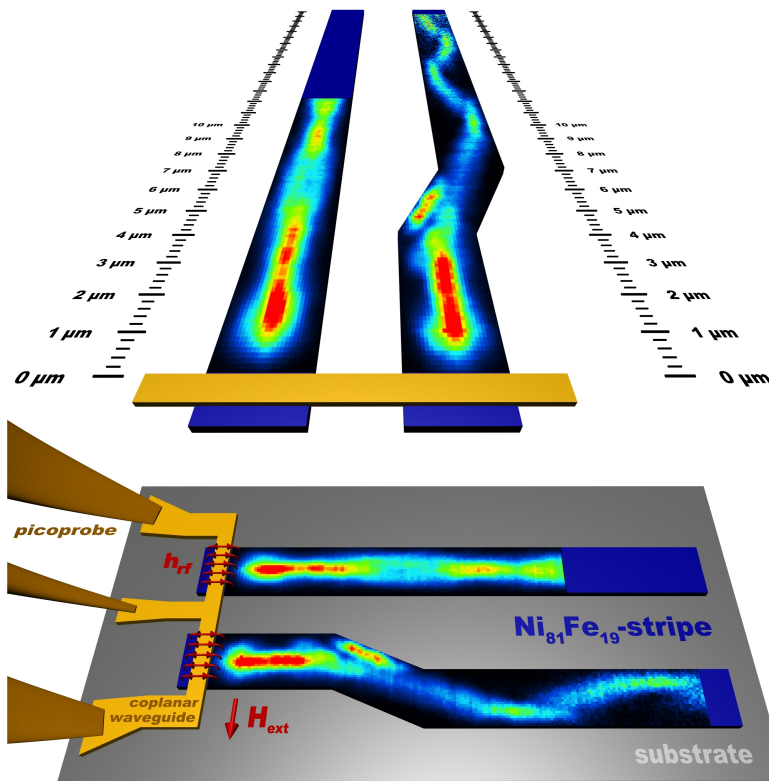
$$\vec{H}_{\text{eff}}(\vec{r}) = \vec{H}_0 + \int_V \tilde{G}(\vec{r}, \vec{r}') \cdot \vec{M}(\vec{r}') d\vec{r}' + \frac{\eta}{\gamma M_S} \nabla^2 \vec{M} + \dots$$

dipolar interaction
exchange interaction



$H_0 = 1710 \text{ Oe}$





Travelling magnons allow one to:

- **transfer** spin information over **centimeter** distances
- **process** the information (using wave nature of magnon)
- **operate** in **insulator**-based technology

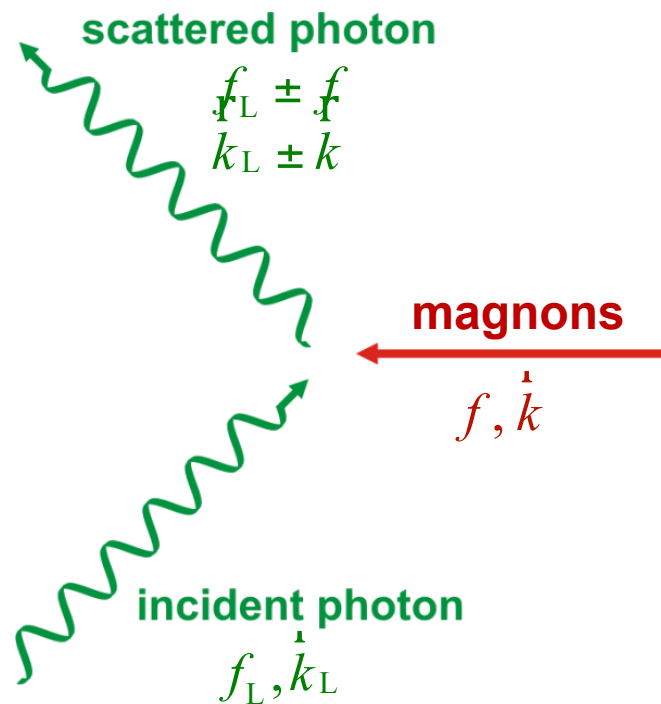
Fundamental properties:

- Minimal wavelength is down to **several nm**
- Frequency is in GHz and up to the **THz range**
- **Energy:** $E_{\text{magnon}} \ll k_B T$
- **Lifetime:** up to several 100 ns

- Basics: Spin Waves
- ➔ Experiment: Brillouin Light Scattering Spectroscopy
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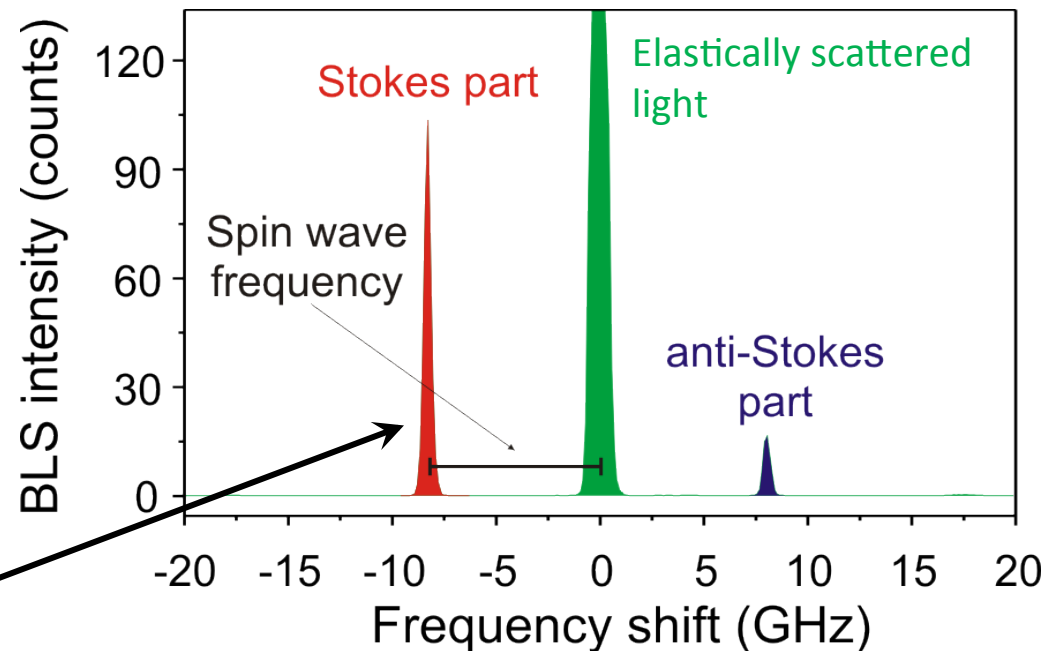
Brillouin light scattering process

= inelastic scattering of photons from spin waves



$$f_{\text{scattered L}} = f_L \pm f$$

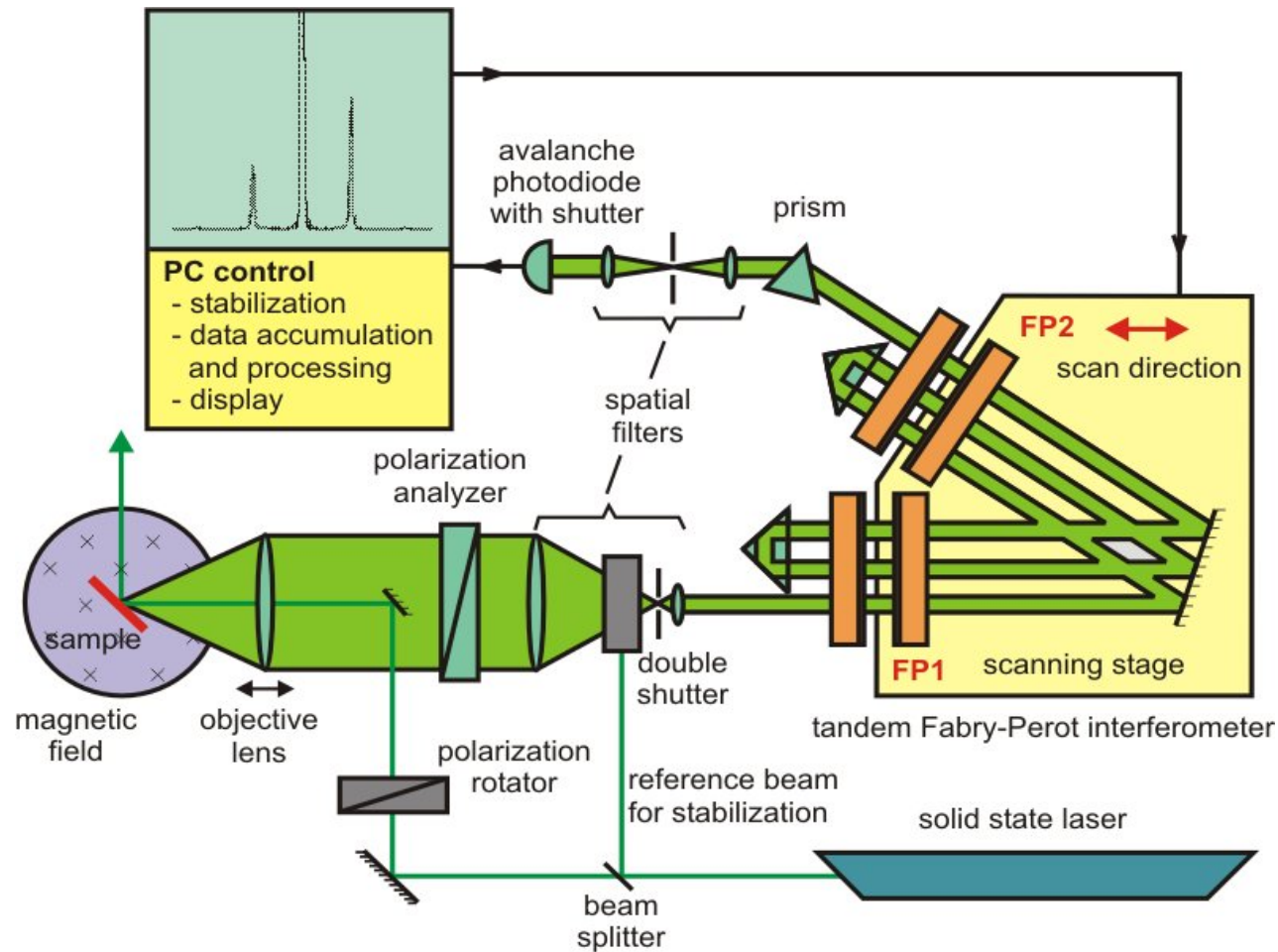
$$\vec{k}_{\text{scattered L}} = \vec{k}_L \pm \vec{k}$$

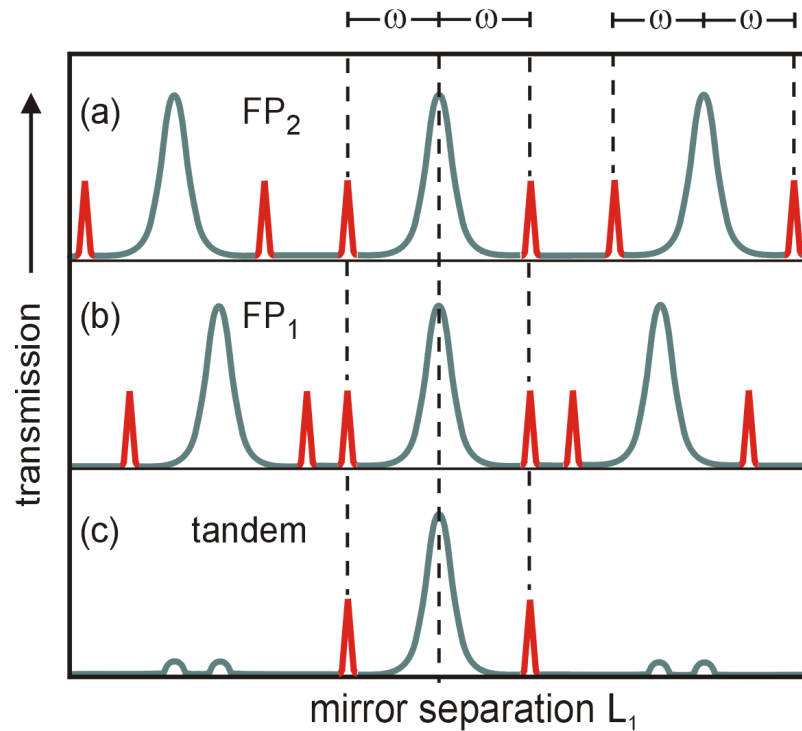


proportional to the spin wave intensity $|\varphi|^2$

Brillouin light scattering spectrometer

**high-resolution interferometry with high contrast
for measurements of acoustic phonons and spin waves**





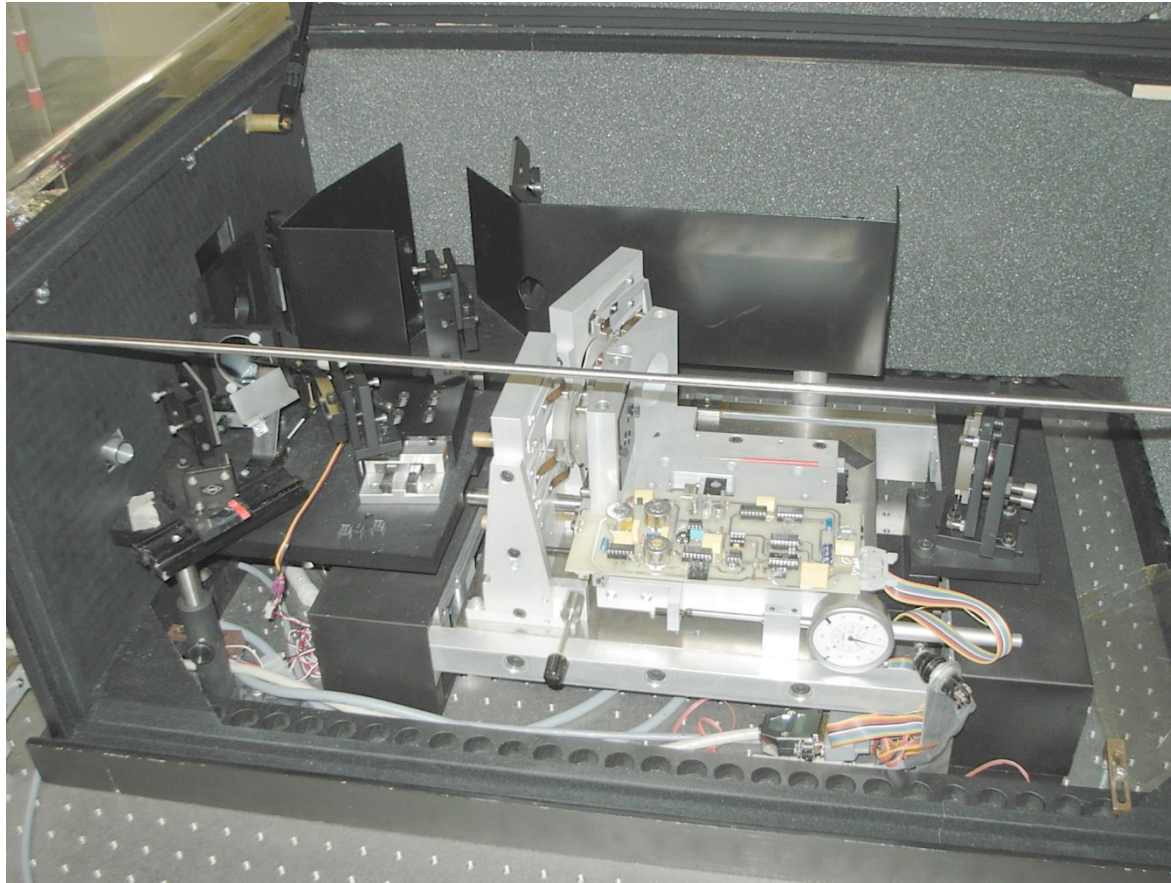
- etalon in transmission if mirror separation L is:

$$L = n \lambda_{\text{Laser}}/2$$

- suppression of neighboring orders if mirror separations L_1, L_2 of both etalons:

