

# Nanomagnetism

## Part 4 – Learn from loops



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<http://neel.cnrs.fr>

**Micro-NanoMagnetism team :** <http://neel.cnrs.fr/mnm>





- ⇒ **Extract loop and moments**
- ⇒ Extract magnetic anisotropy
- ⇒ Extract interactions and distributions
- ⇒ Understand magnetization processes
- ⇒ Analyse thermal effects



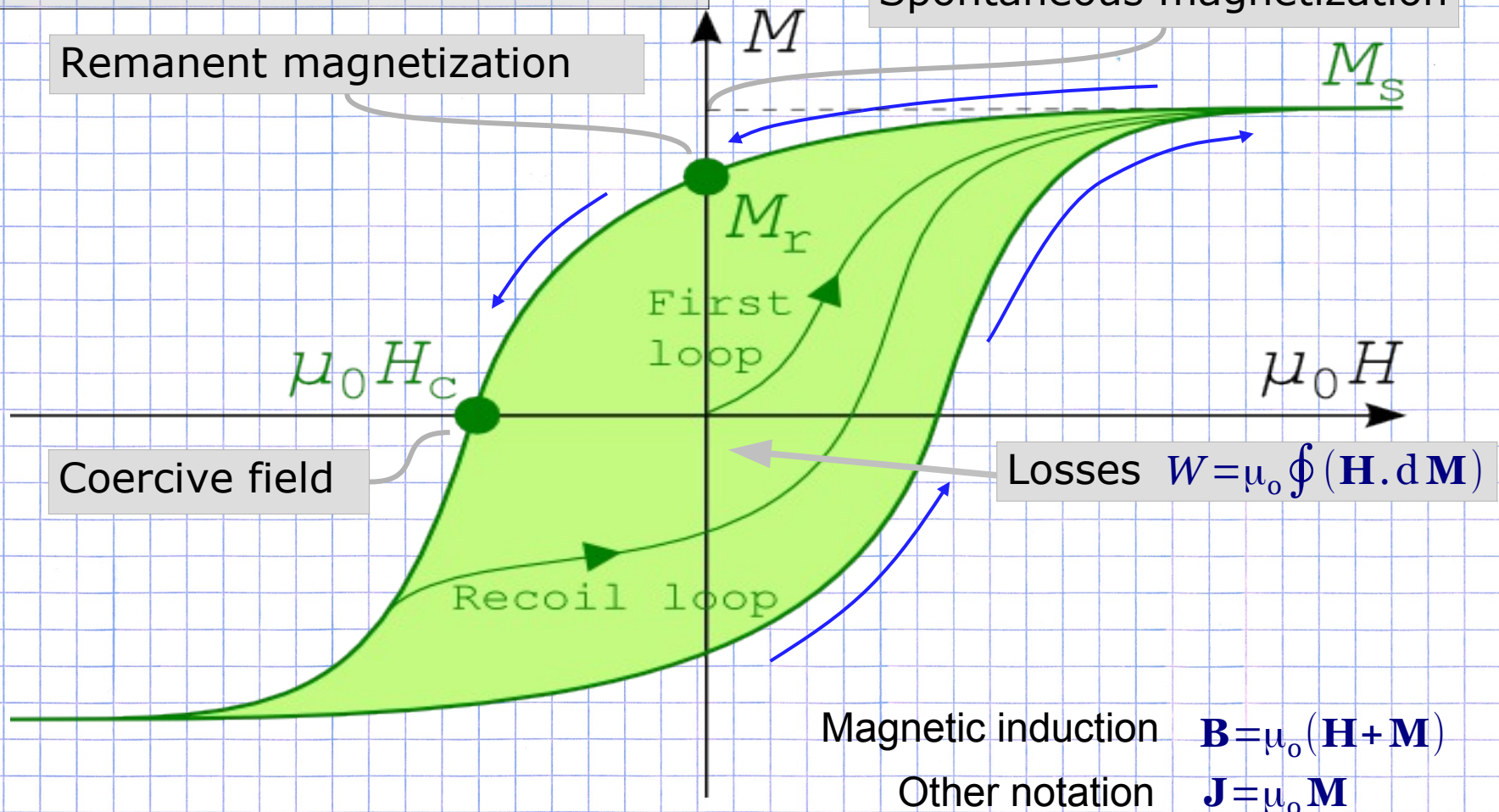
Manipulation of magnetic materials:  
 ↪ Application of a magnetic field

Zeeman energy:  $E_z = -\mu_0 \mathbf{H} \cdot \mathbf{M}$



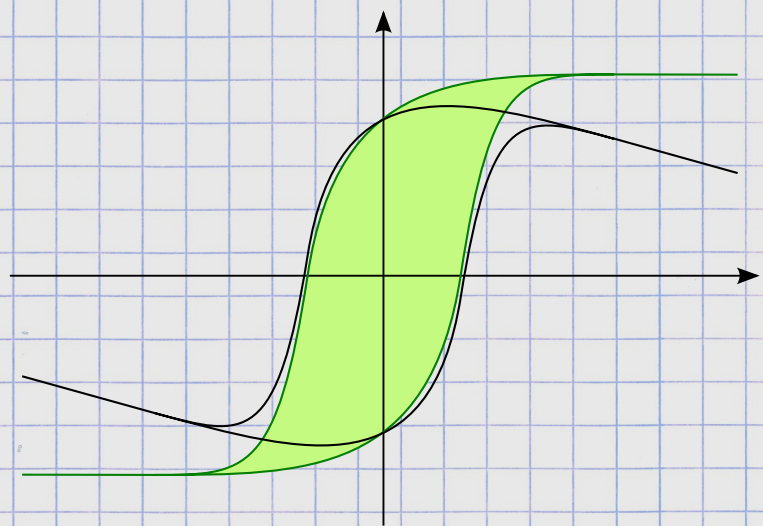
Spontaneous ≠ Saturation

Spontaneous magnetization





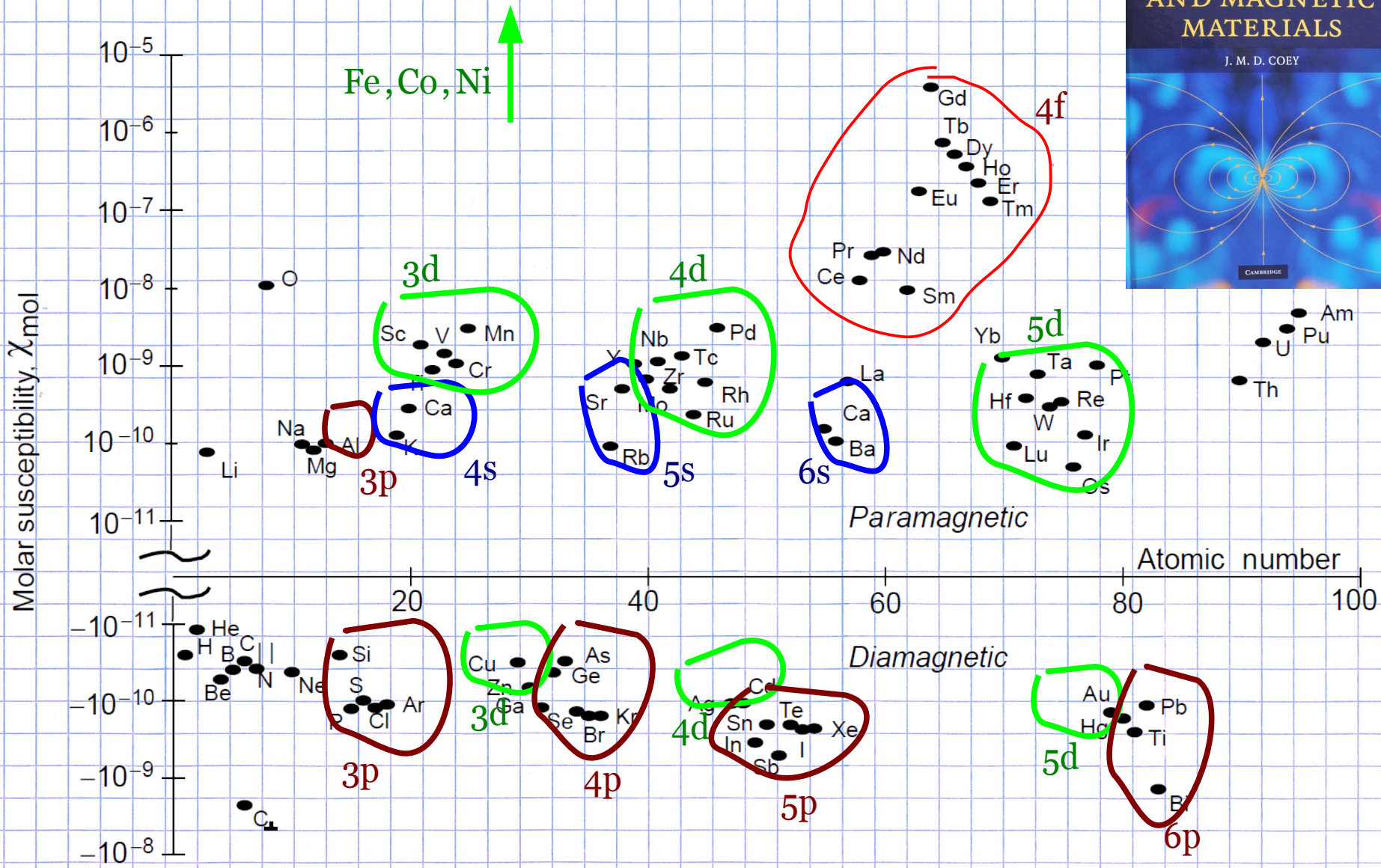
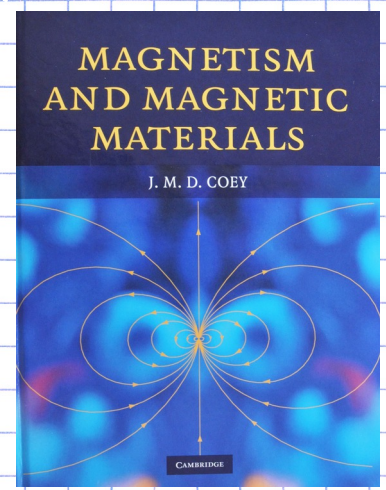
Diamagnetic substrate / holder



$$M(H) \leftarrow M(H) - \chi H$$

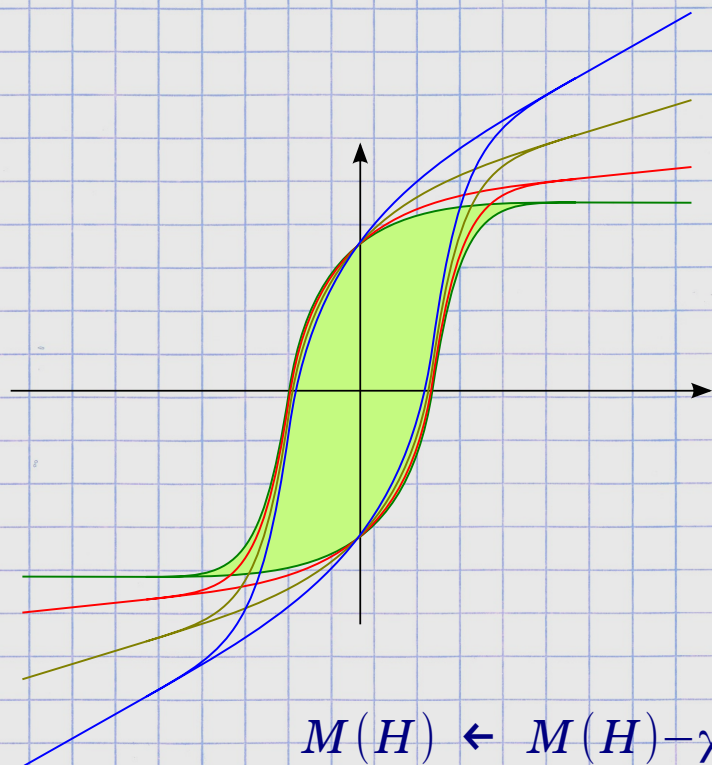
Problems :

- ⇒ Quantitative compensation a priori difficult
- ⇒ Approach to saturation difficult to investigate





Paramagnetic substrate / holder



$$M(H) \leftarrow M(H) - \chi(T)H$$

Example:

- ⇒ 1nm ferro layer
- ⇒ 1ppm in 1mm support

- ⇒ Example : ions impurities in metals, and oxides
- ⇒ Problem : temperature dependent, non-linear



## Paramagnetic substrate / holder



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Journal of Magnetism and Magnetic Materials 301 (2006) 50–66

**M** Journal of  
**M** magnetism  
**M** and  
magnetic  
materials

[www.elsevier.com/locate/jmmm](http://www.elsevier.com/locate/jmmm)

### Magnetism of cigarette ashes

Neli Jordanova<sup>a,\*</sup>, Diana Jordanova<sup>a</sup>, Bernard Henry<sup>b</sup>, Maxime Le Goff<sup>b</sup>,  
Dimo Dimov<sup>c</sup>, Tsenka Tsacheva<sup>d</sup>

⇒ Be careful with : cleanliness, tweezers, holders, ink, etc.

## Artifacts in various techniques

- ⇒ X-ray Magnetic Circular Dichroism (XMCD)
- ⇒ Magneto-Optical Kerr Effect (MOKE)
- ⇒ Lorentz microscopy
- ⇒ Etc.



## Case of a bulk soft magnetic material

Hypotheses:

1. Use an ellipsoid, cylinder or slab along a main direction so that the demagnetizing field may be homogeneous.
2. Domains can be created to yield a uniform and effective magnetization  $M_{\text{eff}}$

Density of energy:

$$E_{\text{tot}} = E_{\text{d}} + E_{\text{z}}$$

$$E_{\text{tot}} = \frac{1}{2} \mu_0 N M_{\text{eff}}^2 - \mu_0 M_{\text{eff}} H_{\text{ext}}$$

Minimization:

$$\frac{\partial E_{\text{tot}}}{\partial M_{\text{eff}}} = \mu_0 N M_{\text{eff}} - \mu_0 H_{\text{ext}} \Rightarrow$$

$$M_{\text{eff}} = \frac{1}{N} H_{\text{ext}}$$



**Conclusion for soft magnetic materials**

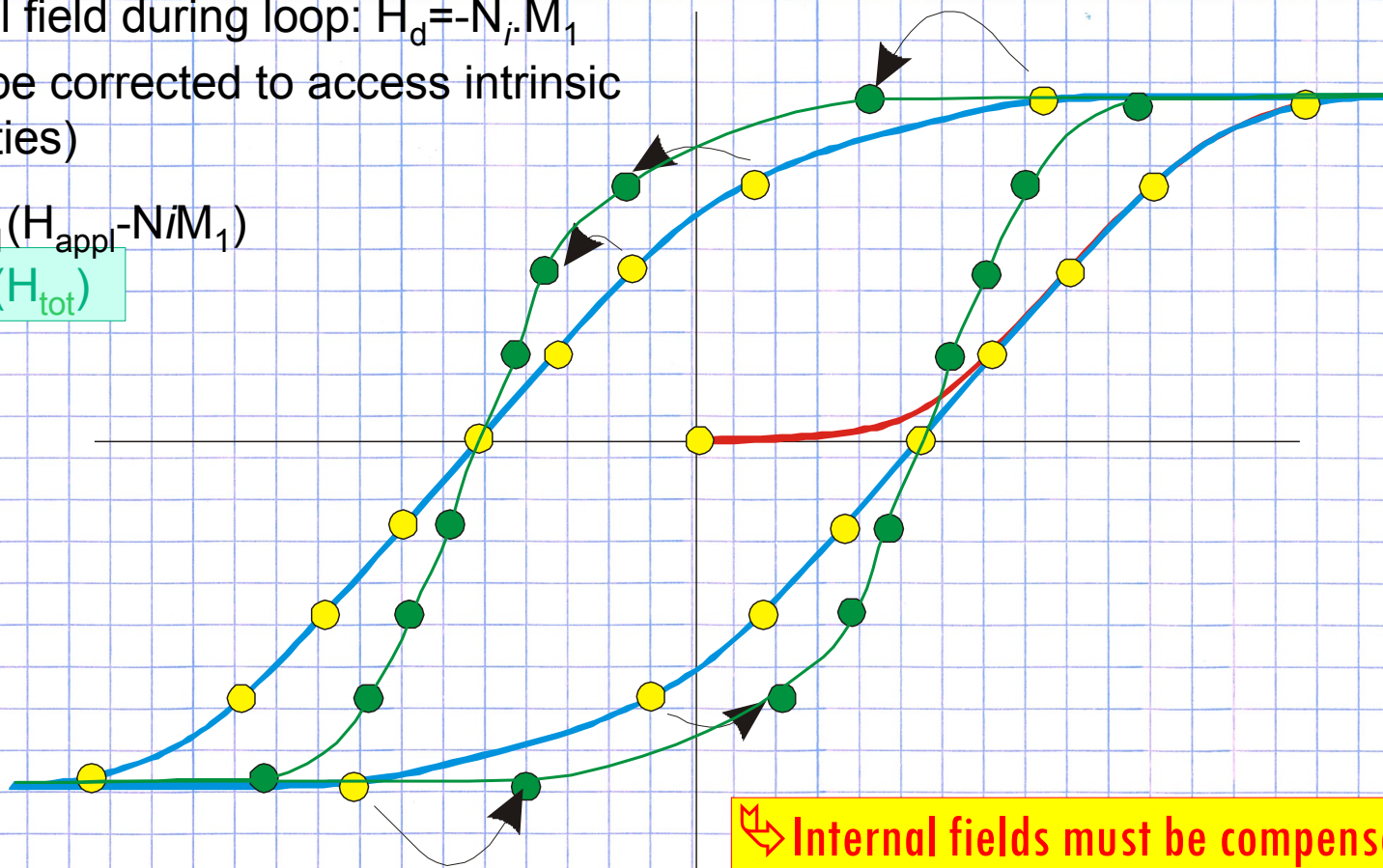
↳ Susceptibility is constant and equal to  $1/N$