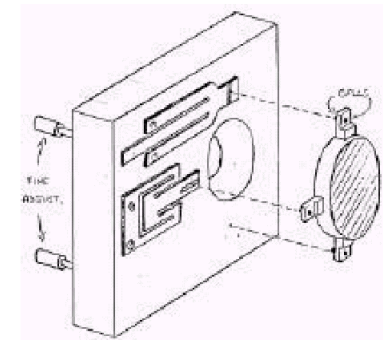
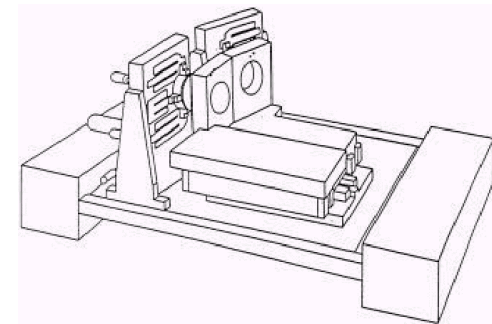
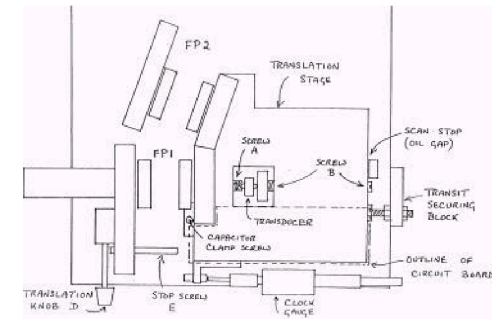
$$L_2 = L_1 \cos \alpha$$

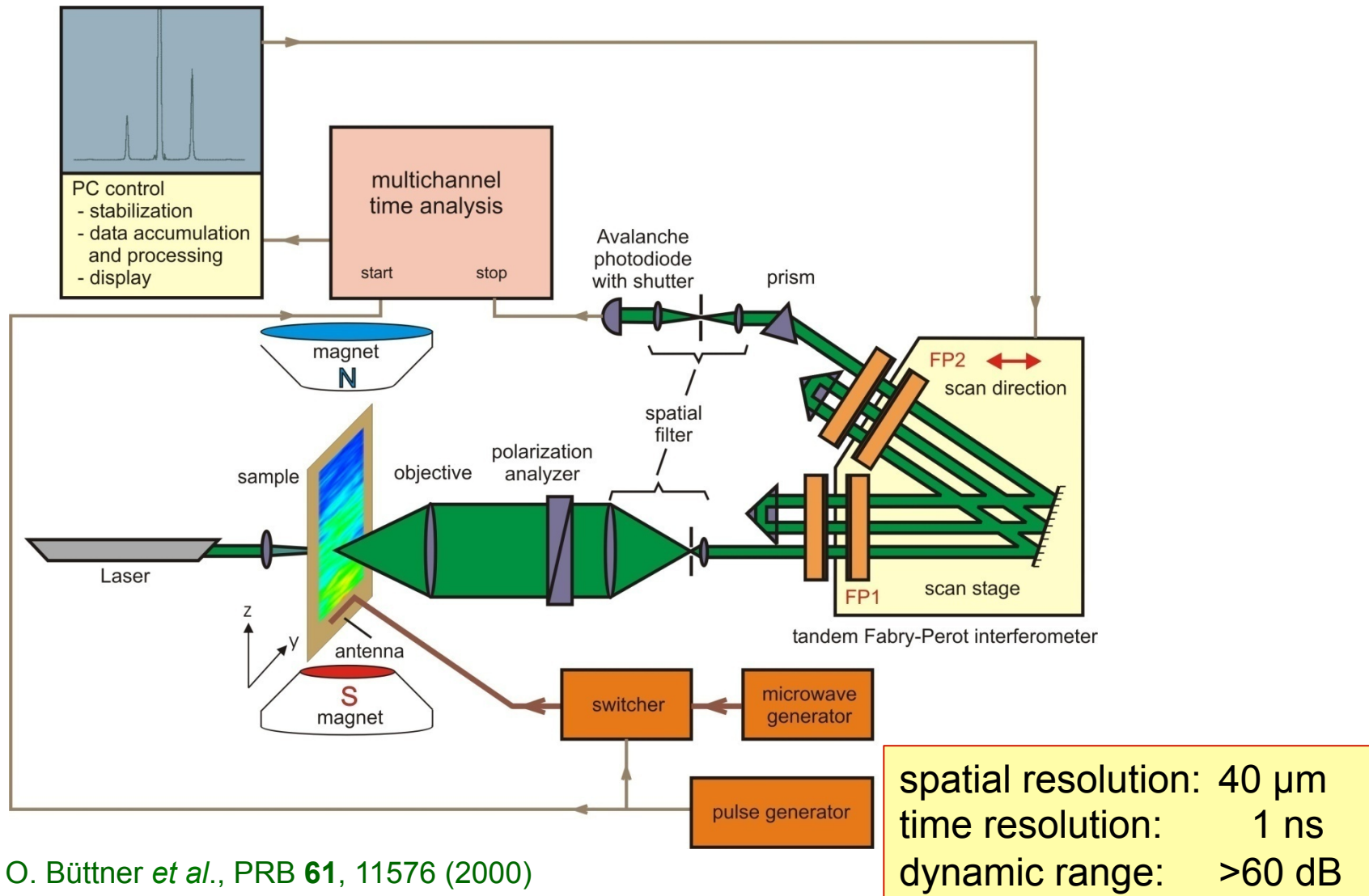
α : angle between etalon axes



Tandem Fabry-Perot Interferometer

Sketch of mechanical stage and mirror mounts
(from John Sandercock's 1993 manual)



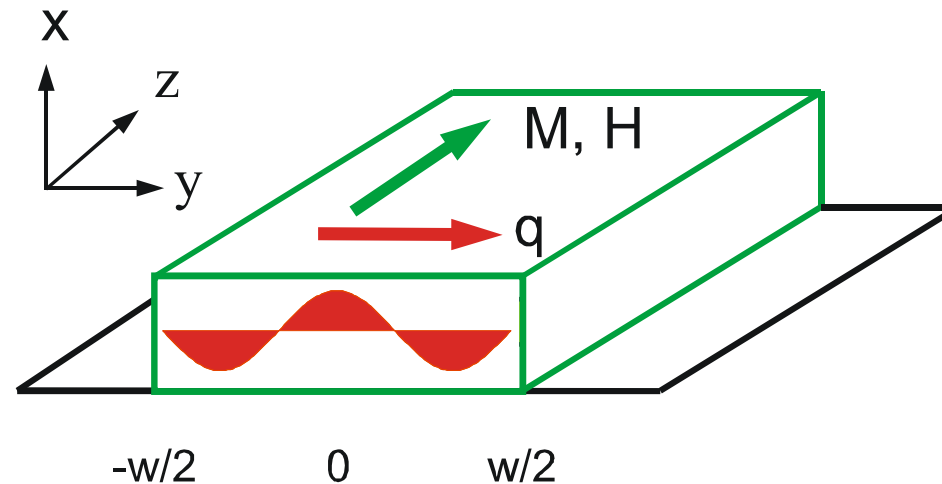


O. Büttner *et al.*, PRB **61**, 11576 (2000)

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- ➔ Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

Confinement to magnetic objects:

quantized eigen modes („standing spin waves“)



⇒ Find dynamic ground state, i.e., eigenmode spectrum

Problems:

- correct boundary conditions
- modes in inhomogeneously magnetized structures

Patterned magnetic films

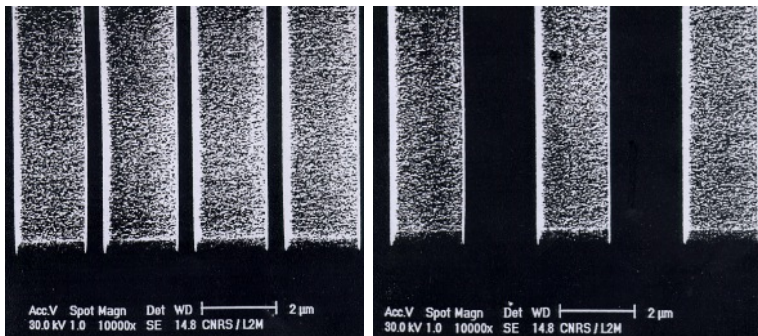
Au / Ni₈₁Fe₁₉ (220nm) / SiO₂ / Si

preparation: e-beam evaporation in UHV

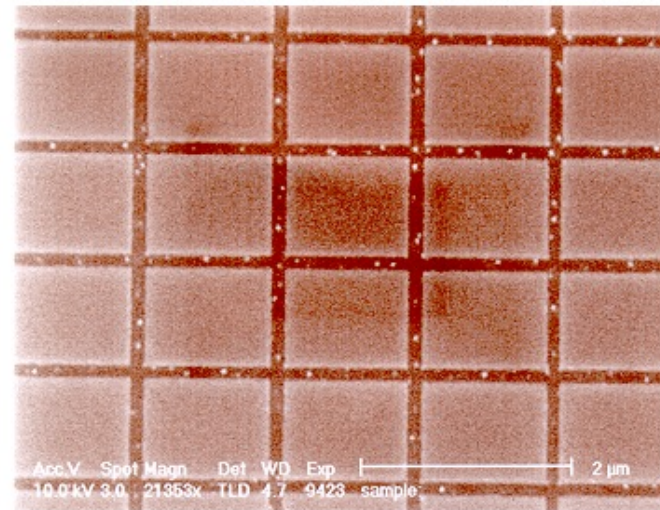
coercivity: H_c = 1-2 Oe

patterning: x-ray lithography (LURE, France)

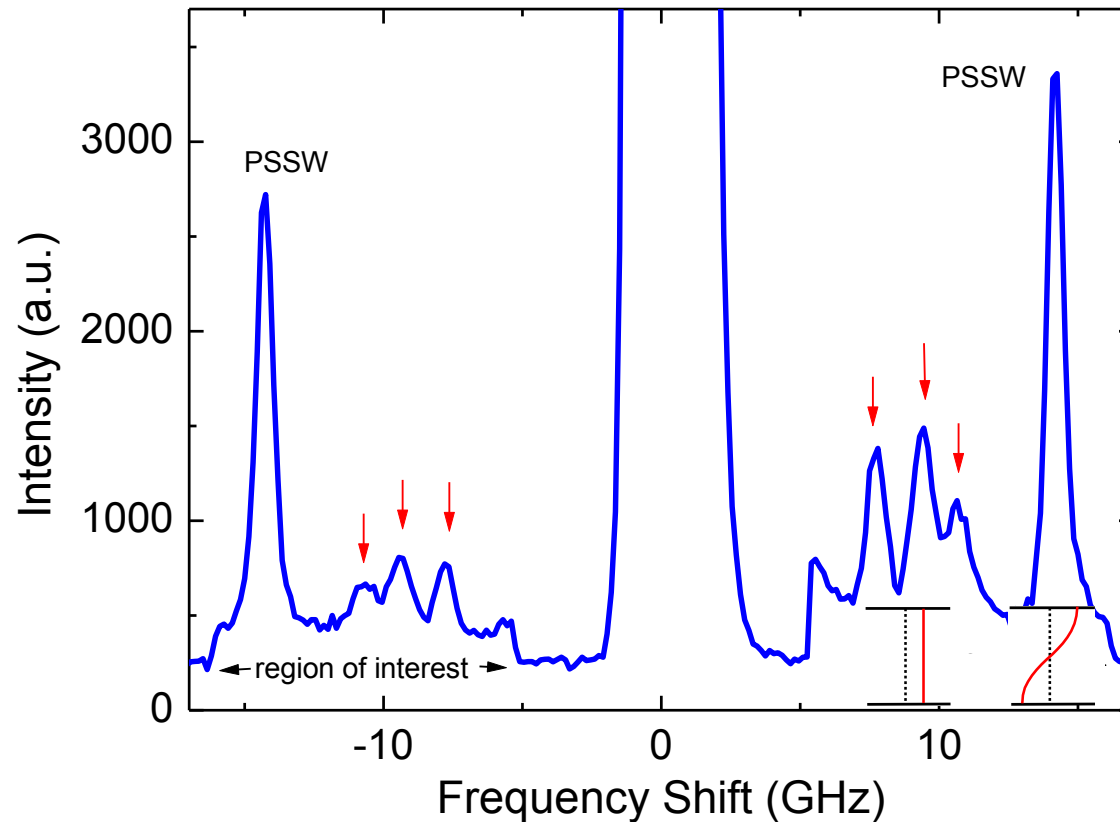
Wires:



Dots:

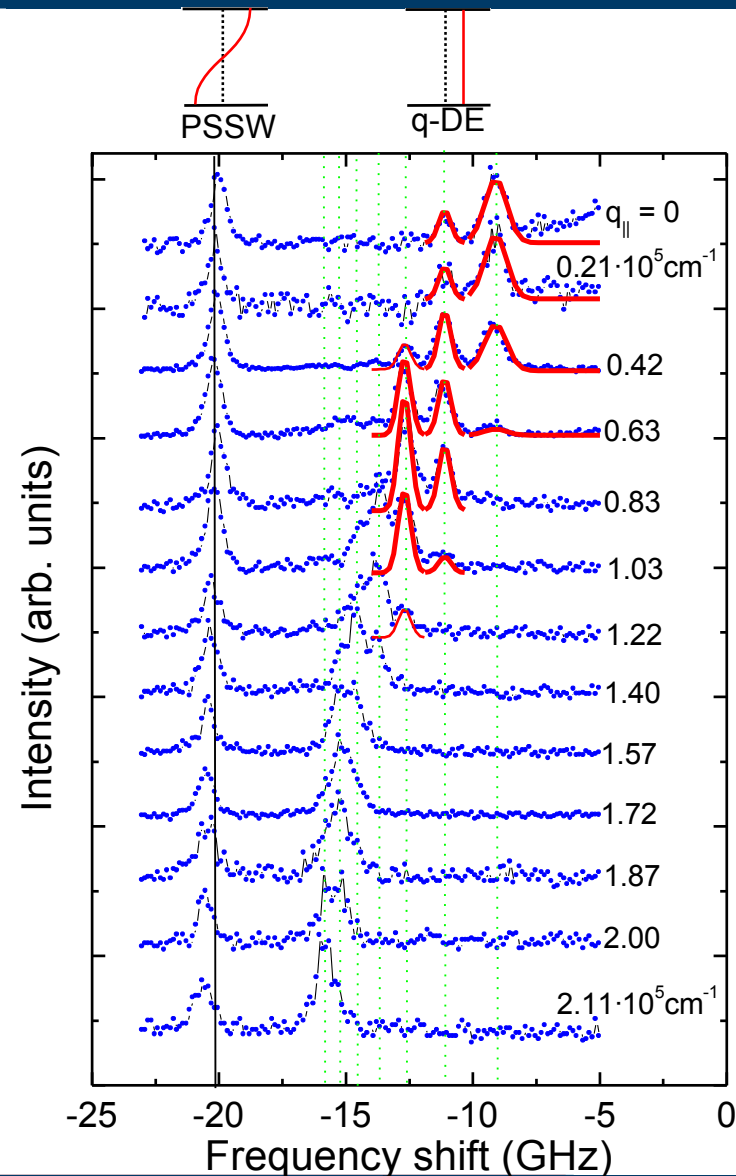


- Damon-Eshbach mode geometry:

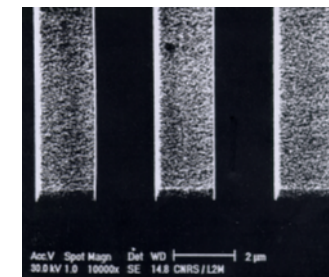
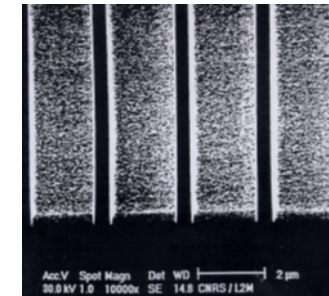
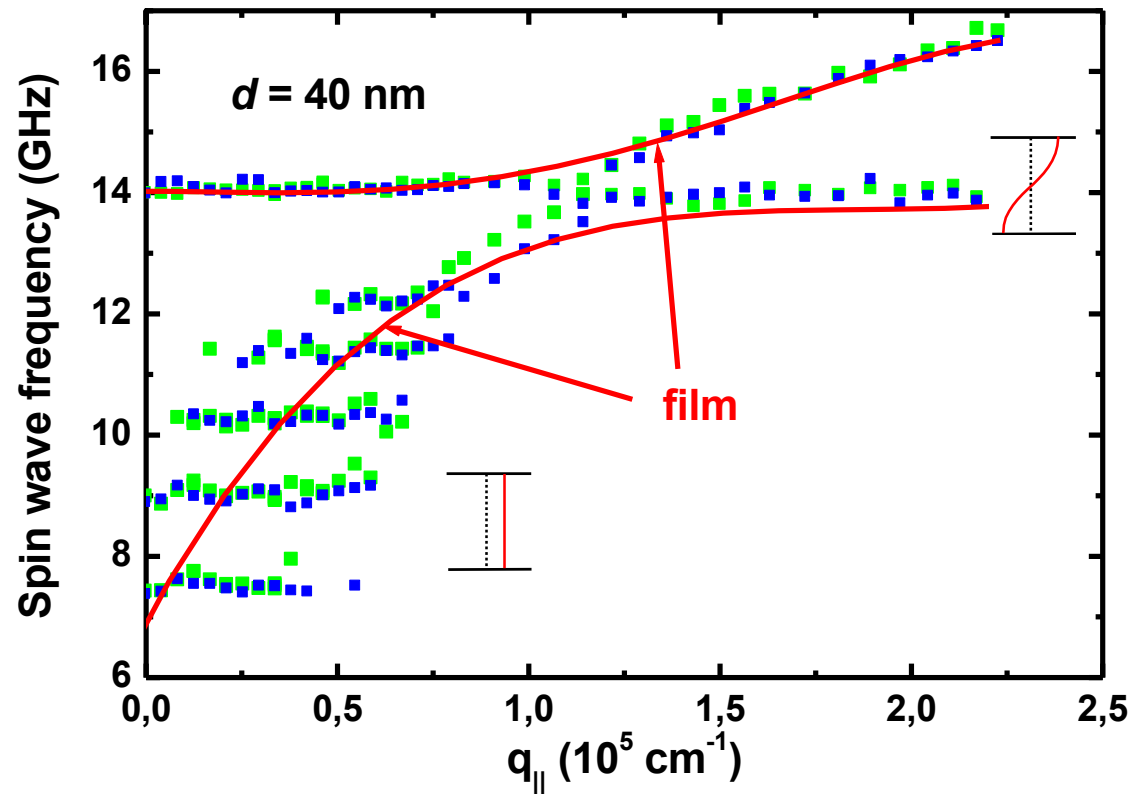


many modes with the same transferred wavevector but with different frequencies

35 mm NiFe wires
 1.75/0.3 μm



localized
 dispersionless modes



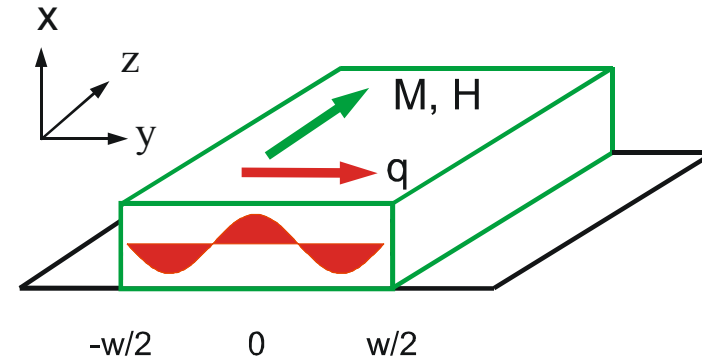
40 nm NiFe stripes

■ 1.8/0.7 μm

■ 1.8/2.2 μm

no interaction, single-stripe effect
 spin wave **quantization** in a single stripe

Standing lateral modes:



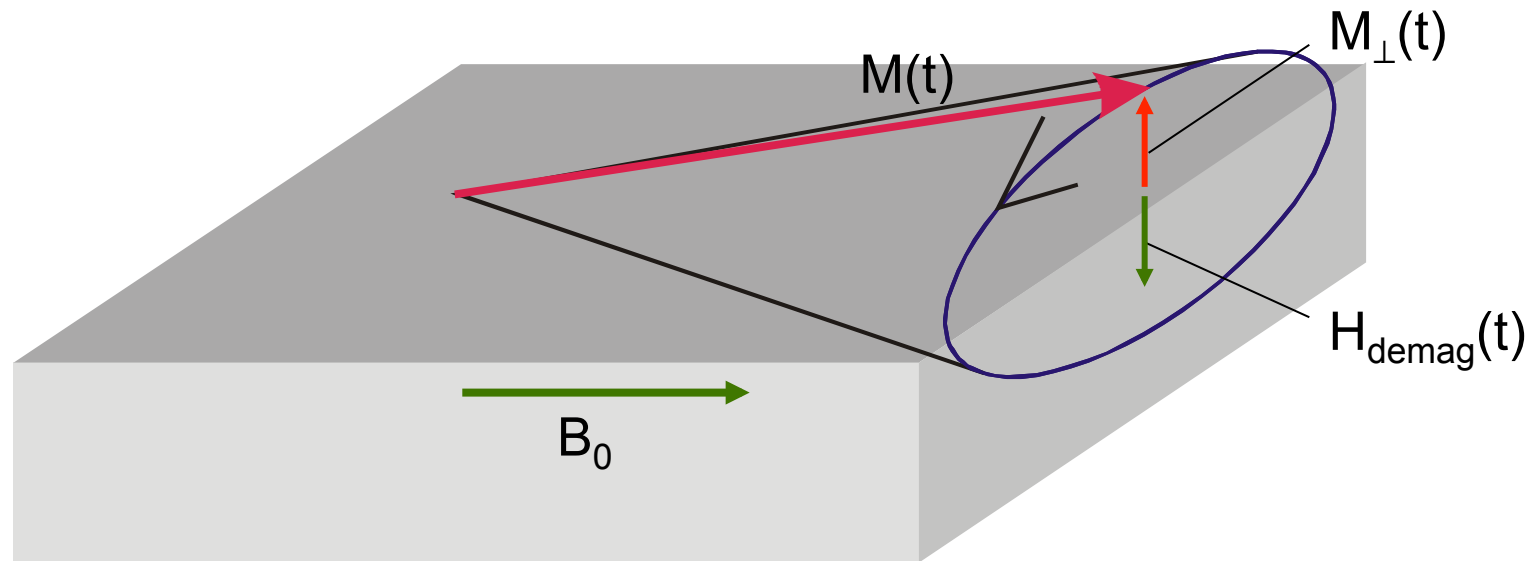
Standing lateral modes

- propagating dipolar modes (Damon-Eshbach modes) perpendicular to wires: "standing lateral modes"
- quantization condition due to the lateral edges:

$$w = n \lambda_{\text{spin wave}}/2;$$

$$q_n = 2\pi/\lambda_{\text{spin wave}} = \pi n/w; \quad n = 1, 2, 3, \dots$$
- boundary conditions (open – pinned)
 - take dynamic stray fields into account
- calculation of frequencies by inserting q_n into Damon-Eshbach equation of motion

Boundary conditions for dynamic magnetization



Precessing magnetization has dynamic out-of-plane component
⇒ dynamic stray fields and thus dynamic surface torque on magnetization

New dynamic dipole boundary condition for non-elliptical elements:

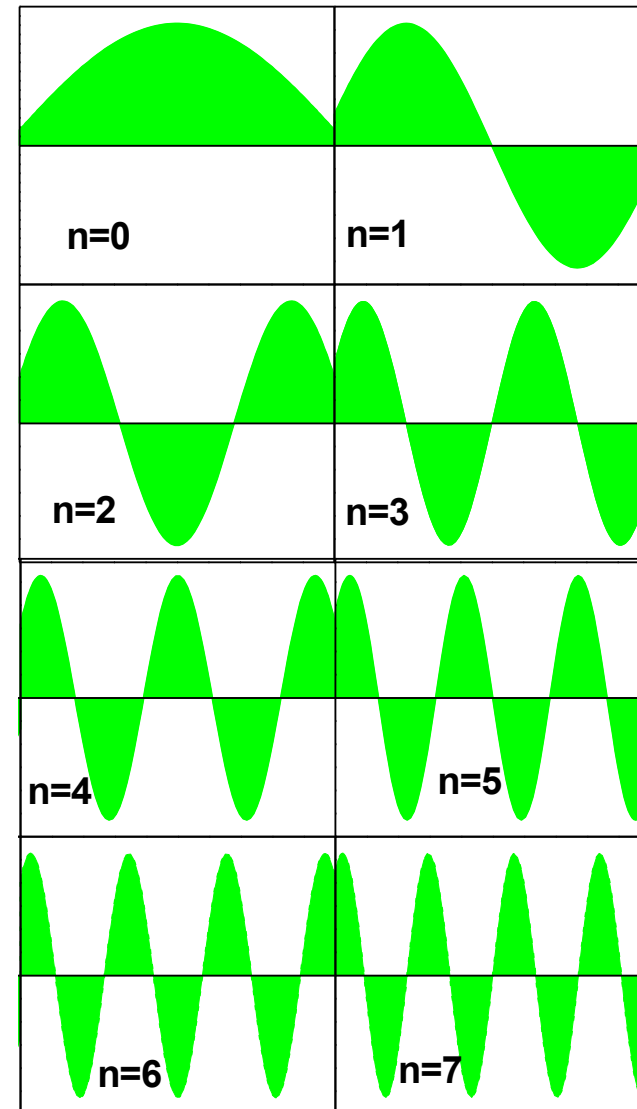
$$\frac{\partial m}{\partial n} + \frac{1}{\xi_D} m = 0$$

$$\xi_D = \frac{t}{2\pi} \left(1 + 2 \ln \frac{w}{t} \right)$$

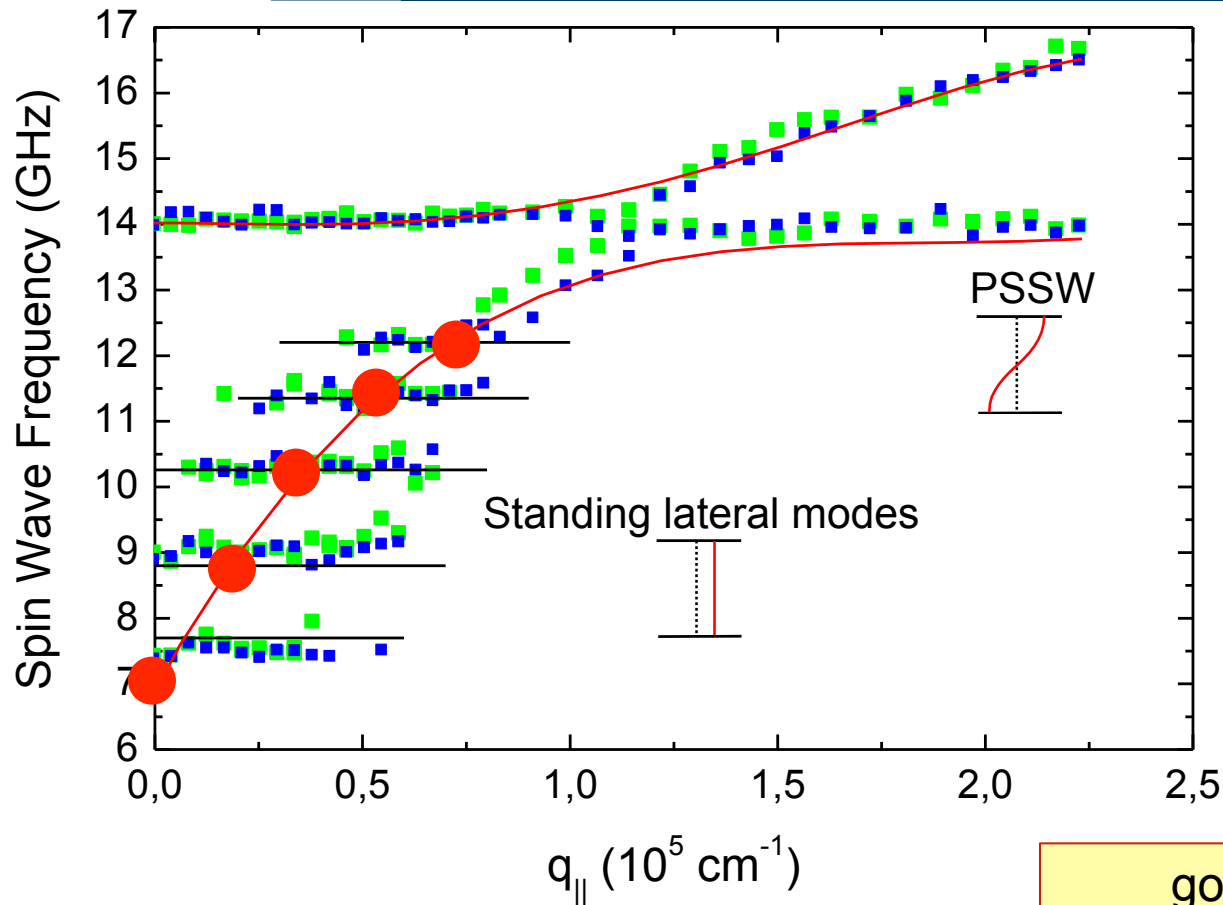
- takes dynamic stray fields into account