Case of an arbitrary material

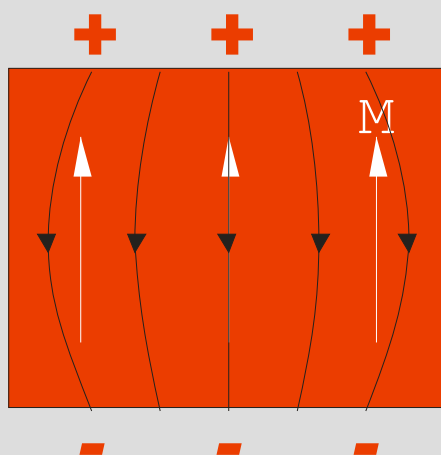
1. Measure a hysteresis loop  $M_1(H_{\text{appl}})$
2. Internal field during loop:  $H_d = -N_i \cdot M_1$   
(must be corrected to access intrinsic properties)
3. Plot  $M_1(H_{\text{appl}} - N_i/M_1)$   
➔  $M_2(H_{\text{tot}})$



➔ Internal fields must be compensated

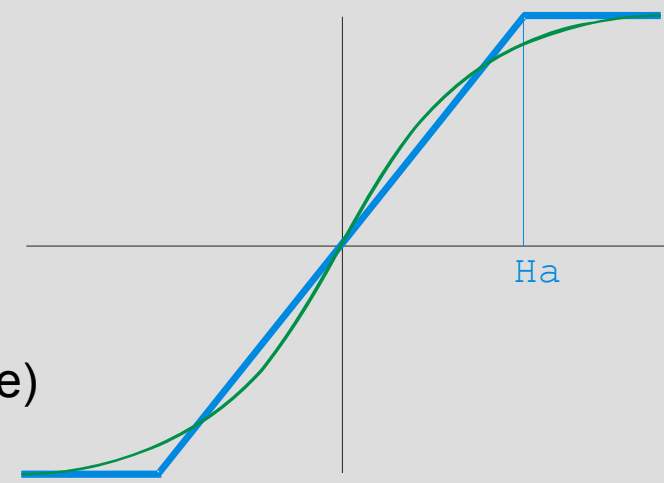


**Specific aspects to systems with non-ellipsoidal shapes**



In a non-ellipsoidal (or cylindrical, slab) system the demagnetizing field is not homogeneous in magnitude nor direction

1. Initial slope higher than  $1/N$   
(demag field smaller than average)
2. Late slope smaller than  $1/N$   
(demag field larger than average)



Demagnetizing energy (thus area above loop) is identical  $E_d = \int_0^{M_s} \mu_0 H_{ext} dM = \frac{1}{2} \mu_0 N M_s^2$

↪ In a non-ellipsoidal sample (or cylinder, slab) the loop is overcompensated at low magnetization and undercompensated at high field, even for soft magnetic materials.

↪ This effect adds up to the previous effect of grain shape

P. O. Jubert, O. Fruchart et al., *Europhys. Lett.* **63**, 102-108 (2003)



- ⇒ Extract loop and moments
- ⇒ **Extract magnetic anisotropy**
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**Magnetization loop of a macrospin along a hard axis**

$$e = \sin^2(\theta) - 2h \cos(\theta - \theta_H)$$

Dipolar energy:  $H = h \cdot H_a$   
 $H_a = 2K / \mu_0 M_s$   
 $K = N_j K_d$

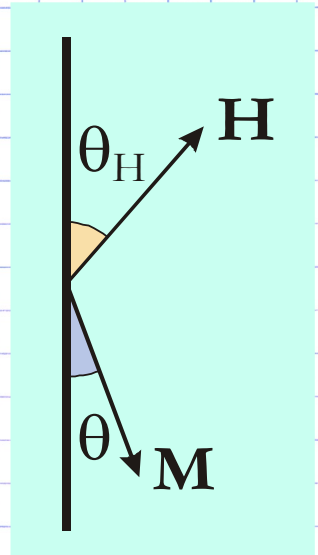
Hard axis:  $\theta_H = \pi / 2$

$$e = \sin^2(\theta) - 2h \sin(\theta)$$

$$\frac{\partial e}{\partial \theta} = 2 \cos \theta (\sin \theta - h)$$

$$h = \sin \theta = \cos(\theta - \theta_H) = \mathbf{m} \cdot \mathbf{u}_h$$

Equilibrium position

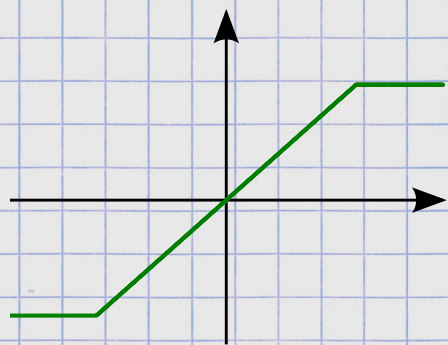




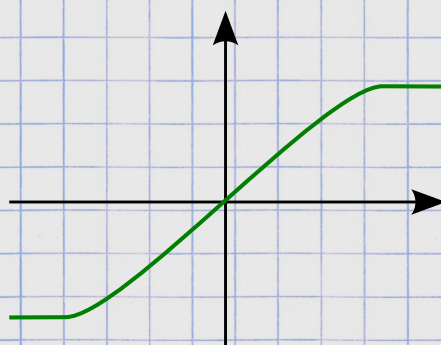
## Loops

$$E_{mc} = K_1 \sin^2 \theta + K_2 \sin^4 \theta + \dots$$

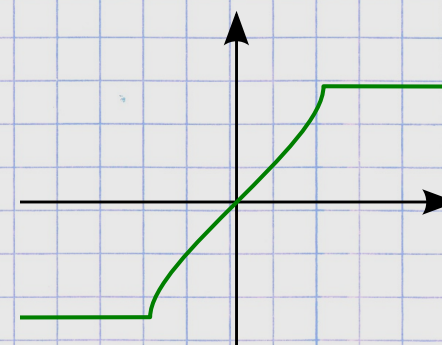
$$K_1 > 0$$



$K_2 = 0$



$K_2 > 0$

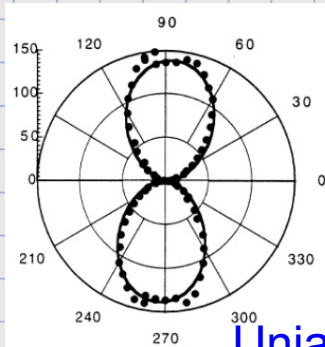
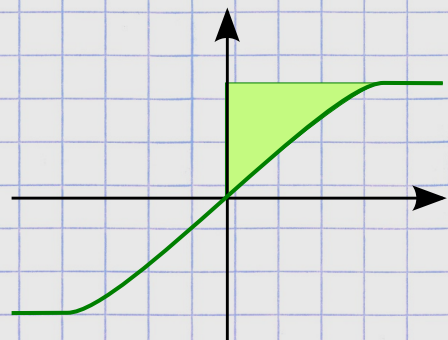


$K_2 < 0$

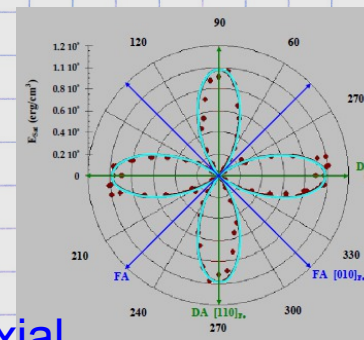
## Means of analysis

⇒ Fit curve  $H(M)$  (is analytical)

⇒ Initial susceptibility + saturation or area above curve as  $E_{sat} - E_0 = \mu_0 \oint (\mathbf{H} \cdot d\mathbf{M})$



Uniaxial

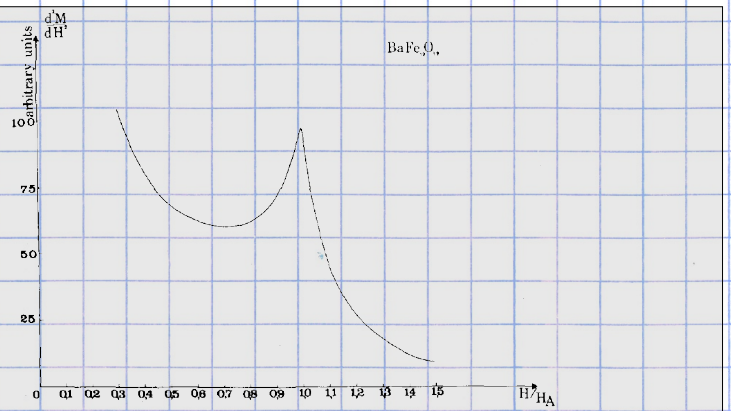


Biaxial



## Distribution

- ⇒ Use area above curve
- ⇒ Singular point detection for saturation field

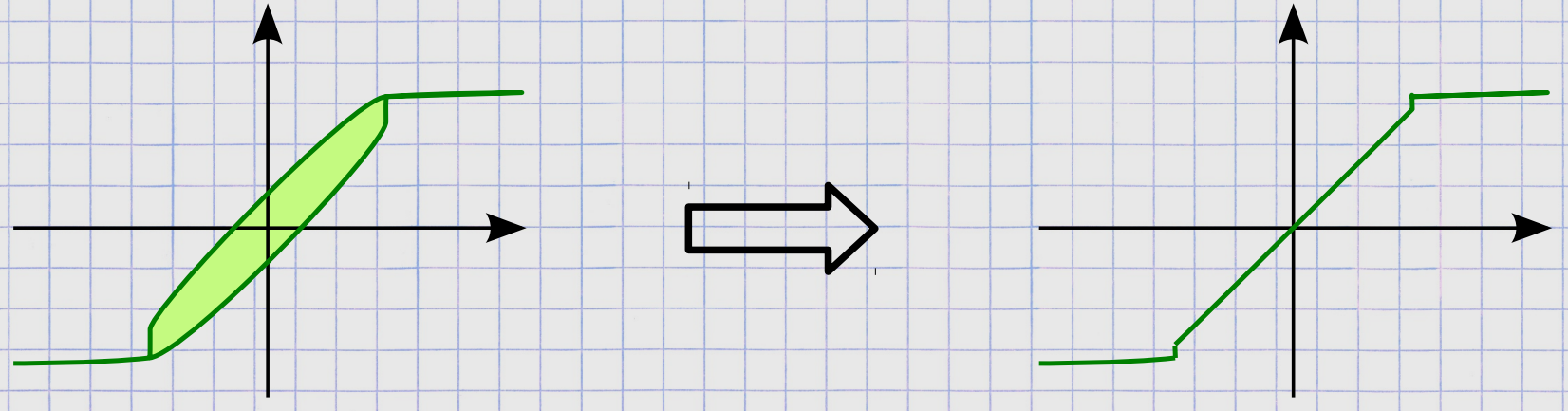


G. Asti et al., J. Appl. Phys. 45, 3600 (1974)

FIG. 5. Experimental plot of  $d^2M/dH^2$  vs  $H = H_{\text{ext}} - NM$  for an isotropic polycrystalline sample of  $\text{BaFe}_{12}\text{O}_{19}$ .  $H_{\text{ext}}$  is the applied field, and  $N$  denotes the demagnetizing factor of the sample.

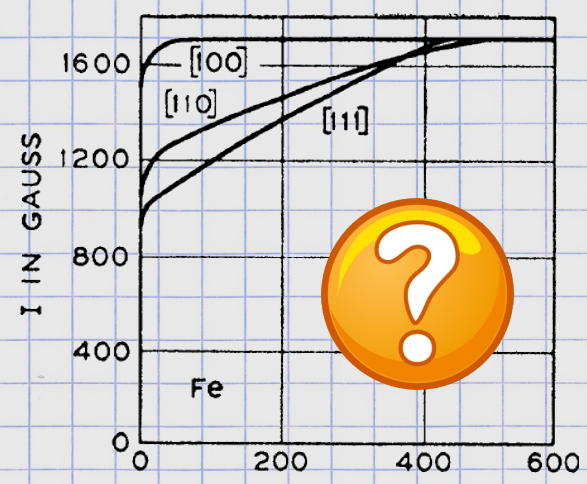
## Residual hysteresis

⇒ Compute anhysteretic curve  $M(H) \rightarrow [M(H_{\text{up}}) + M(H_{\text{down}})]/2$





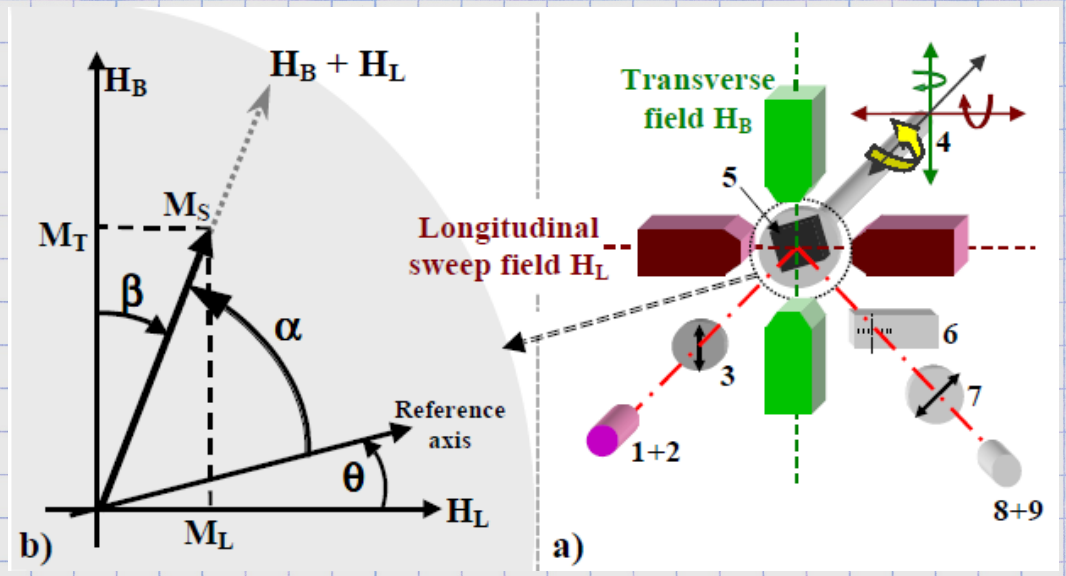
**Issue**



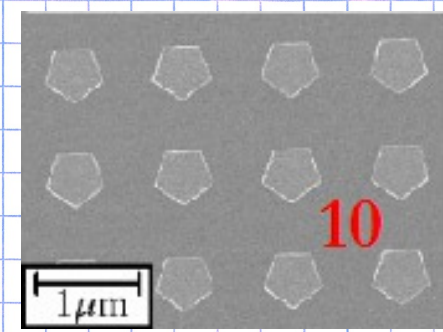
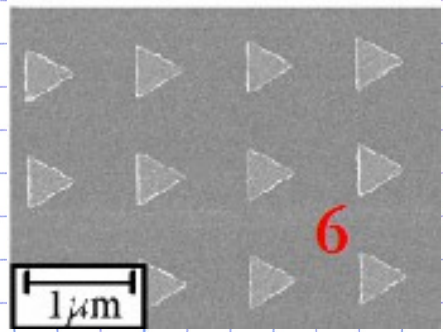
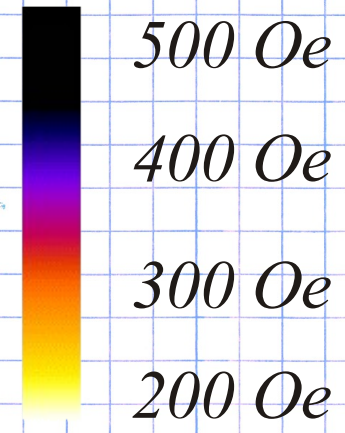
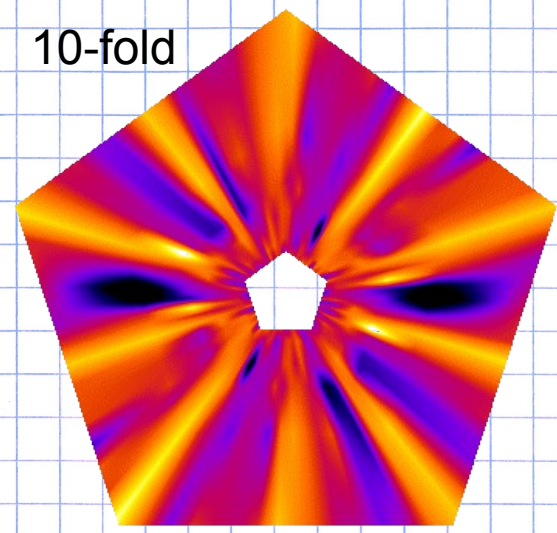
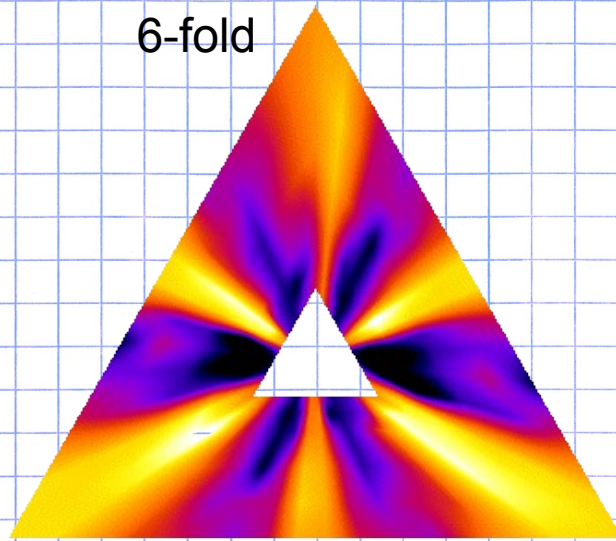
- ➡ High remanence in all directions
- ➡ Fit over small part of loop  
→ sensitive to imperfections
- ➡ Solution : loop under transverse bias field

**Solution**

TBIIST: Transverse Bias Initial Inverse Susceptibility and Torque



D. Berling, J. Magn. Magn. Mater. 297, 118 (2006)



R.P. Cowburn, *J.Phys.D:Appl.Phys.*33, R1–R16 (2000)