low-index modes ($\lambda \gg \xi_D$) „pinned“

high-index modes ($\lambda \approx \xi_D$) „unpinned“



Frequencies of the quantized modes



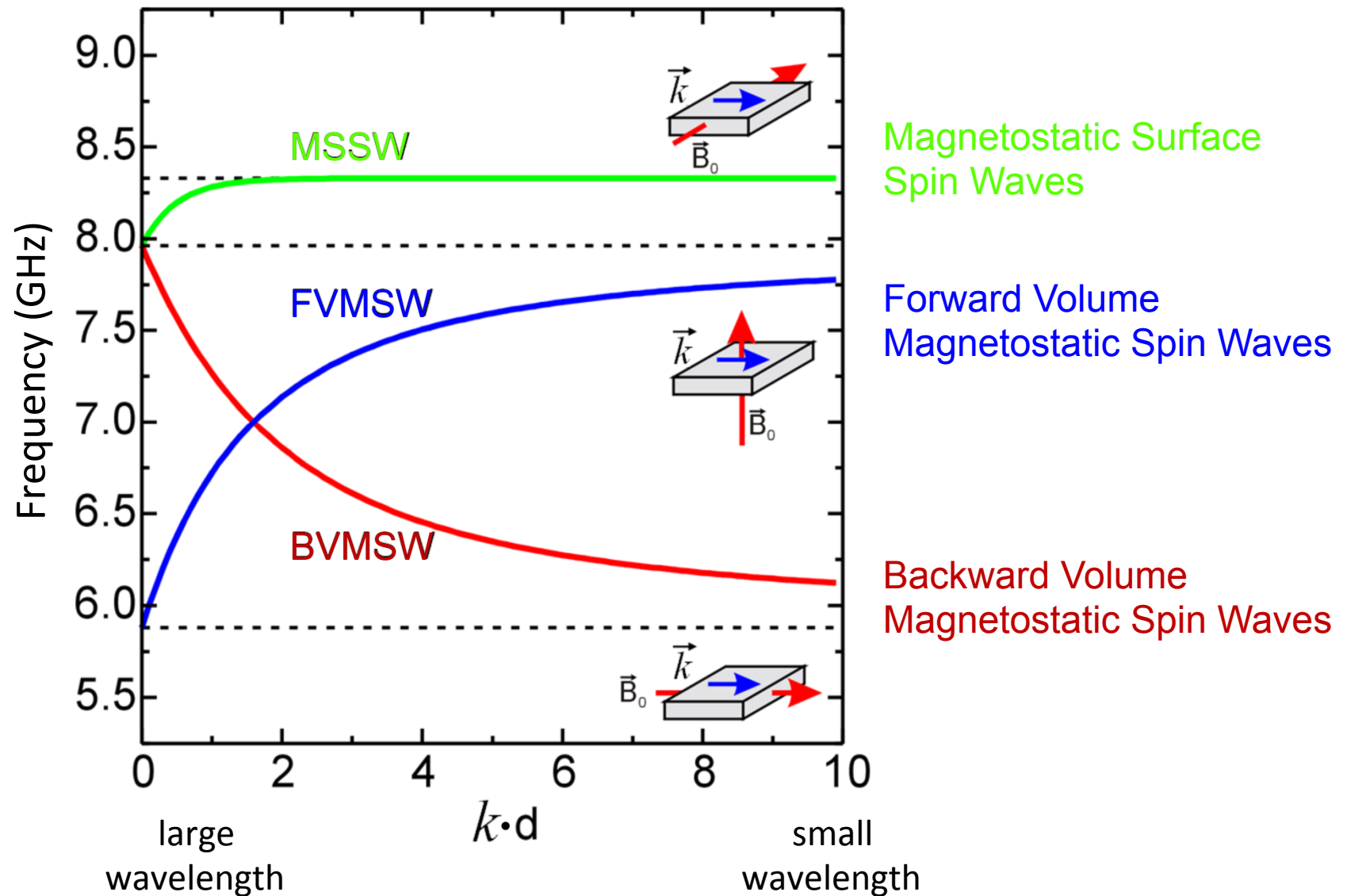
width:
 1.8 μm
 separation:
 0.7 μm (green)
 2.2 μm (blue)
 thickness: 50 nm

good quantitative
 agreement between the
 theory and the experiment
 is obtained

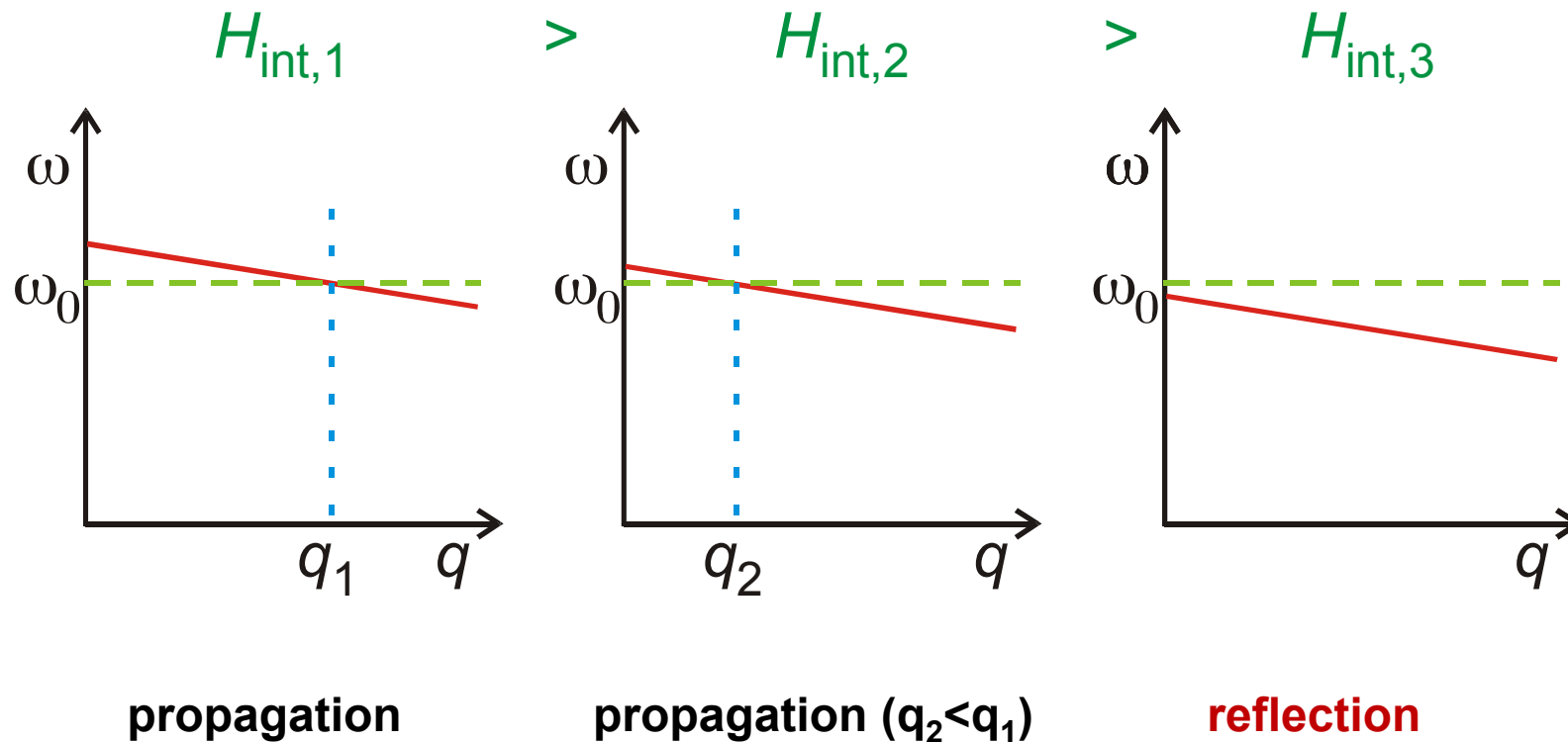
C. Mathieu et al., PRL **81**, 3968 (1998)

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- ➔ Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

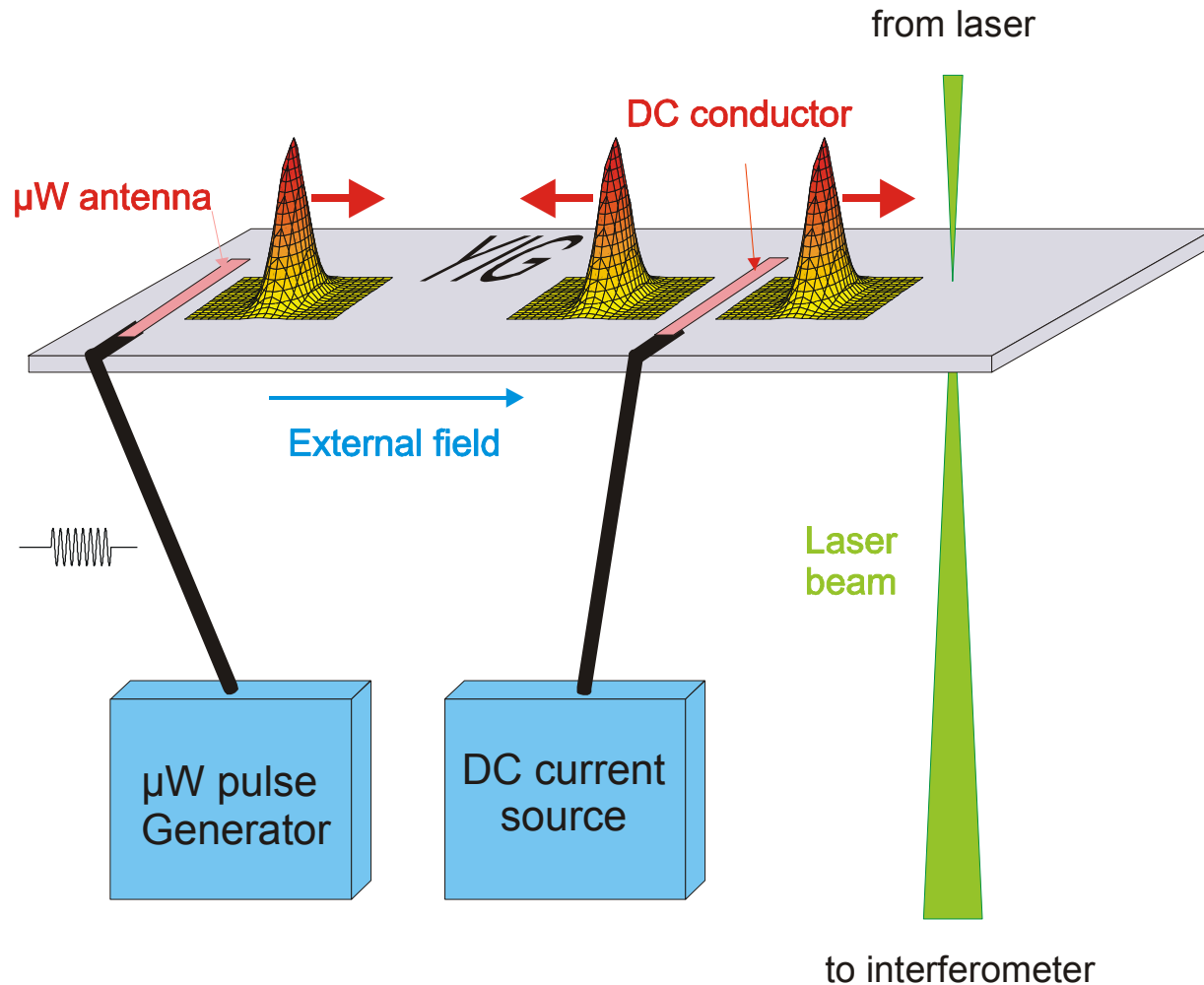
Dipolar spin waves



Motion of a spin wave packet in varying field



Real-time observation of spin wave propagation



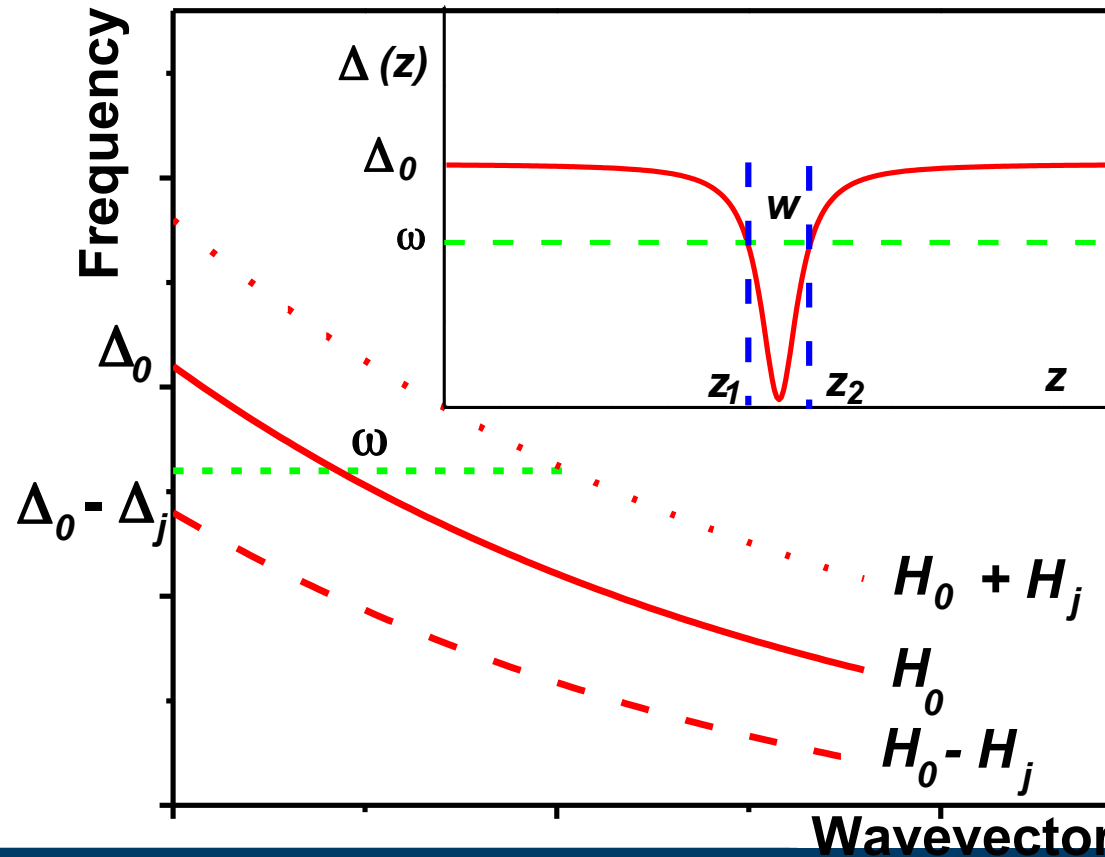
SW-pulses created by
microwaves and
 detected by
light scattering with
time and space
 resolution