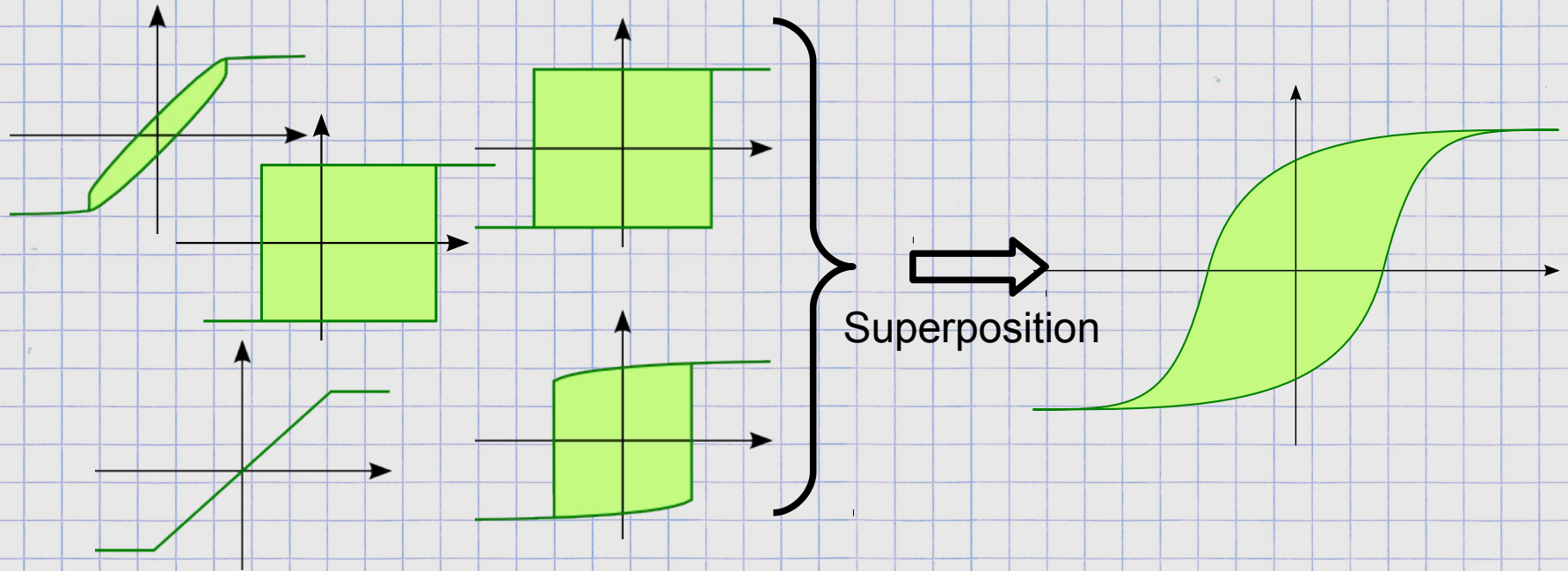




- ⇒ Extract loop and moments
- ⇒ Extract magnetic anisotropy
- ⇒ **Extract interactions and distributions**
- ⇒ Understand magnetization processes
- ⇒ Analyse thermal effects

## Physics : coercivity determined dual grains

⇒ Different loops with distribution



### Possible effects that may arise

- Distribution of coercive fields
- (Dipolar) interactions
- The loops of the macropins are slanted

**Textbook case**

⇒ Uniaxial anisotropy, second order :  $E_{mc} = K \sin^2 \theta$

⇒ Fully remanent grains

**3D distribution**

⇒ Remanence :  $m_r^{3D} = 1/2$

⇒ Measured anisotropy :  $\langle K^{3D} \rangle = 2K/3$

**2D distribution**

⇒ Remanence :  $m_r^{3D} = 2/\pi$

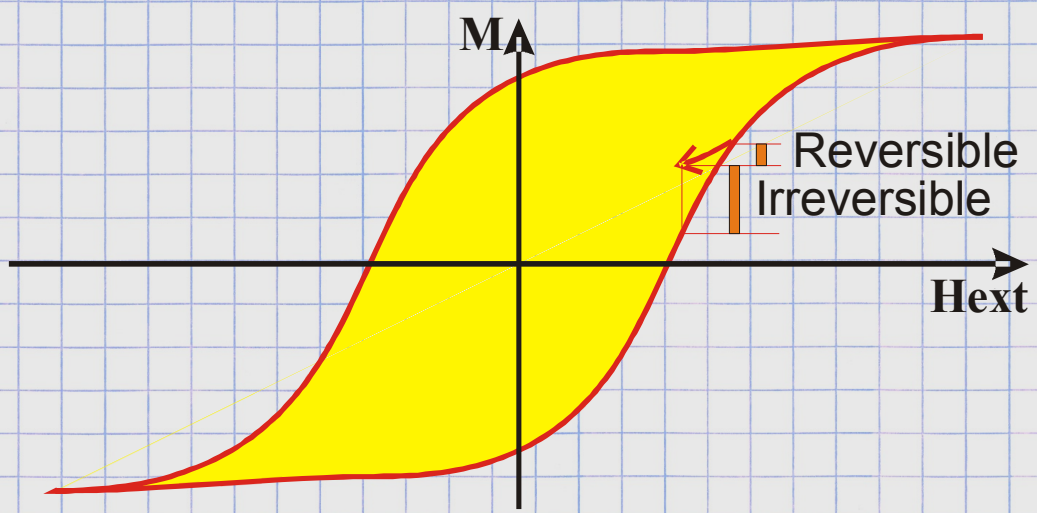
⇒ Measured anisotropy :  $\langle K^{3D} \rangle = K/2$

**Use**

⇒ Distribution : estimate K

⇒ Interactions : impact on increased or decreased remanence

Distribution of properties



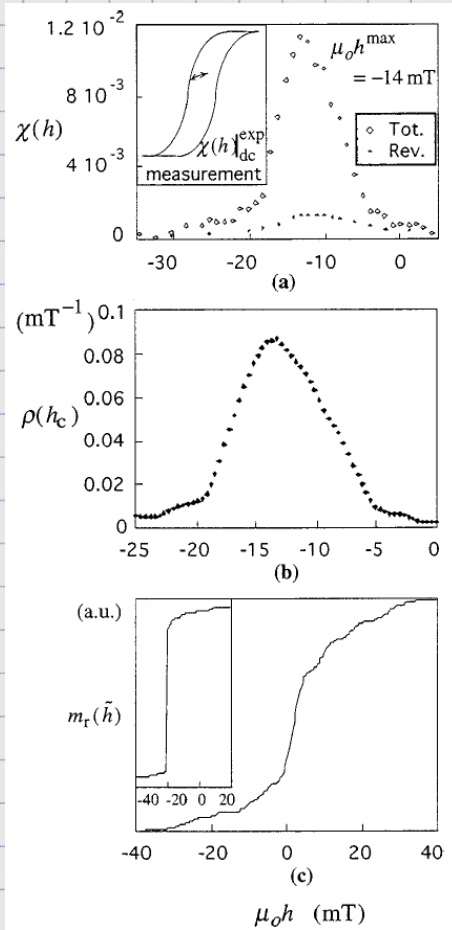
$$\rho(H_r) = \left. \frac{dm}{dH} \right|_{\text{irreversible}}$$

**Hc(T) for a given population of the distribution can be studied at a given stage of the reversal (10%, 20% etc.)**

**Effect of distributions and dipolar interactions are sometimes difficult to disentangle**

### Reconstruct average single hysteresis loop

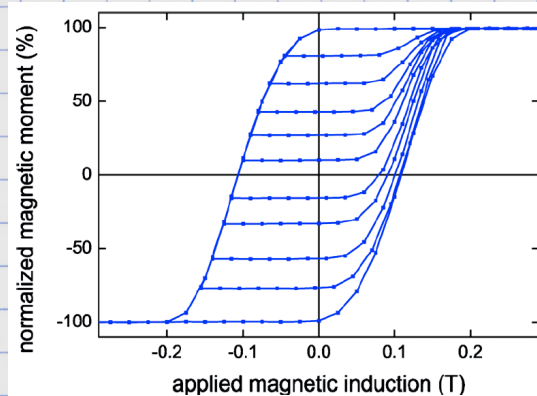
#### Ultrathin Fe(110) dots



O. Fruchart et al.,  
Phys. Rev. B 57, 2596 (1998)

### Susceptibility or distribution ?

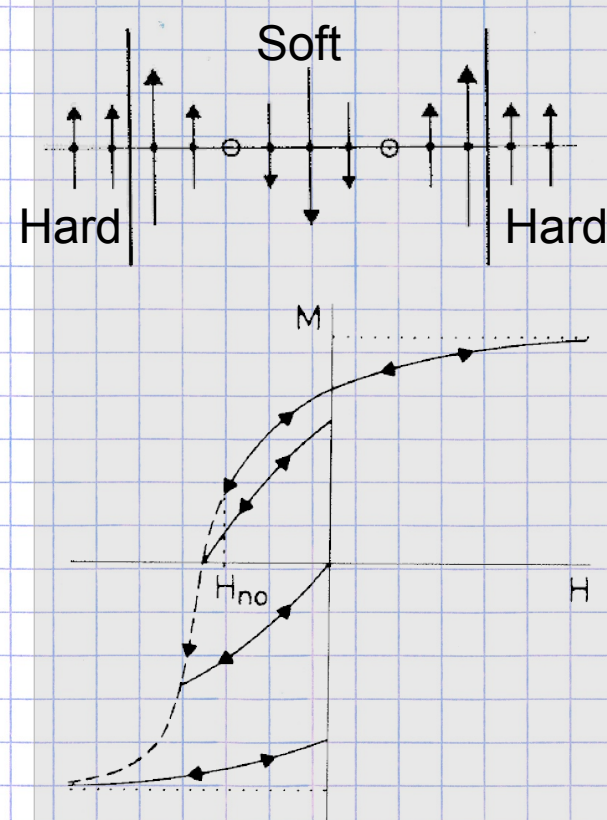
#### Arrays of electroplated parallel Ni nanowires



S. Da-Col et al.,  
Appl. Phys. Lett. 98,  
112501 (2011)

### Scrutinize multiphased materials

#### Exchange-spring magnets

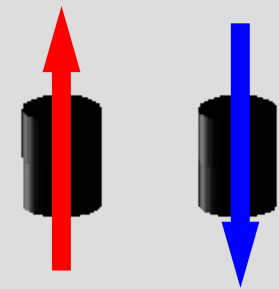
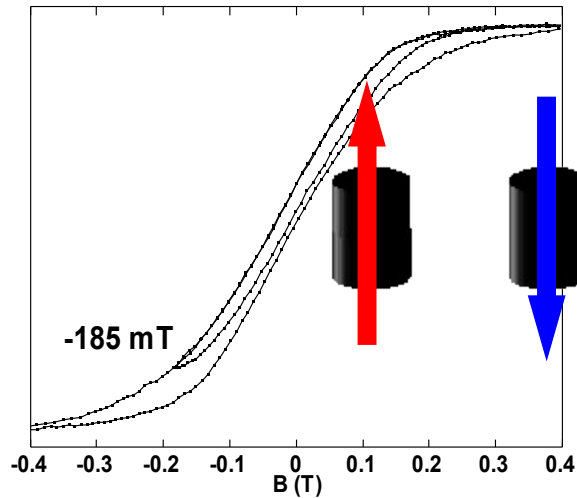
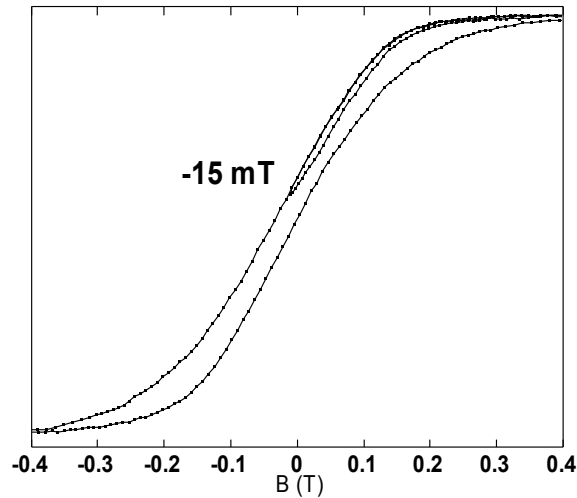


E. Kneller et al.,  
IEEE Trans. Magn. 27,  
3588 (1991)

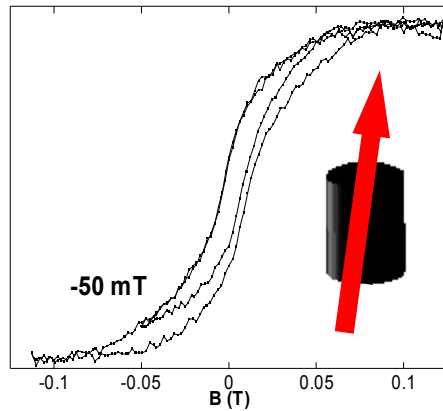
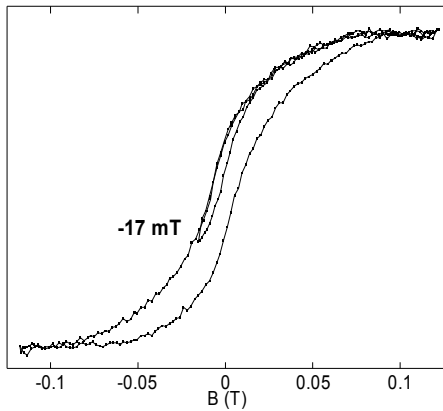


Minor loops: negative interactions

Example: dipolar interactions in arrays of Co/Au(111) pillars



Minor loops: negligible interactions

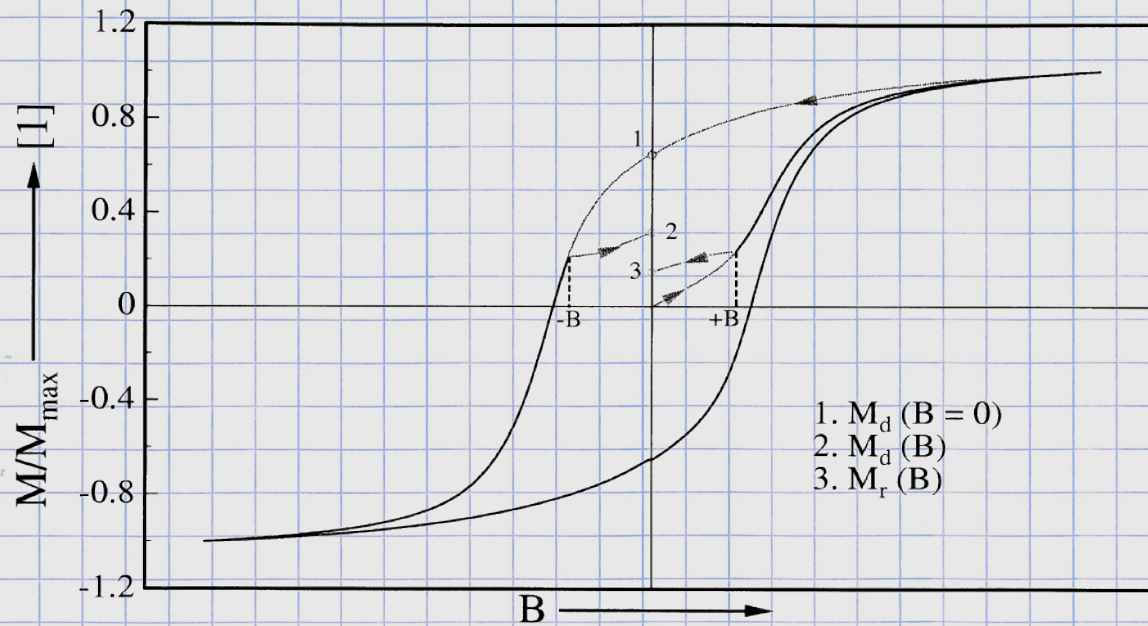


↪ Faster than Henkel and Preisach  
 ↪ Other applications: characterization of exchange bias

O. Fruchart et al., unpublished



## Henkel plots



O. Henkel,  
 Phys. Stat. Sol. 7, 919 (1964)  
 S. Thamm et al.,  
 JMMM184, 245 (1998)

Fig. 1. Explanation of how to measure the two different remanent magnetisations  $M_r$  and  $M_d$ .

### Measure of dipolar interactions

$$\Delta M_H(x) = M_d(x) - [1 - 2M_r(x)]$$

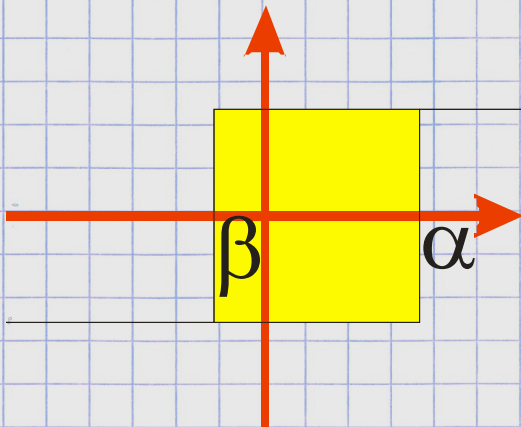
↪ The analysis of interactions on qualitative  
 ↪ Long experiments (ac demagnetization)



## Preisach model

G. Biorci et al., *Il Nuov. Cim. VII, 829 (1958)*

I. D. Mayergoyz, *Mathematical models of hysteresis, Springer (1991)*

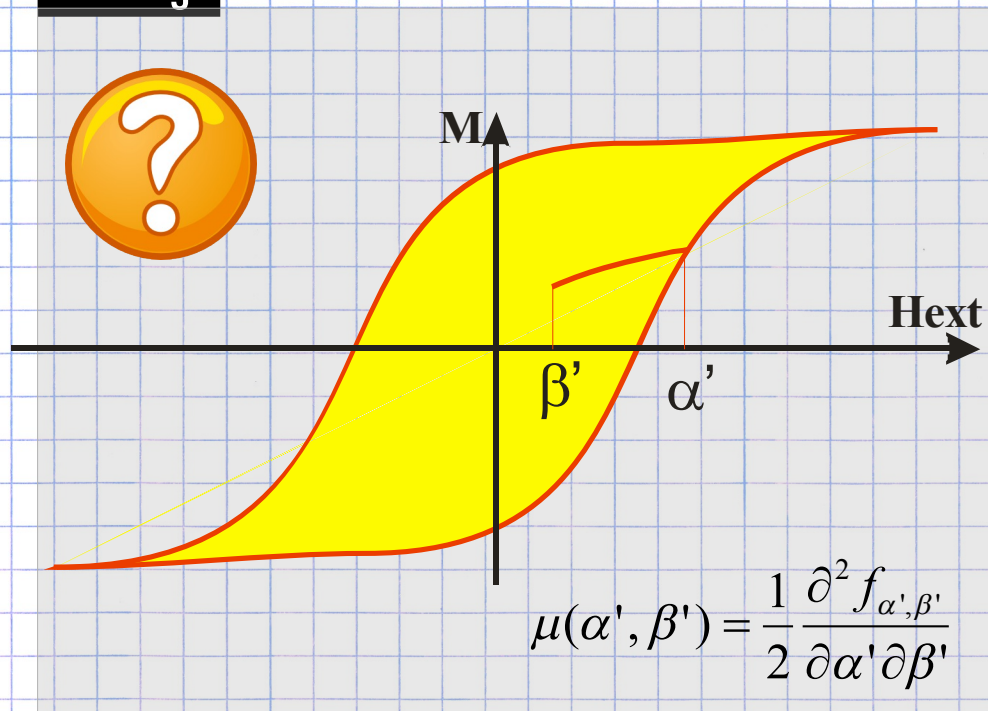


⇒ Distribution function

$$\mu(\alpha, \beta) \text{ with } \alpha > \beta$$

⇒ No true link between real particles and  $\mu$

## Solving



⇒ Long experiments (1D set of hysteresis curves)