DC conductor provides
 a local
 field inhomogeneity

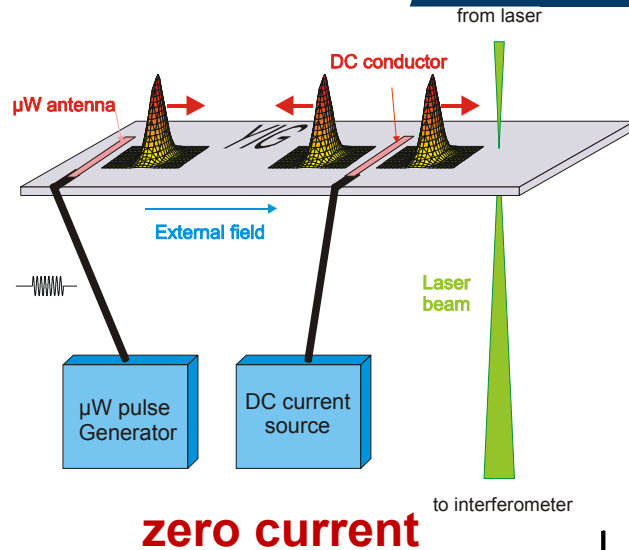
Spin wave tunneling



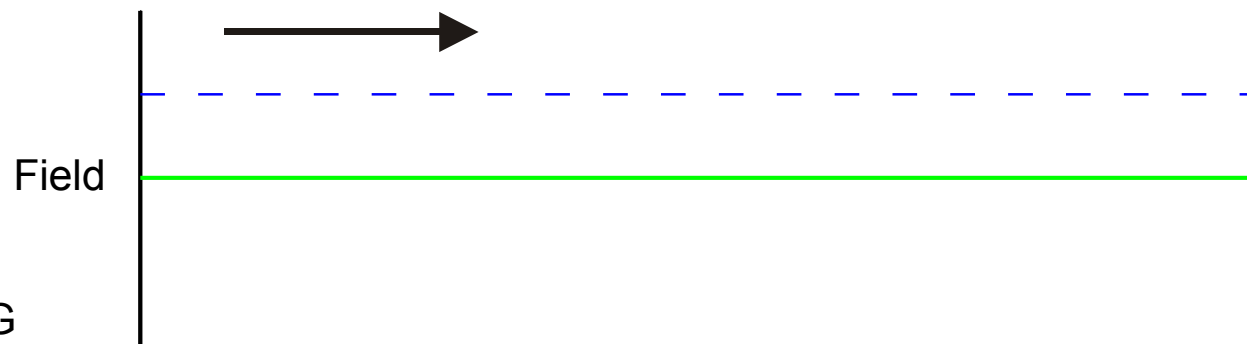
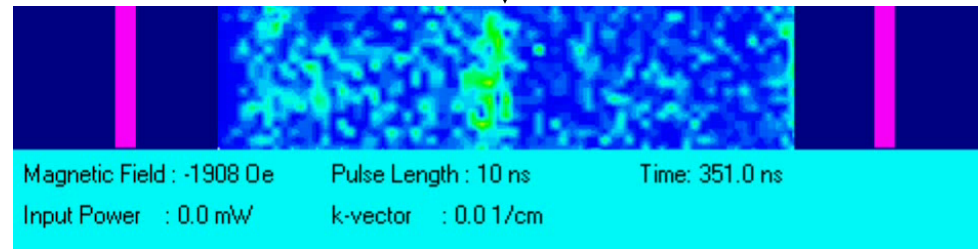
Δ : zero-wavevector gap



Spin wave pulse propagation



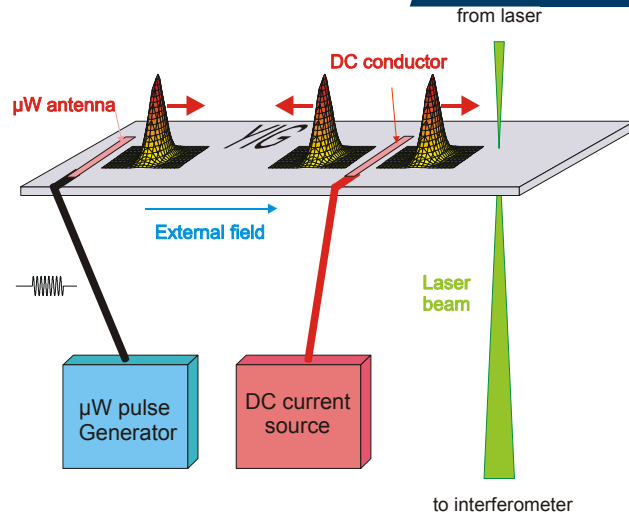
position of the dc conductor



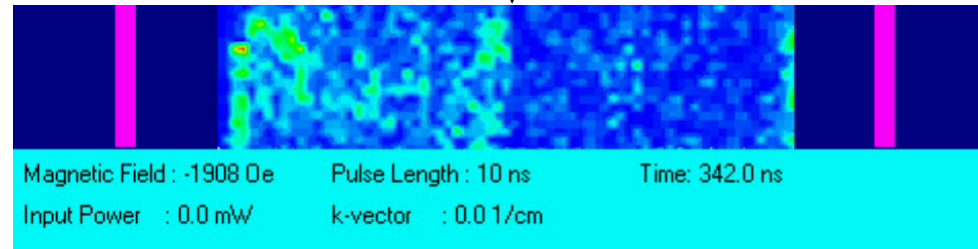
magnetic material: YIG

S.O. Demokritov et al., Phys. Rev. Lett. **93**, 047201 (2004)

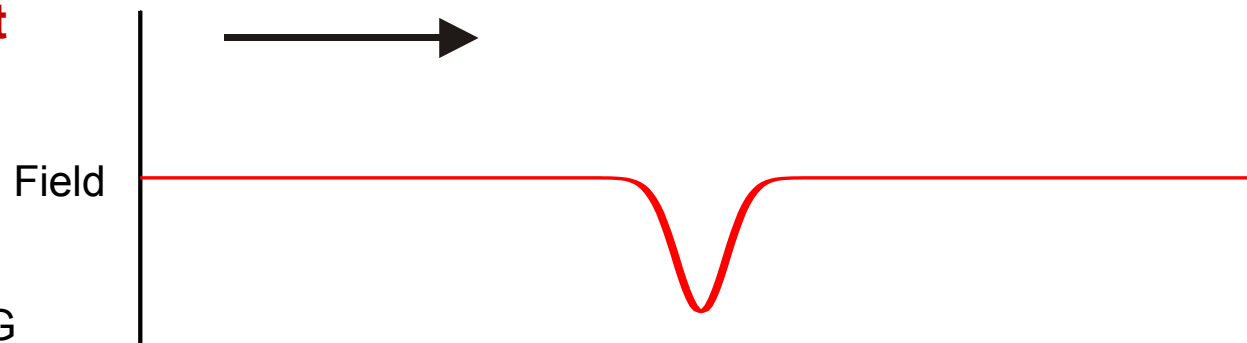
Spin wave pulse propagation



position of the dc conductor



positive current



magnetic material: YIG

dip in field acts like potential barrier

Potential barrier: reflection and tunneling

S.O. Demokritov et al., Phys. Rev. Lett. **93**, 047201 (2004)

Reflection of spin wave at barrier and spin wave tunneling

Carrier frequency:
7.125 GHz

Bias field:
1836 Oe

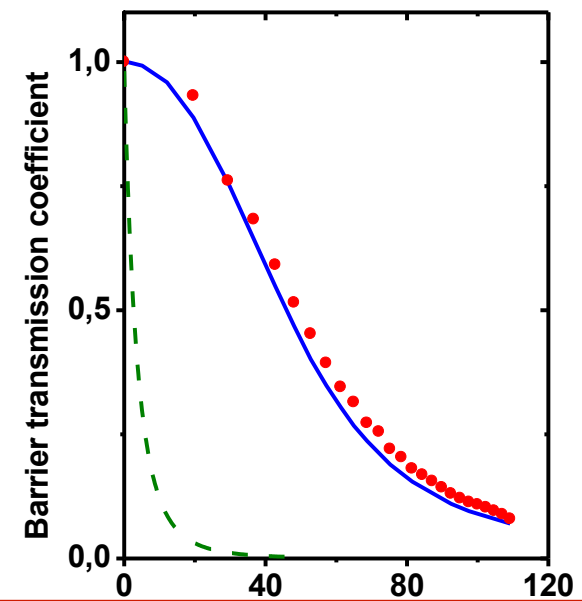
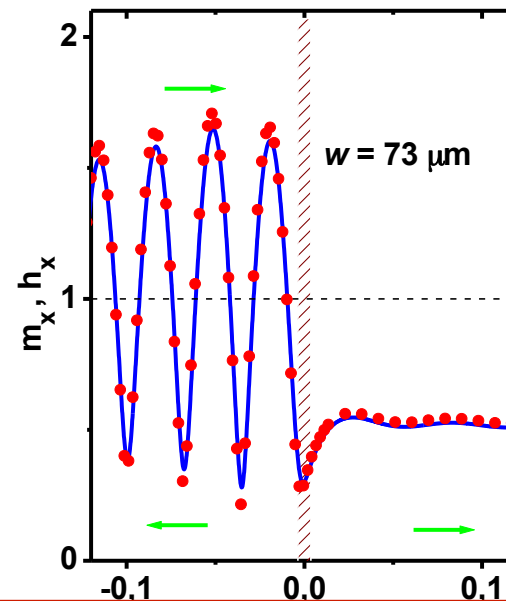
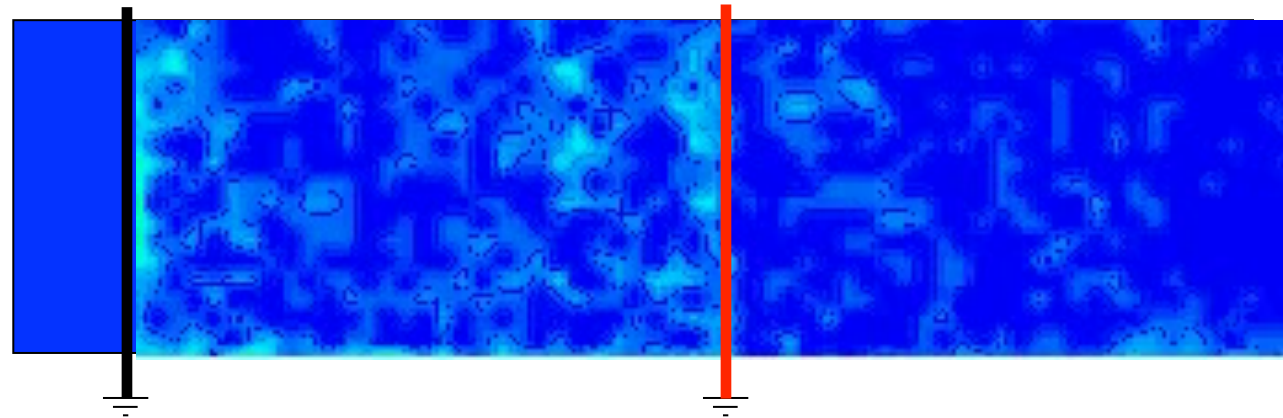
Wave number:
112 rad/cm

Group velocity:
 ≈ 30 km/s

Film thickness:
5.7 μm

Scan region:
 6.0×1.8 mm²

Logarithmic scale



Non-exponential decrease of spin wave intensity with barrier size

Spin wave Fabry-Perot

Carrier frequency:
7.125 GHz

Bias field:
1836 Oe

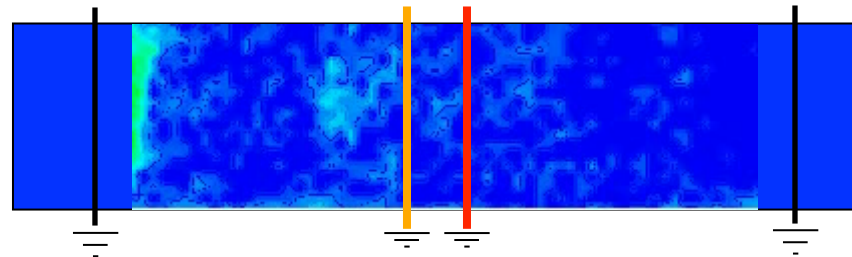
Wave number:
112 rad/cm

Group velocity:
 ≈ 30 km/s

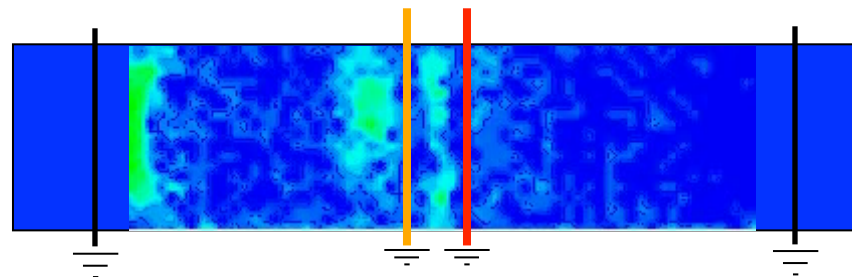
Film thickness:
5.7 μm

Scan region:
 6.0×1.8 mm²

Logarithmic scale



**Short SW pulse
18 ns**



**Long SW pulse
40 ns**

Spin wave tunneling through mechanical gap

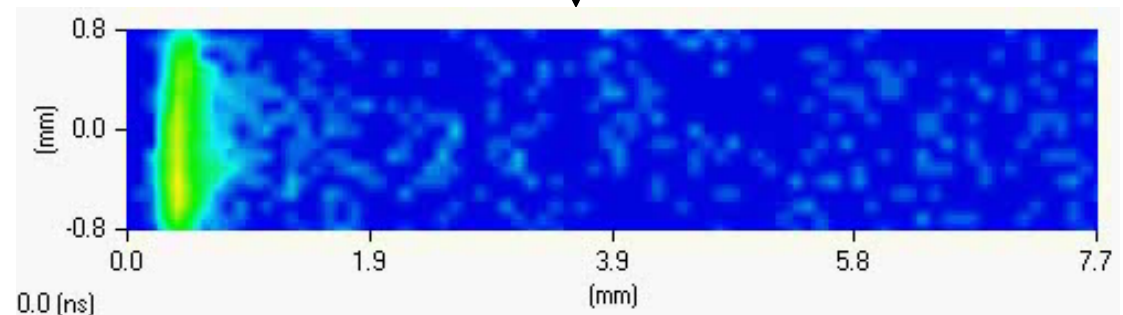
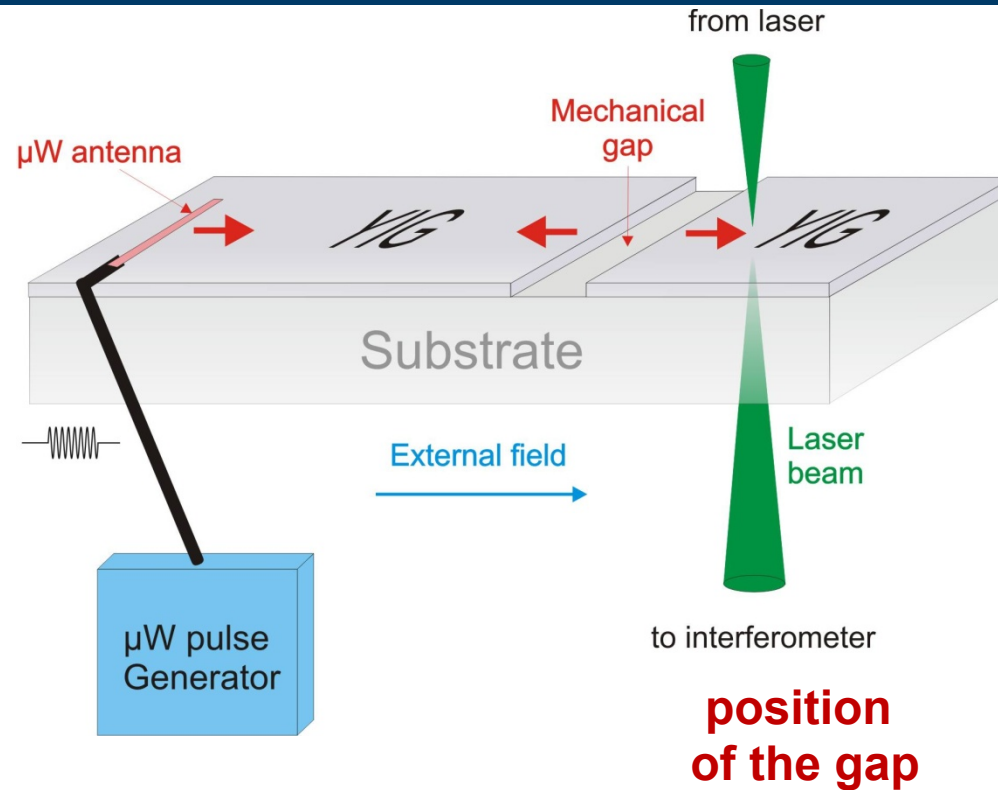
Film thickness:
6 μm

Gap width:
20 μm

Frequency:
7.125 GHz

Magnetic field:
1835 Oe

Logarithmic scale



Spin wave cavity

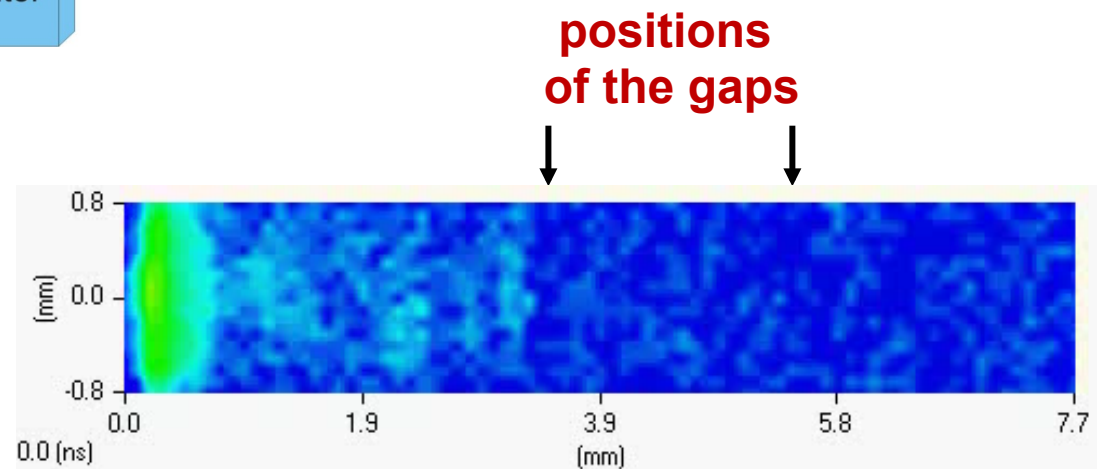
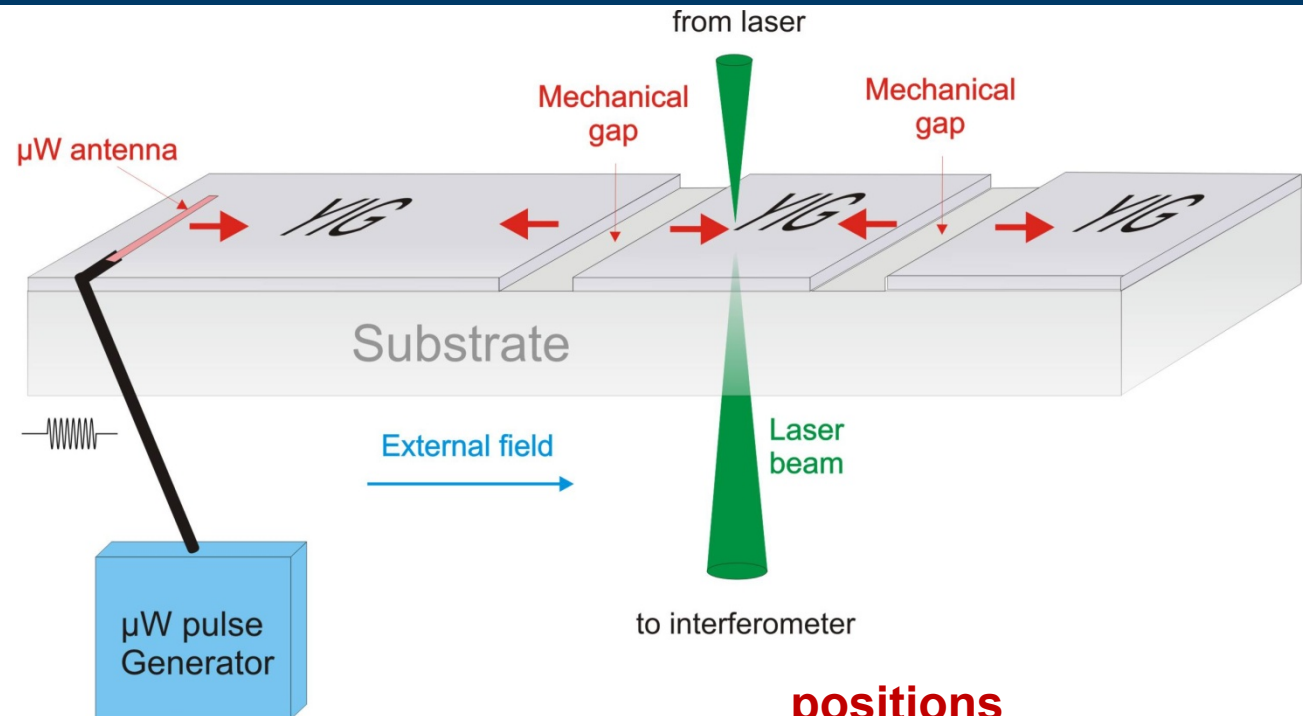
Film thickness:
6 μm

Gap width:
20 μm

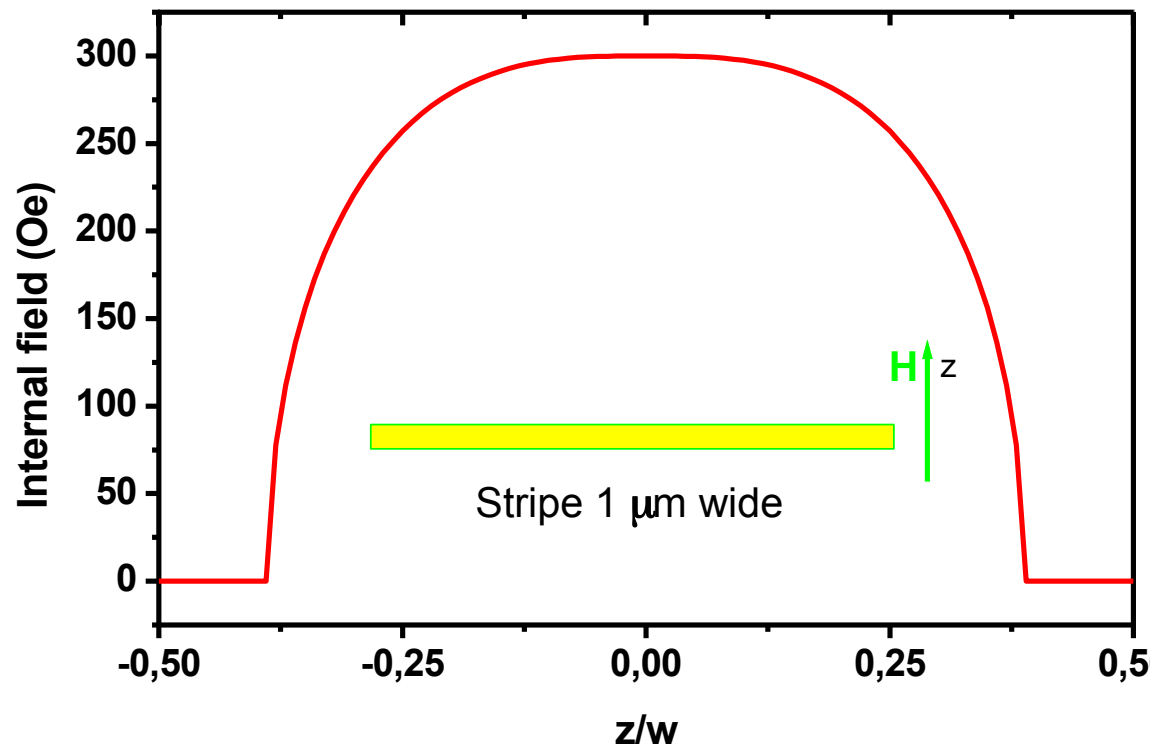
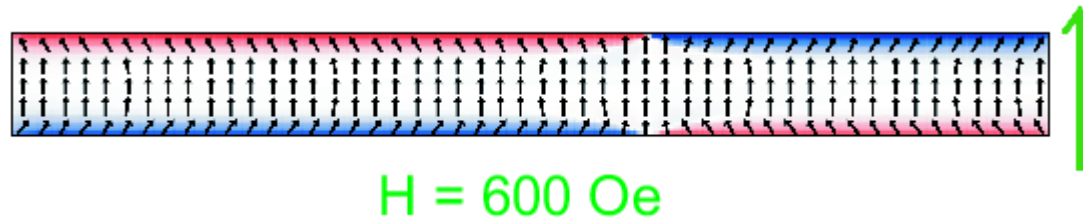
Frequency:
7.125 GHz

Magnetic field:
1839 Oe

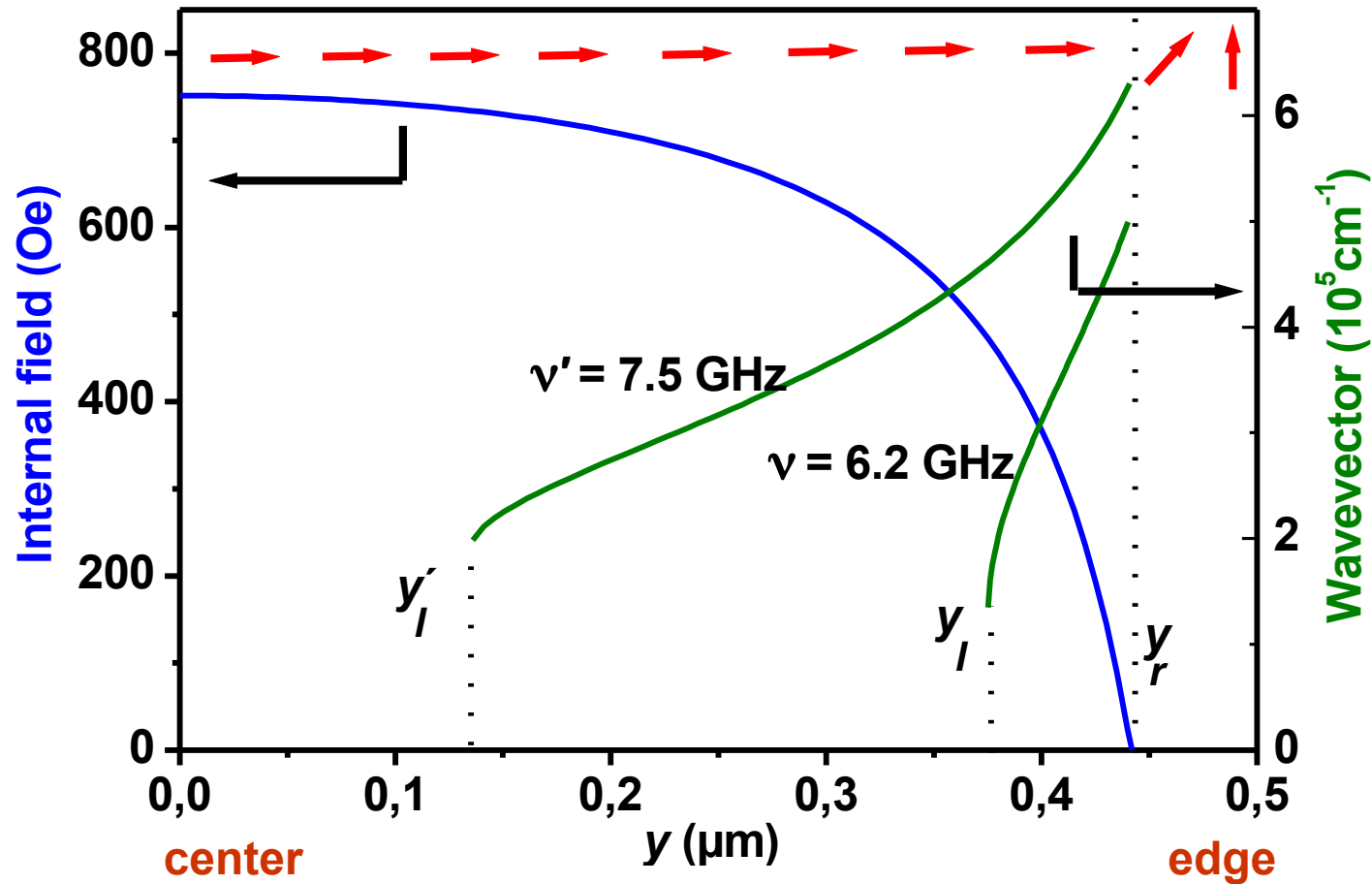
Logarithmic scale



Application: Spin waves in films with internal field distribution

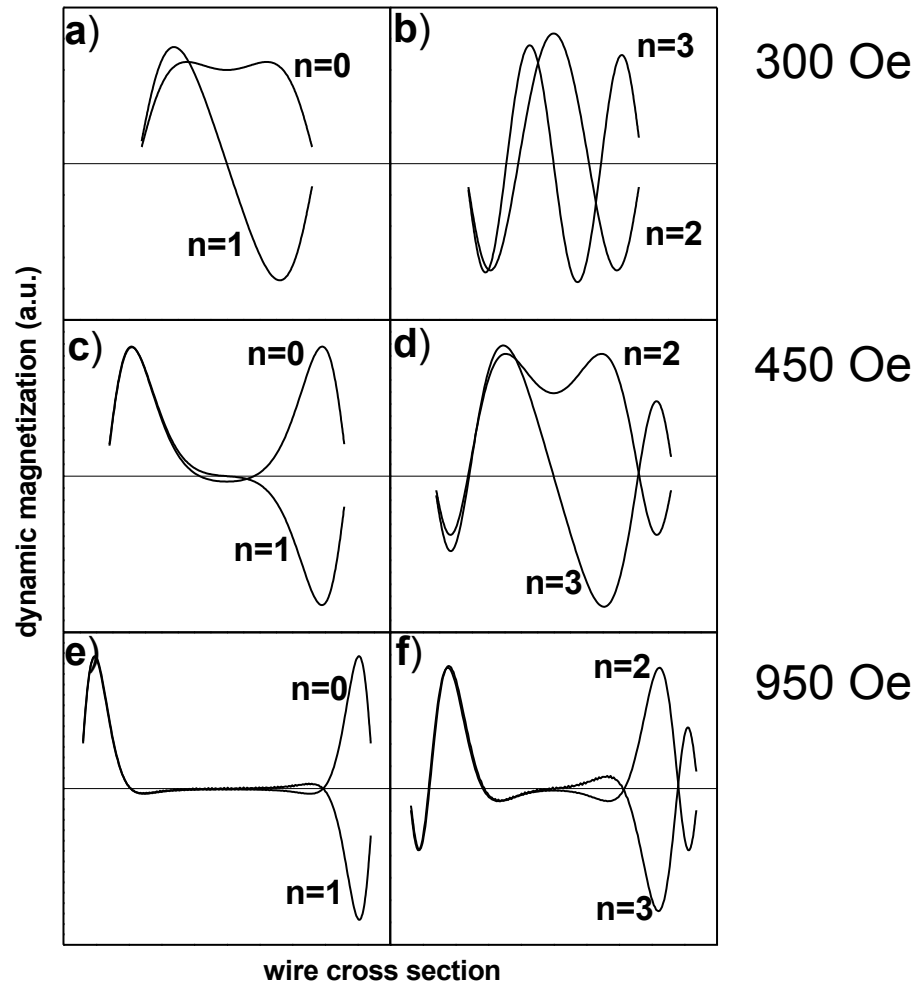


Regions with canted magnetization and zero internal field are located near the edges of the stripe



y_l', y_l : turning points
 $y_r - y_l$: localization length

Calculated mode profiles



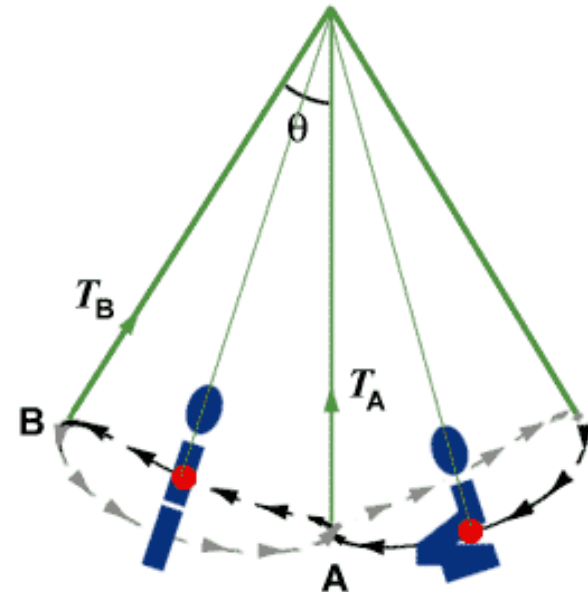
Green's function approach
using correct internal field
distribution