⇒ Better suited to bulk materials with strong interactions





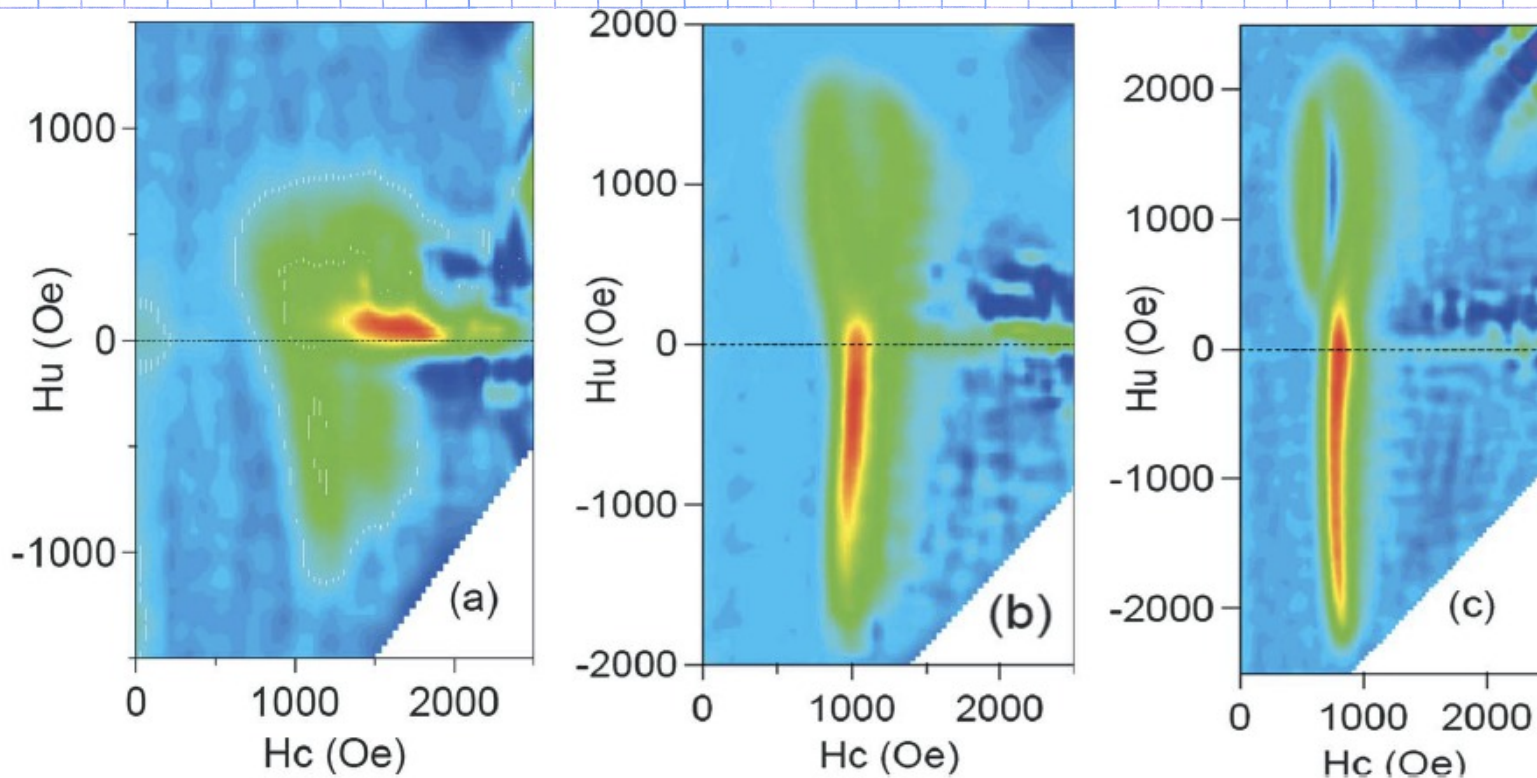
Recent 'rediscovery' or 're-interpretation' : the FORC diagrams:

First-Order Reversal Curves

→ Outline distribution of switching field and bias field

C. Pike et al., *J. Appl. Phys.* **85**, 6668 (1999)

Ex : arrays of parallel permalloy nanowires wire increasing diameter



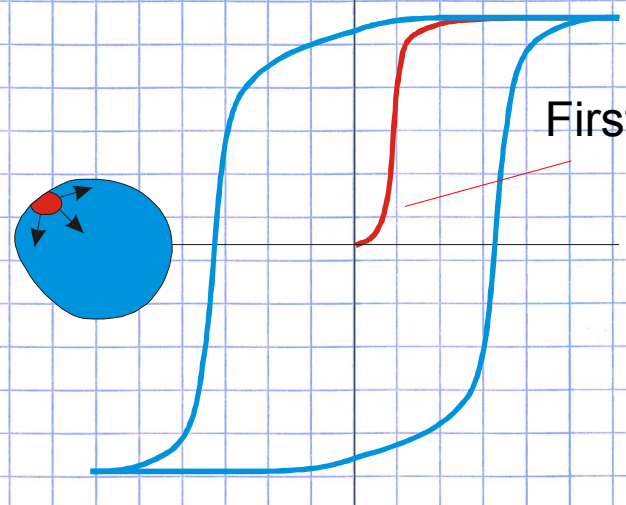
M. S. Salem et al., *J. Mater. Chem.* **22**, 8549 (2012)



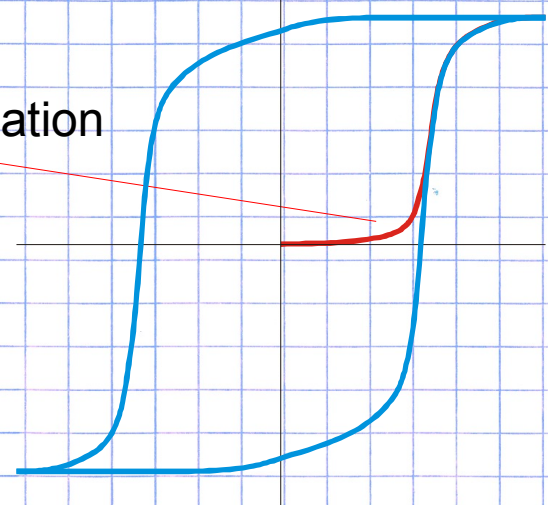
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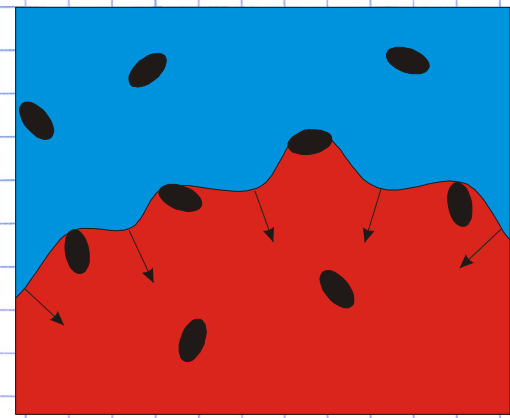
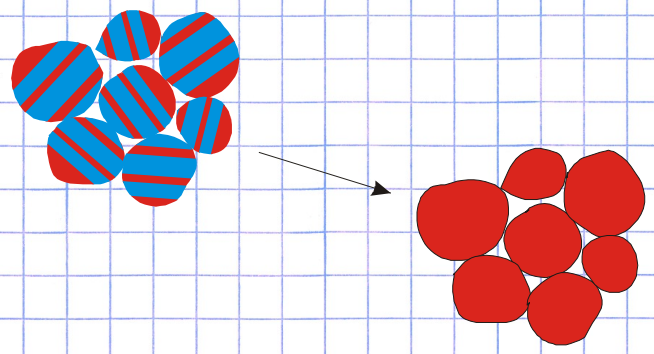
Use first-magnetization curves to determine the type of coercivity



Nucleation-limited  
Ex:  $\text{Sm}_2\text{Co}_{17}$



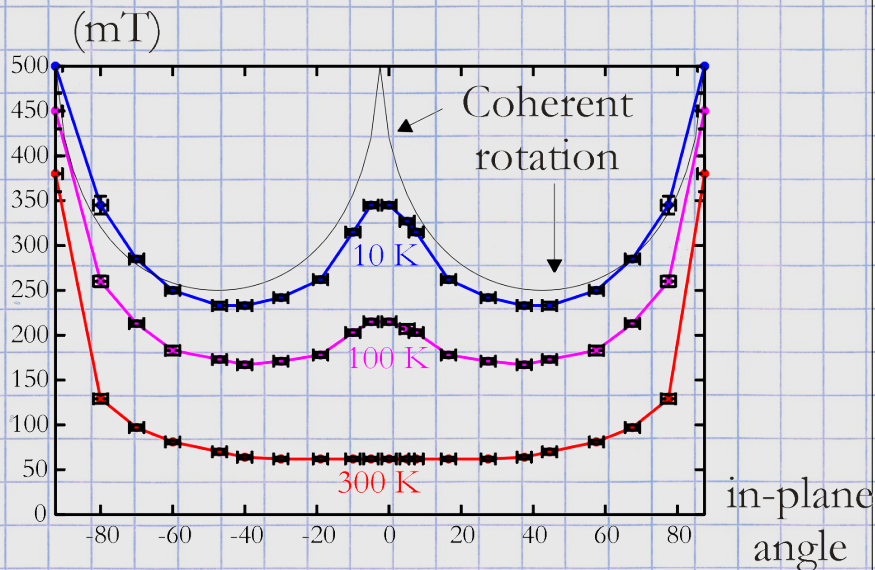
Propagation-limited  
Ex:  $\text{SmCo}_5$





**Max(Hc) for easy axis → nucleation**

Ultrathin uniaxial Fe(110) dots



O. Fruchart et al.,  
Phys. Rev. Lett. 82, 1305-1308 (1999)

**1/cosθ<sub>H</sub> law → propagation**

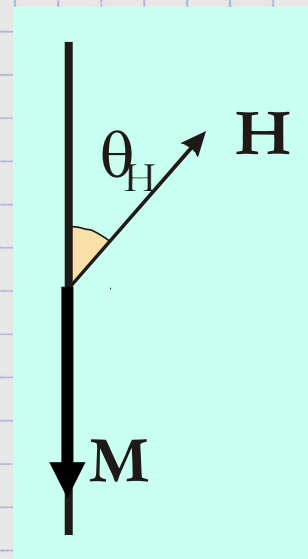
E. J. Kondorsky, J. Exp. Theor. Fiz. 10, 420 (1940)

Hypothesis:

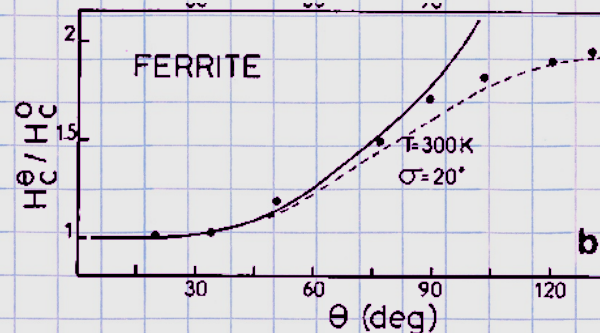
⇒ Based on nucleation volume

⇒ Hc << H<sub>a</sub>

Energy barrier  $E_0$  overcome by gain in Zeeman energy plus thermal energy



$$E_0 = -\mu_0 M_s v_a H \cos(\theta_H) + 25k_B T$$



D. Givord et al., JMMM72, 247 (1988)



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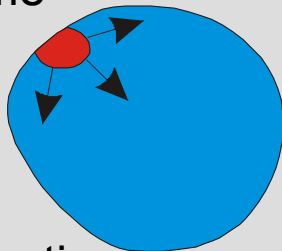


## Activation volume

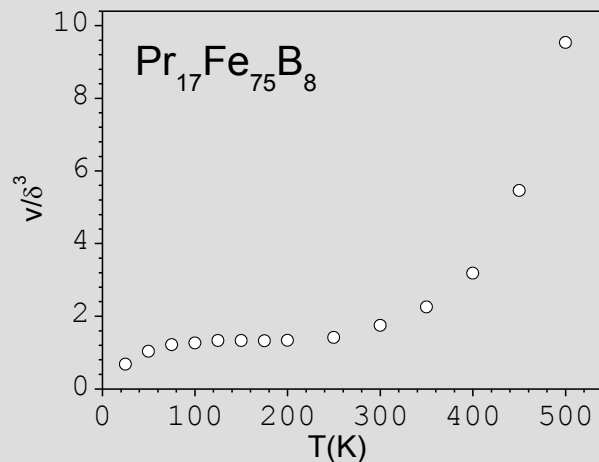
Also called: nucleation volume

Can be used for:

- ⇒ Estimating  $H_c(T)$
- ⇒ Estimating long-time relaxation
- ⇒ Determination of dimensionality



Note: of the order of domain wall width  $\delta$



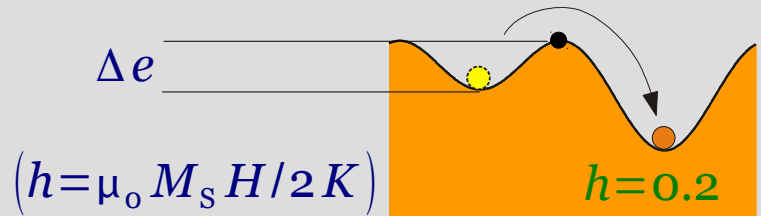
More detailed models:

D. Givord et al., JMMM258, 1 (2003)

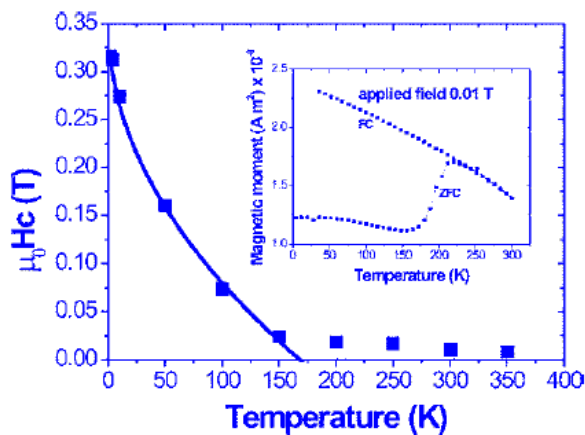


### Barrier height

$$\Delta e = e(\theta_{\max}) - e(0) = (1-h)^2$$



J. Appl. Phys. 99, 08Q514 (2006)



### Thermal activation

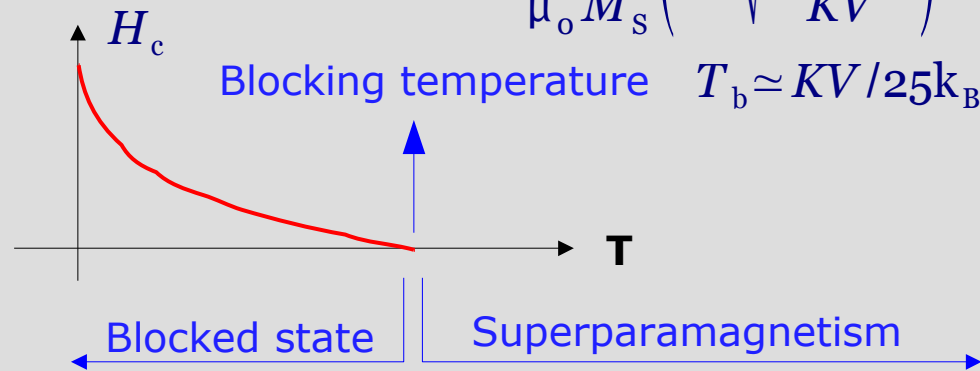
Brown, Phys.Rev.130, 1677 (1963)

$$\tau = \tau_0 \exp\left(\frac{\Delta \mathcal{E}}{k_B T}\right) \implies \Delta \mathcal{E} = k_B T \ln(\tau/\tau_0)$$

$$\tau_0 \approx 10^{-10} \text{ s}$$

Lab measurement:  $\tau \approx 1 \text{ s} \implies \Delta \mathcal{E} \approx 25 k_B T$

$$\implies H_c = \frac{2K}{\mu_0 M_s} \left(1 - \sqrt{\frac{25 k_B T}{KV}}\right)$$



E. F. Kneller, J. Wijn (ed.) Handbuch der Physik XIII/2: Ferromagnetismus, Springer, 438 (1966)

M. P. Sharrock, J. Appl. Phys. 76, 6413-6418 (1994)

Notice, for magnetic recording :  $\tau \approx 10^9 \text{ s}$      $KV_b \approx 40 - 60 k_B T$



**Formalism**

C. P. Bean & J. D. Livingston, *J. Appl. Phys.* **30**, S120 (1959)

Energy

$$E = KV.f(\theta, \varphi) - \mu_0 \mu H$$

Partition function

$$Z = \sum \exp(-\beta E)$$

Average moment

$$\langle \mu \rangle = \frac{1}{\beta \mu_0 Z} \frac{\partial Z}{\partial H}$$

**Isotropic case**

$$Z = \int_{-M}^M \exp(-\beta E) d\mu$$

Note: equivalent to integration over solid angle

$$\langle \mu \rangle = M [\text{cotanh}(x) - 1/x]$$

Langevin function

**Note:**

Use the moment  $M$  of the particule, not spin  $1/2$ .

$$x = \beta \mu_0 M H$$



**Infinite anisotropy**

$$Z = \exp(\beta \mu_0 M H) + \exp(-\beta \mu_0 M H)$$

$$\langle \mu \rangle = M \cdot \tanh(x)$$

Brillouin  $1/2$  function