Numerical evaluation of the
resulting integro-differential
equations

Increasing localization
with increasing
internal field

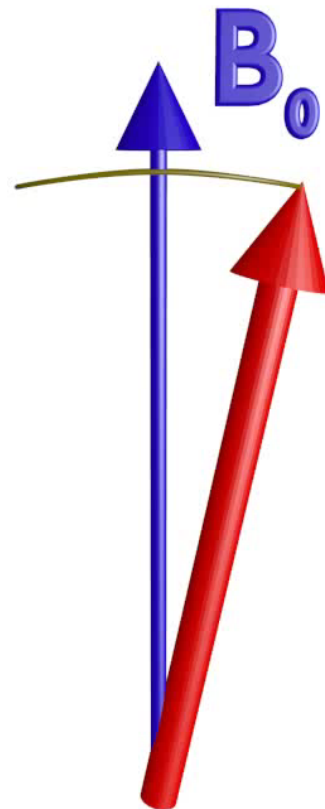
- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- ➔ Parametric Generation and Amplification of Spin Waves

Parametric amplification of spin waves

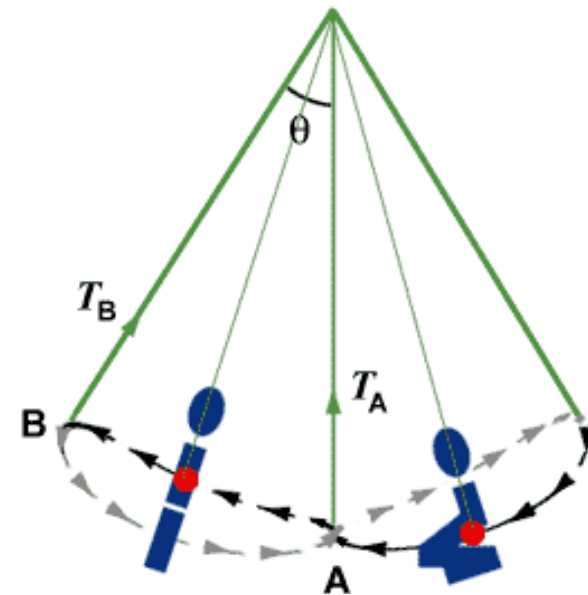


www.hk-phy.org/articles/swing/swing_e.html

Parametric amplification of spin waves



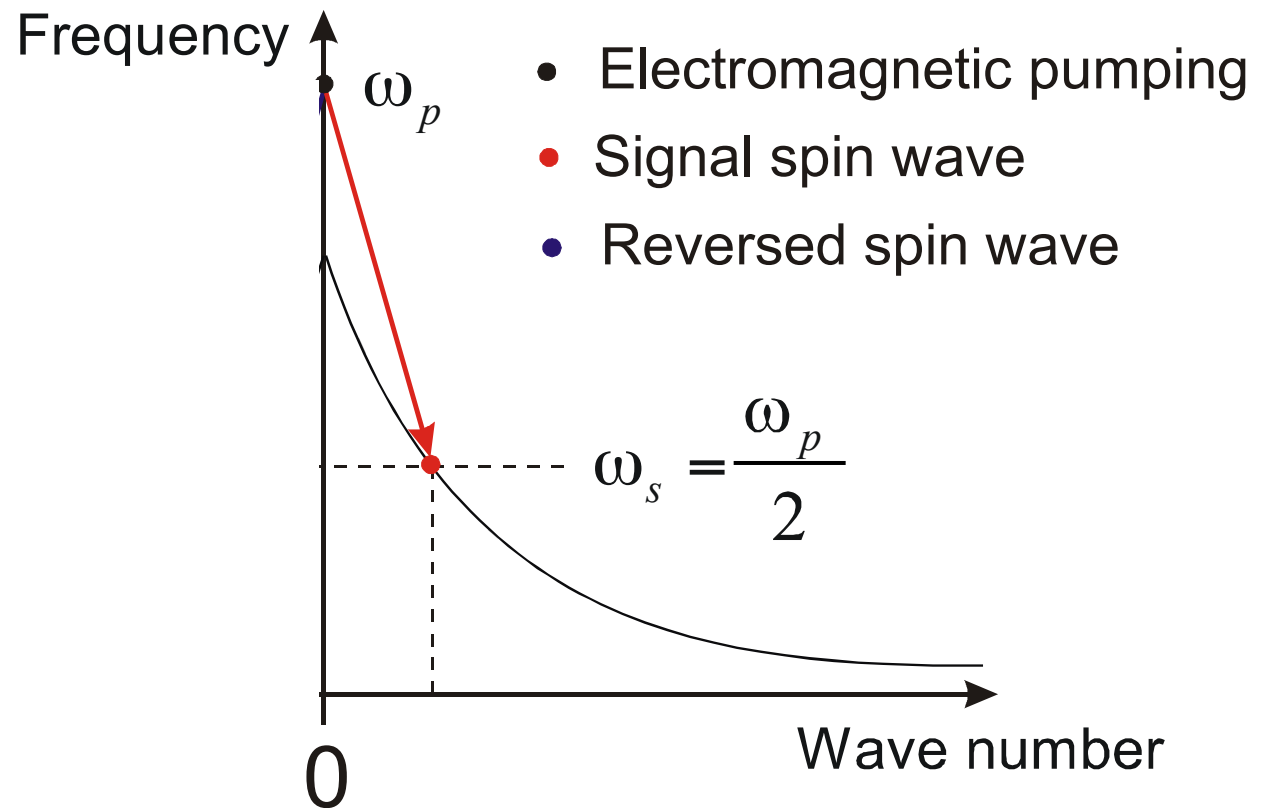
Parametric amplification is
swinging of spin waves



www.hk-phy.org/articles/swing/swing_e.html

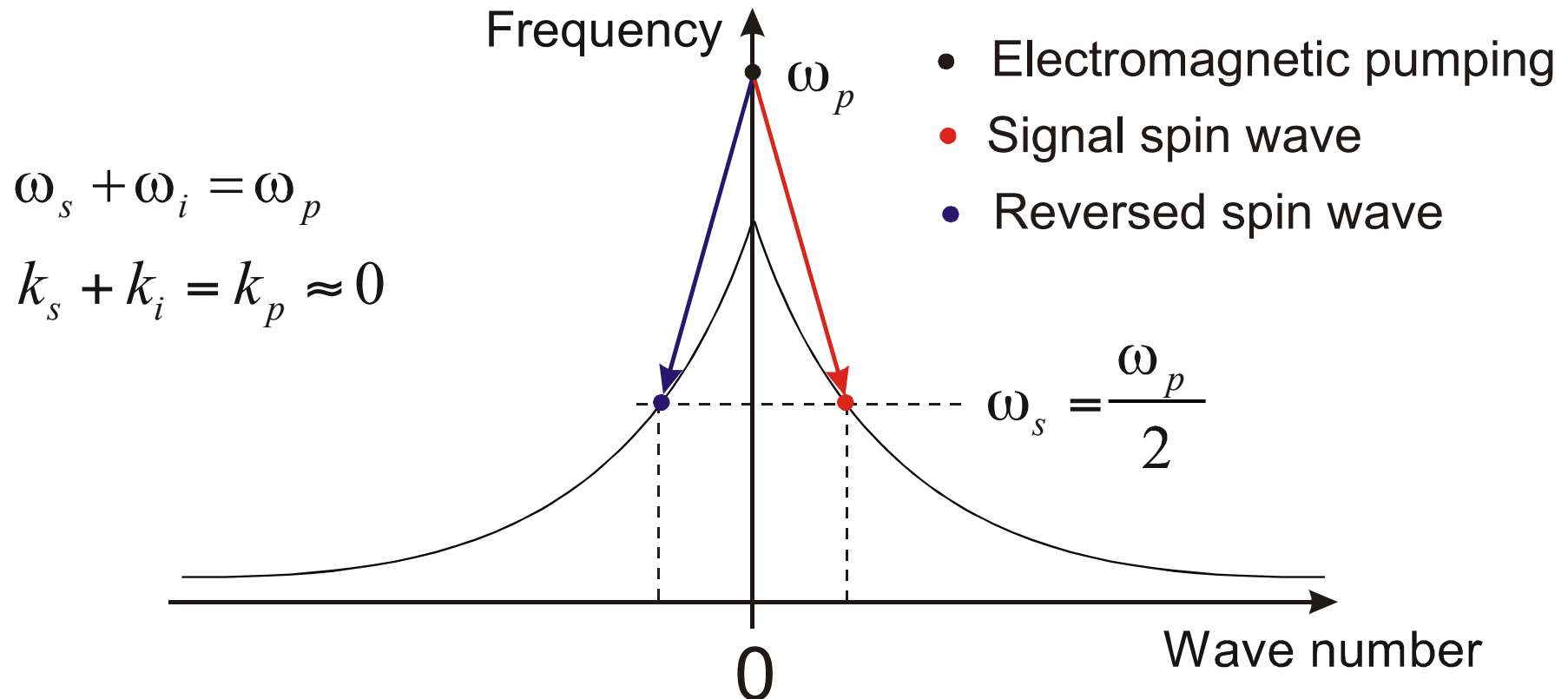
Parametric amplification and phase conjugation

Splitting of a quasi-uniform electromagnetic pumping wave in two contra-propagating spin waves



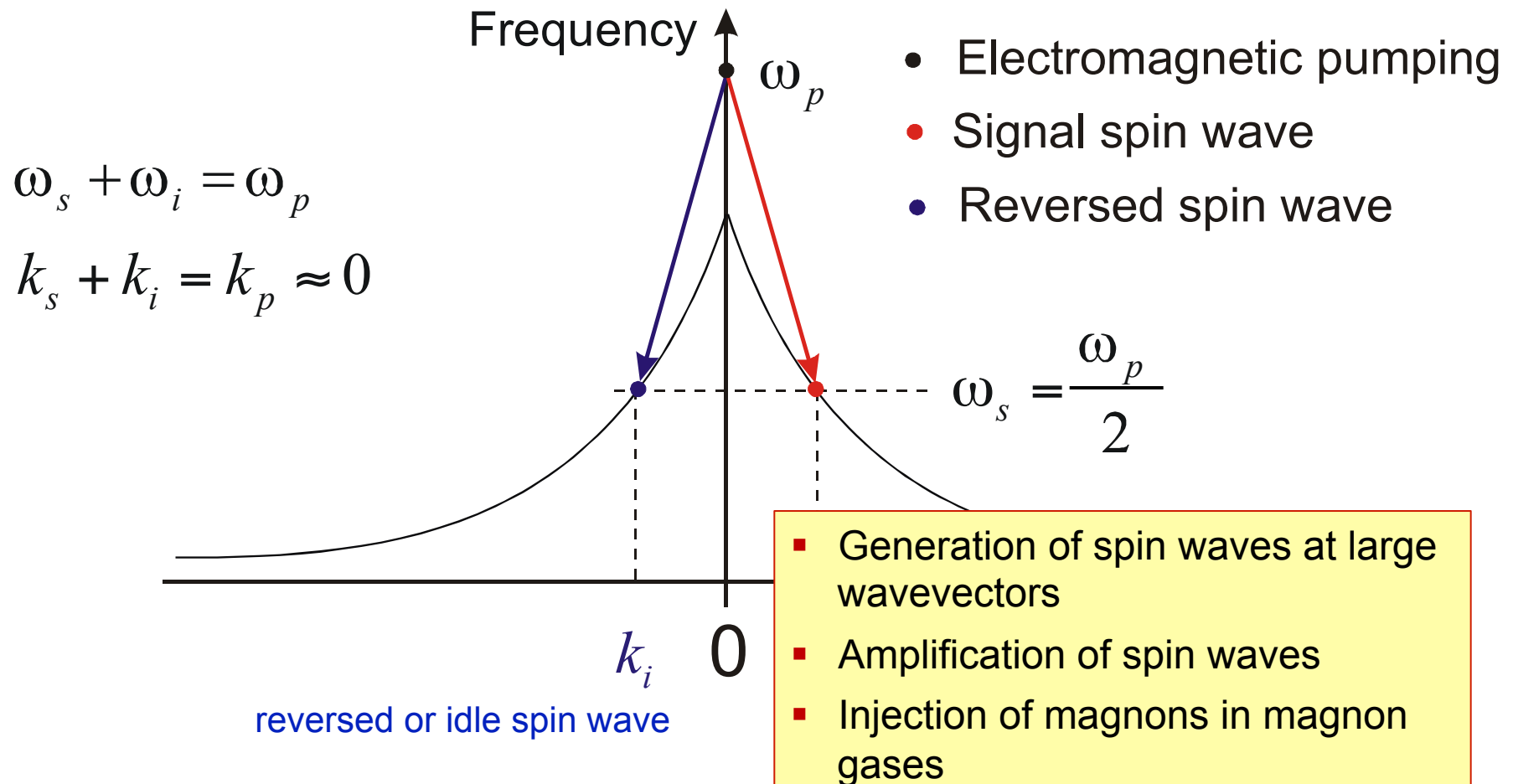
Parametric amplification and phase conjugation

Splitting of a quasi-uniform electromagnetic pumping wave in two contra-propagating spin waves

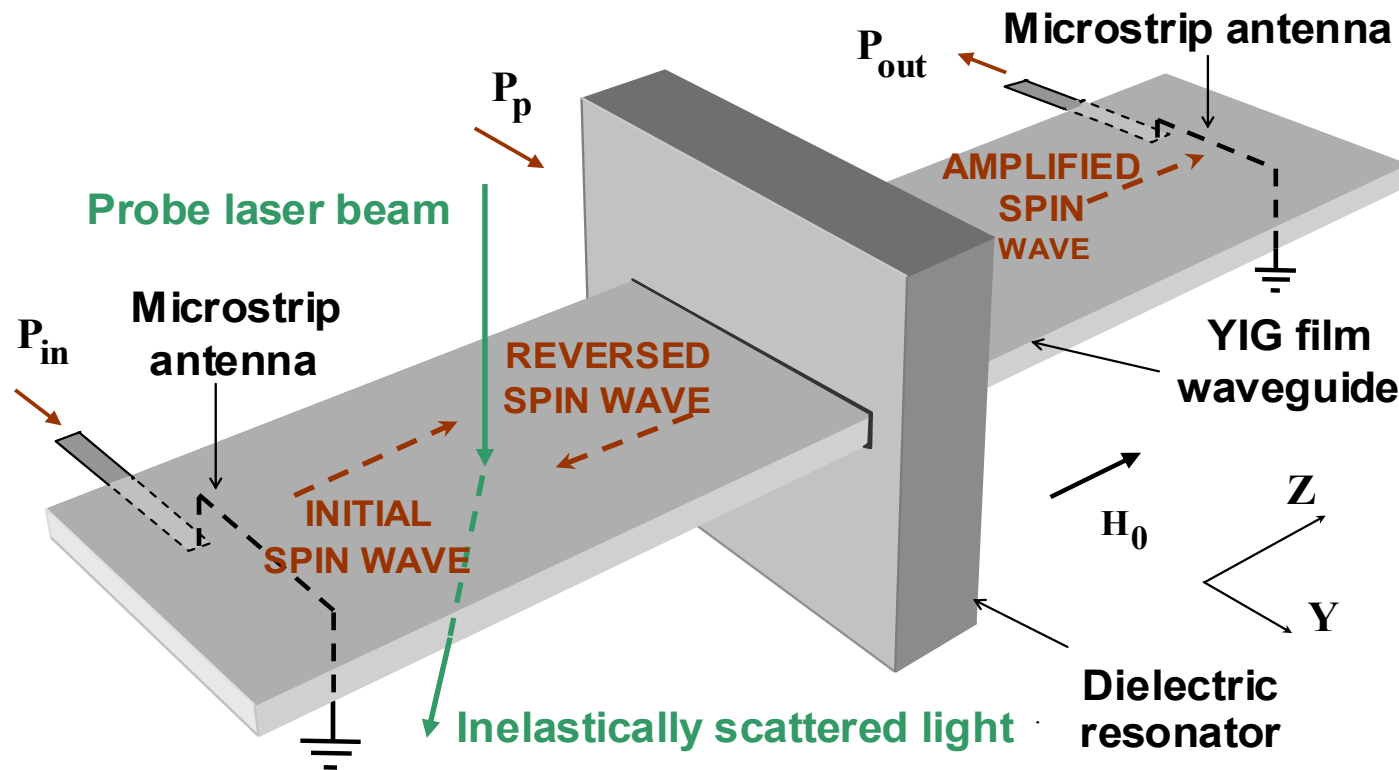


Parametric amplification and phase conjugation

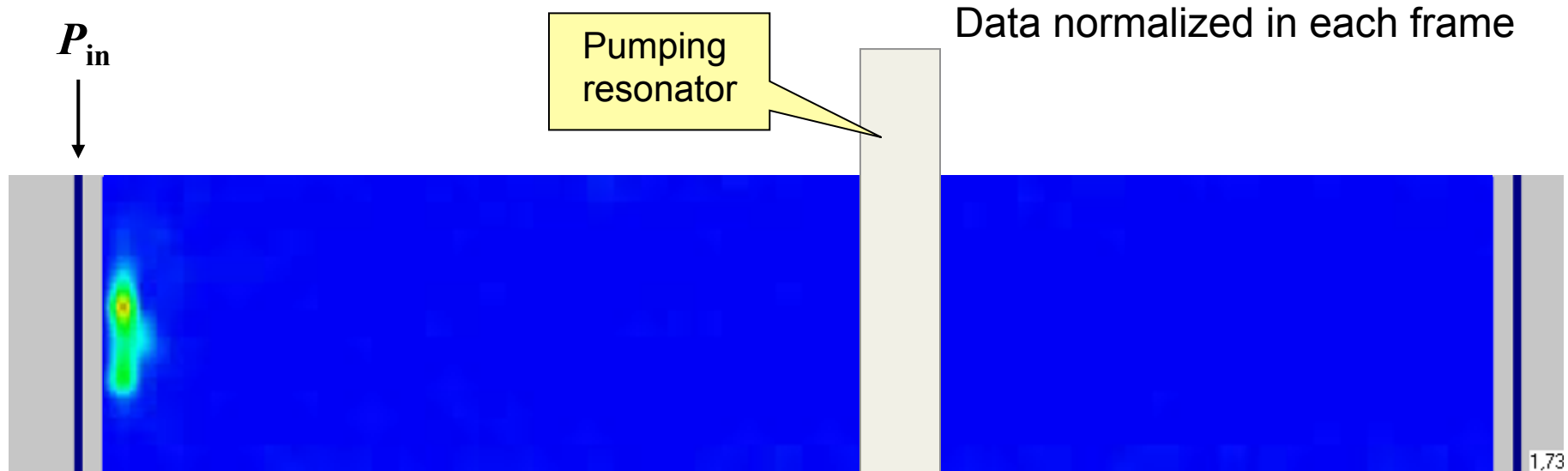
Splitting of a quasi-uniform electromagnetic pumping wave in two contra-propagating spin waves



Bullet amplification (setup)



Parametric amplification and phase conjugation



Input spin wave :

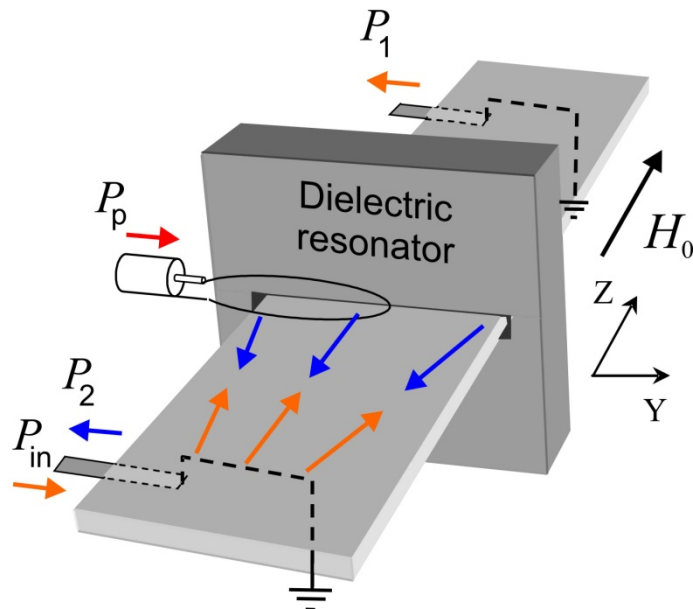
Bias magnetic field	$H_0 = 1827$ Oe
Carrier frequency of SW packet	$f_s = 7.125$ GHz
Carrier wave number of SW packet	$k_s = 180$ rad/cm
Group velocity of SW packet	$v_s = -2.5 \cdot 10^4$ m/s
Signal pulse duration	$\tau_s = 28$ ns
Signal pulse power	$P_{in} = 44$ mW

Pumping pulse :

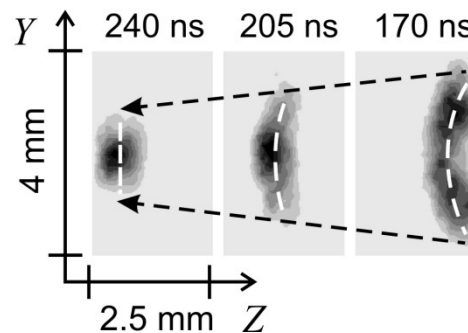
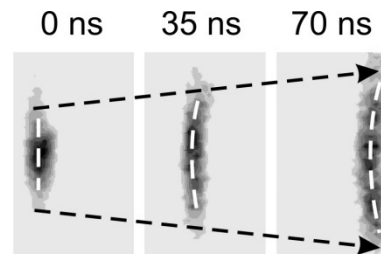
Carrier frequency	$f_p = 2f_s = 14.250$ GHz
Power	$P_p = 8$ W
Duration	$\tau_p = 80$ ns

A.A. Serga et al., J. Appl. Phys. **93**, 8585 (2002)

2D wave front reversal



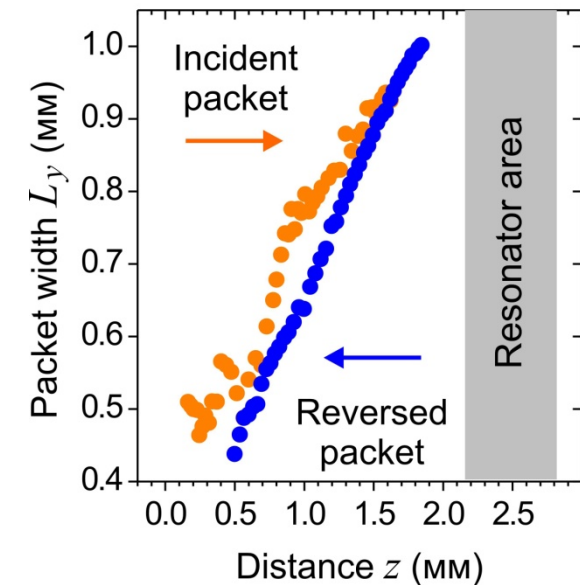
wave front
reversal
observed in
linear regime



Input spin wave :

Bias magnetic field	$H_0 = 800$ Oe
Carrier frequency	$f_s = 3.966$ GHz
Carrier wave number	$k_s = 95$ rad/cm
Group velocity	$v_s = -2.0 \cdot 10^4$ m/s
Pulse duration	$\tau_s = 30$ ns
Pulse power	$P_{in} = 22$ mW

Width of the linear BVMSW packet
as a function of propagation distance



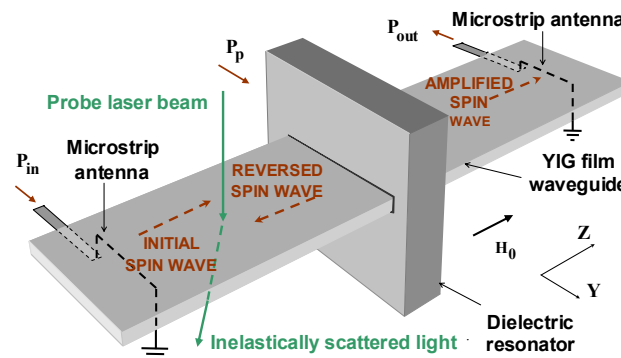
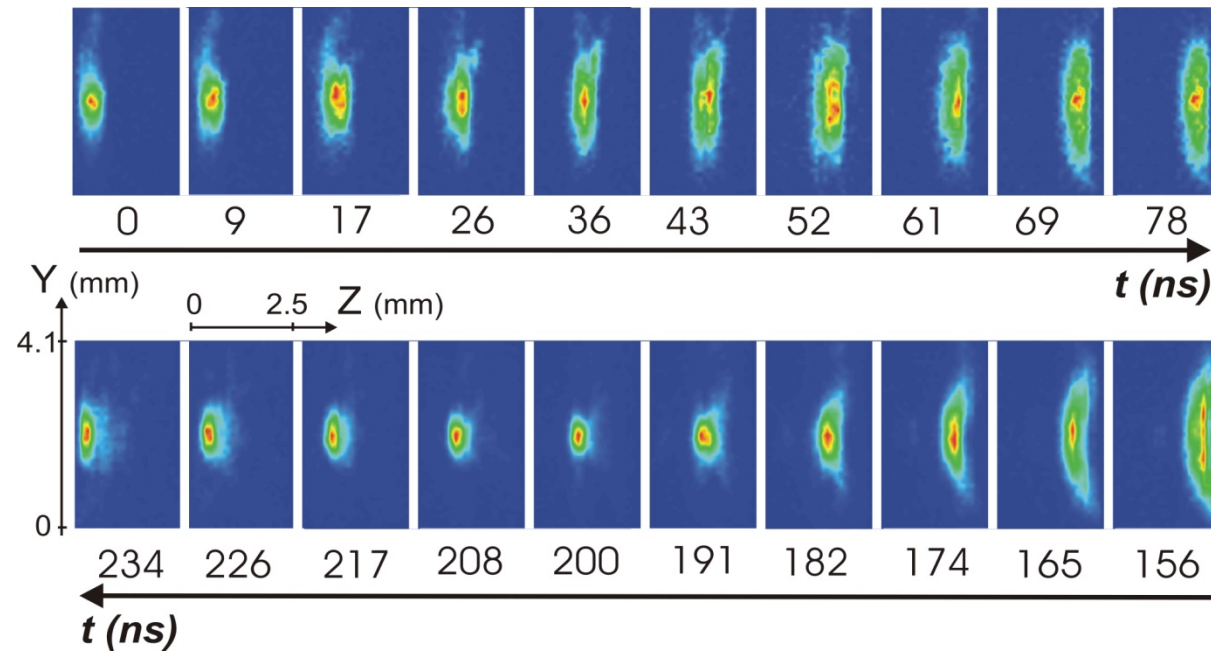
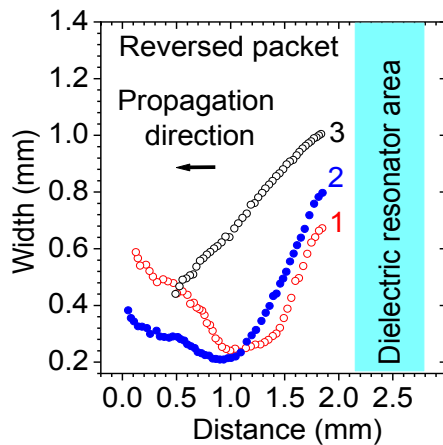
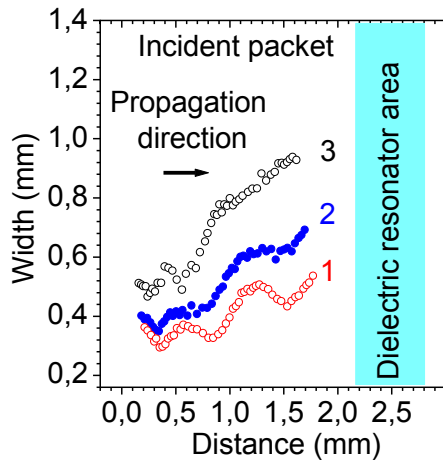
Pumping pulse :

Carrier frequency	$f_p = 2f_s$
Power	$P_p = 30$ W
Duration	$\tau_p = 40$ ns

A.A. Serga *et al.*, Phys. Rev. Lett. **94**, 167202 (2005)

2D Wave Front Reversal and Bullet Formation

amplification up to nonlinear level



Lecture 1: What did we address so far:

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves