

# Nanomagnetism

## Part 2 – Domains and domain walls



**Olivier Fruchart**

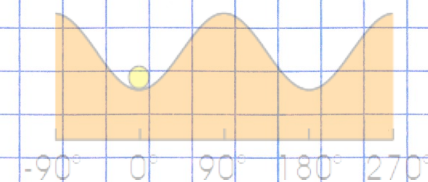
Institut Néel (Univ. Grenoble Alpes – CNRS)  
Grenoble – France

<http://neel.cnrs.fr>

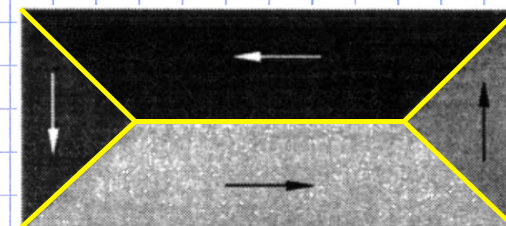
**Micro-NanoMagnetism team :** <http://neel.cnrs.fr/mnm>



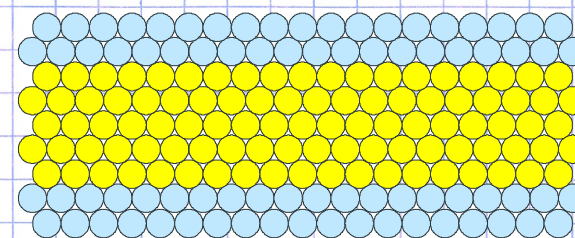
➔ **Part 1 : basics of micromagnetism –  
Simple models of magnetization reversal**



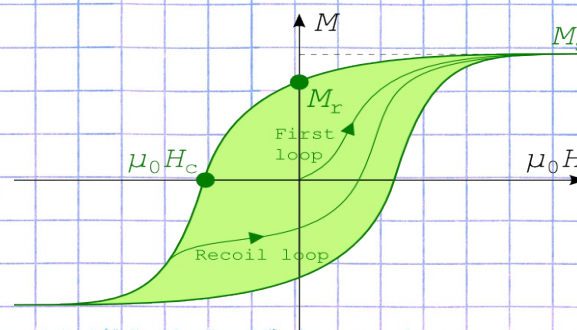
➔ **Part 2 : non-uniform magnetization in  
nanostructure: domains, domain walls**

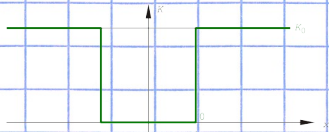


➔ **Part 3 : Low-dimensions,  
interfaces and heterostructures**

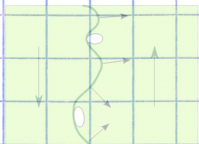


➔ **Part 4 : Learn from  
hysteresis loops**

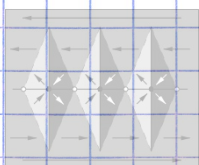




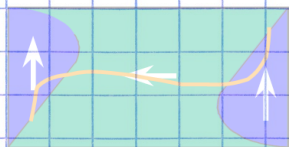
⇒ Brown paradox



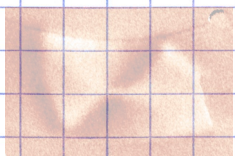
⇒ Nucleation and propagation



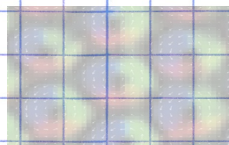
⇒ Walls and domains in films and nanostructures



⇒ Near single domains



⇒ Domain walls in tracks

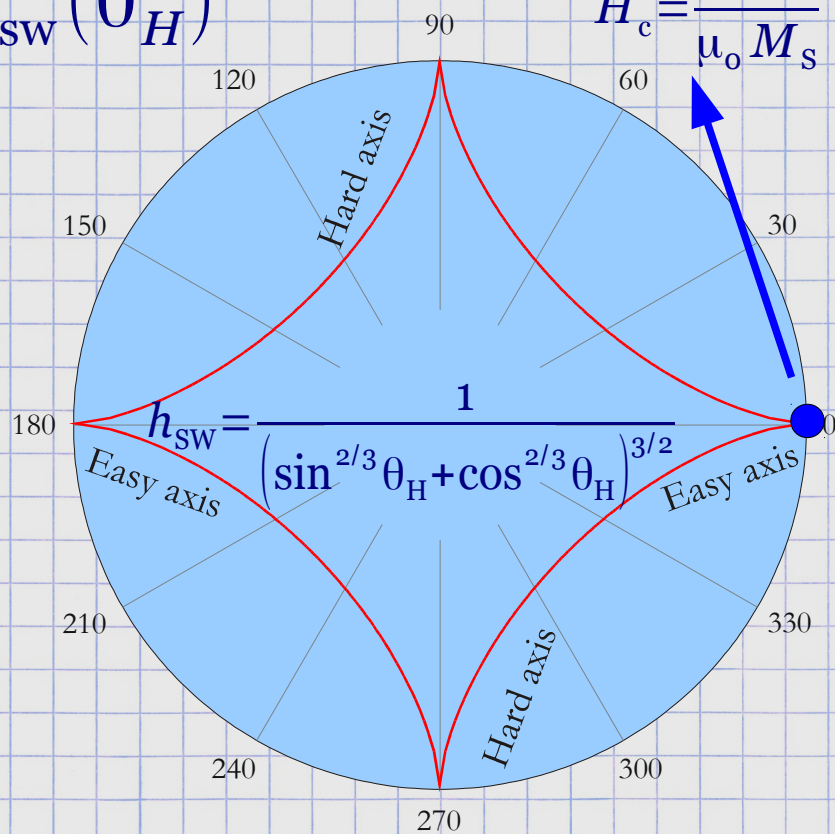


⇒ Skyrmions



Theory, 'Astroid' curve

$$h_{sw}(\theta_H)$$



Experiments

$$H_c \ll \frac{2K}{\mu_0 M_s}$$

Known as Brown paradox

W. F. Brown, Jr.,  
 Micromagnetics (Wiley, New York, 1963)

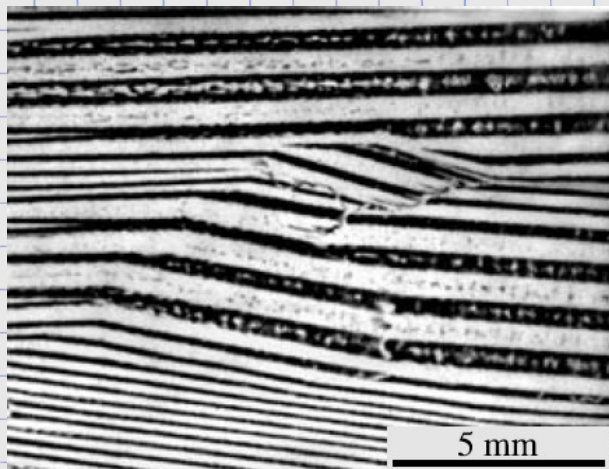
Seeking understanding

- ↪ Non-uniform distributions of magnetization
- ↪ Origin may be intrinsic (eg shape) or extrinsic (defect)



**Bulk material**

Numerous and complex magnetic domains

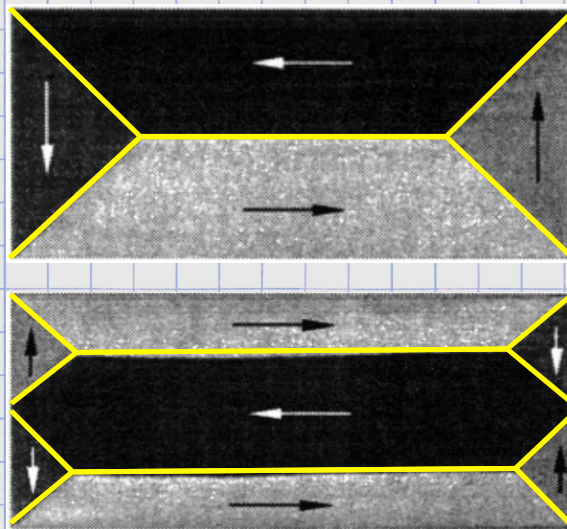


FeSi soft sheet

A. Hubert, *Magnetic domains*

**Mesoscopic scale**

Small number of domains, simple shape

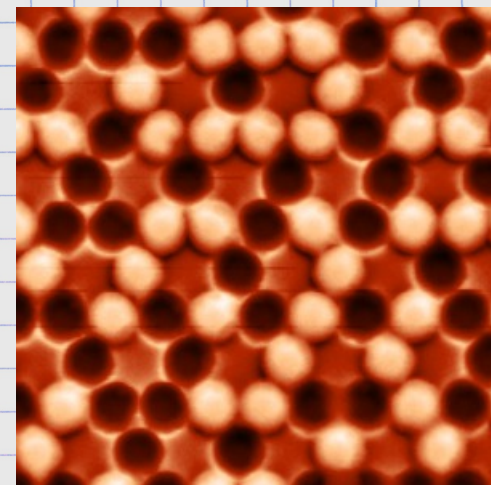


Microfabricated dots  
Kerr magnetic imaging

A. Hubert, *Magnetic domains*

**Nanometric scale**

Magnetic single-domain



Nanofabricated dots  
MFM

Sample courtesy :  
N. Rougemaille, I. Chioar

➤ Domain walls play a crucial role in magnetization processes

➤ Domain walls define length scales



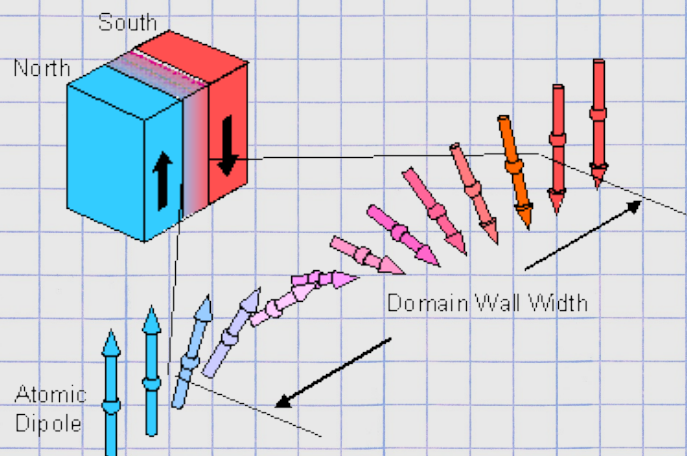
### Anisotropy exchange length

$$E = A(\partial_x \theta)^2 + K \sin^2 \theta$$

Exchange  $\rightarrow$  J/m      Anisotropy  $\rightarrow$  J/m<sup>3</sup>

Anisotropy exchange length:  $\Delta_u = \sqrt{A/K}$

$\Delta_u \approx 1 \text{ nm} \rightarrow \Delta_u \geq 100 \text{ nm}$   
 Hard                      Soft



Often called *Bloch parameter* or *domain-wall width*

### Dipolar exchange length

$$E = A(\partial_x \theta)^2 + K_d \sin^2 \theta$$

Exchange  $\rightarrow$  J/m      Dipolar energy  $\rightarrow$  J/m<sup>3</sup>

$$K_d = \frac{1}{2} \mu_0 M_s^2$$

Dipolar exchange length:

$$\Delta_d = \sqrt{A/K_d}$$

$$= \sqrt{2A/\mu_0 M_s^2}$$

$\Delta_d \approx 3 - 10 \text{ nm}$

Single-domain critical size relevant for nanoparticles made of soft magnetic material



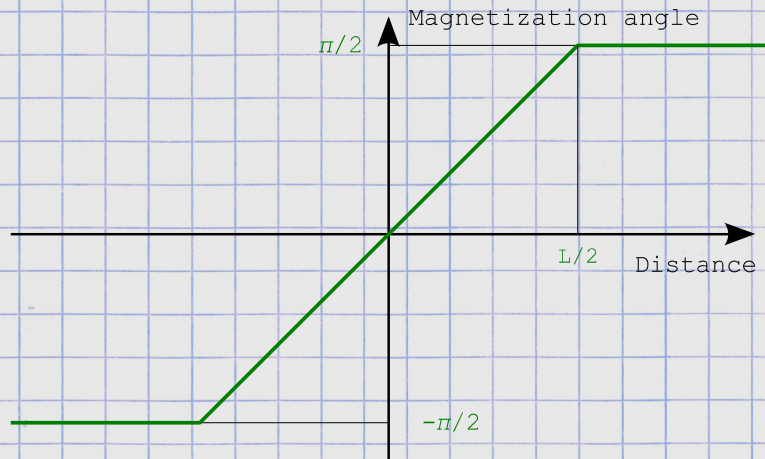
Often called *Exchange length*

**Notice:**  
 Other length scales: with field etc.



### Linear model

Naïve however provides all physics

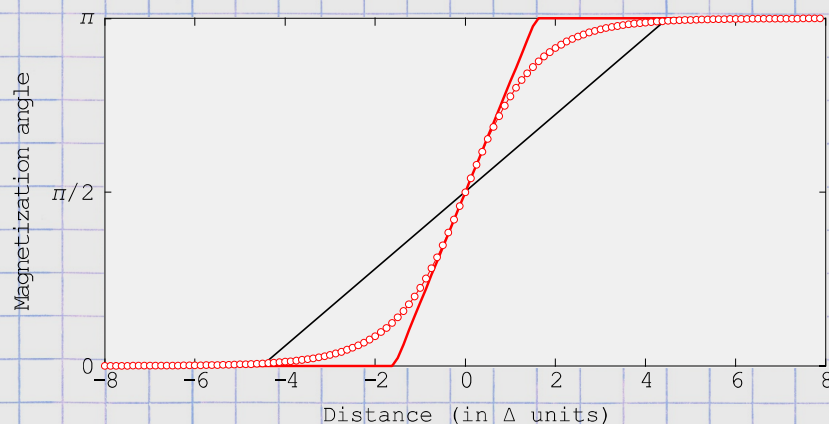


$$E_{ex} = A(\nabla \cdot \mathbf{m})^2 = A(\pi/L)^2$$

$$\langle E_u \rangle = K_u \langle \cos^2 \theta \rangle = K_u/2$$

### Feature of domain walls

	Linear	Exact
Width	$\Lambda_W = \pi\sqrt{2}\sqrt{A/K_u}$	$\Lambda_W = \pi\sqrt{A/K_u}$
Energy	$\mathcal{E} = \pi\sqrt{2}\sqrt{AK_u}$	$\mathcal{E} = 4\sqrt{AK_u}$



### Domain walls and coercivity

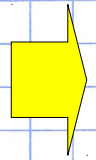
↳ Soliton-like propagation : no deformation, requires no energy (NB : under quasistatic conditions)

↳ Contribution to coercivity requires geometrical or material inhomogenities



**Brown's paradox**

In most systems  $H_c \ll \frac{2K}{\mu_0 M_s}$



**Micromagnetic modeling**

Exhibit analytic however realistic models for magnetization reversal

PHYSICAL REVIEW

VOLUME 119, NUMBER 1

JULY 1, 1960

**Reduction in Coercive Force Caused by a Certain Type of Imperfection**

A. AHARONI

*Department of Electronics, The Weizmann Institute of Science, Rehovot, Israel*

(Received February 1, 1960)

As a first approach to the study of the dependence of the coercive force on imperfections in materials which have high magnetocrystalline anisotropy, the following one-dimensional model is treated. A material which is infinite in all directions has an infinite slab of finite width in which the anisotropy is 0. The coercive force is calculated as a function of the slab width. It is found that for relatively small widths there is a considerable reduction in the coercive force with respect to perfect material, but reduction saturates rapidly so that it is never by more than a factor of 4.

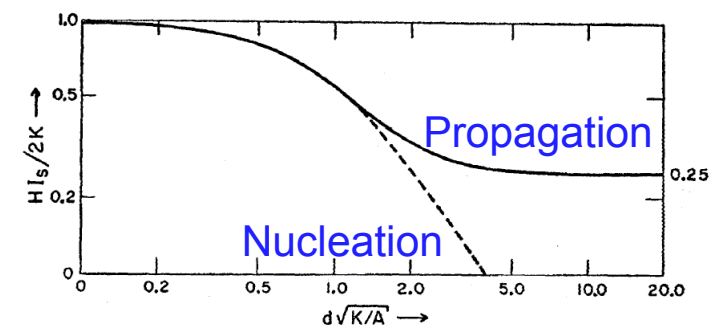
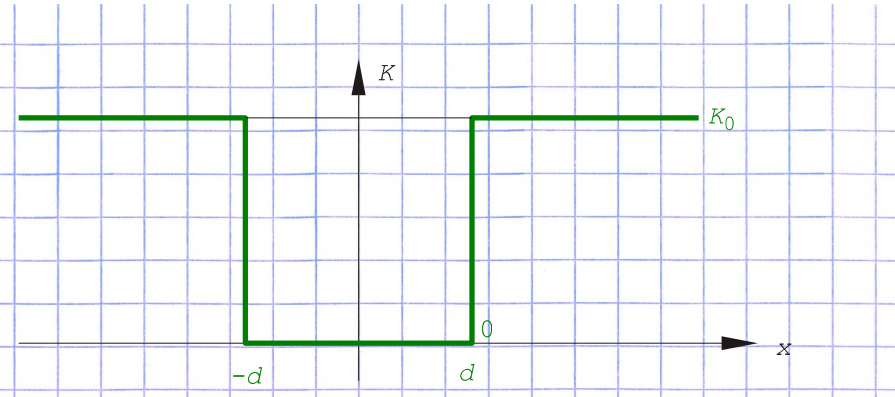
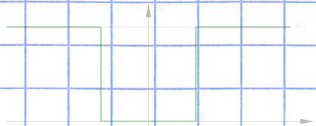
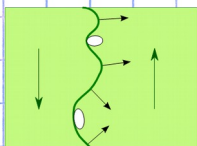


FIG. 1. The nucleation field (dashed) and coercive force (full curve) in terms of the coercive force of perfect material,  $HI_s/2K$ , as functions of the defect size,  $d$ .

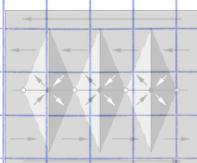




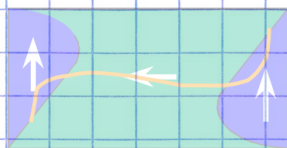
⇒ Brown paradox



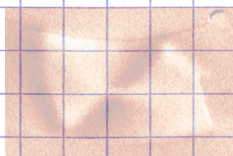
⇒ Nucleation and propagation



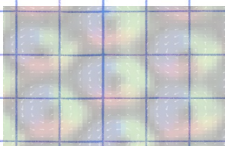
⇒ Walls and domains in films and nanostructures



⇒ Near single domains



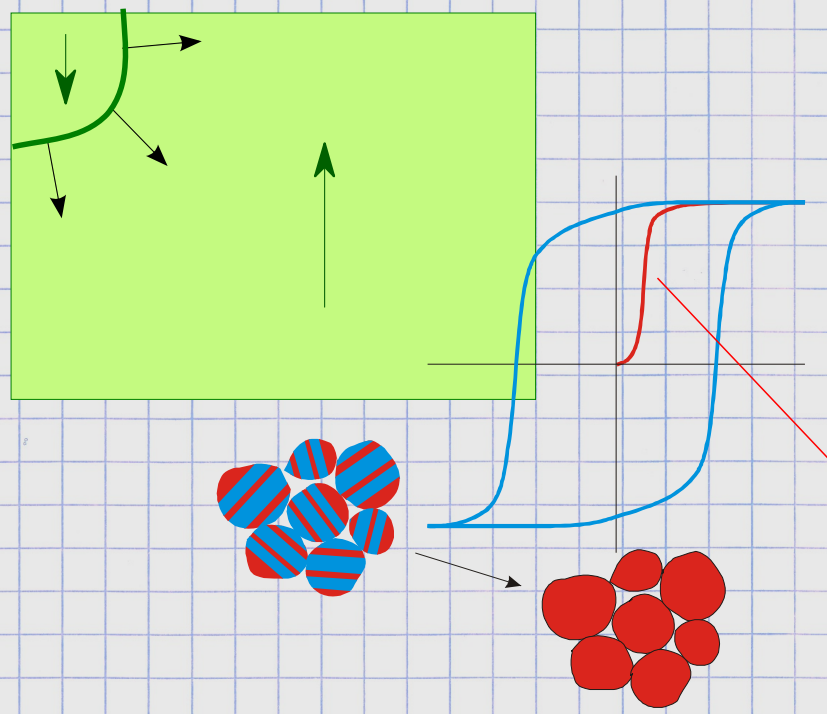
⇒ Domain walls in tracks



⇒ Skyrmions

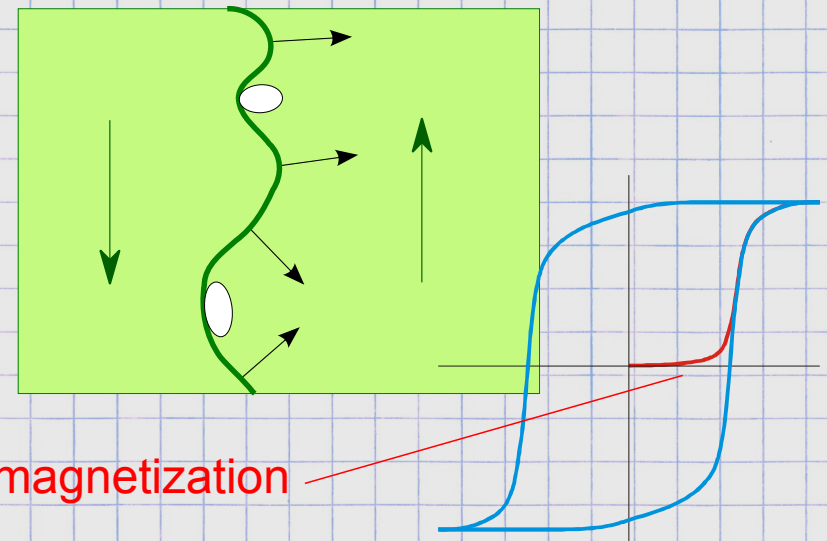


**Coercivity determined by nucleation**



**H**

**Coercivity determined by propagation**



First magnetization

- ⇒ Physics has some similarity with that of grains
- ⇒ Concept of nucleation volume

- ⇒ Physics of surface/string in heterogeneous landscape
- ⇒ Modeling necessary

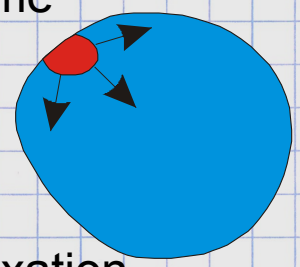


## Activation volume

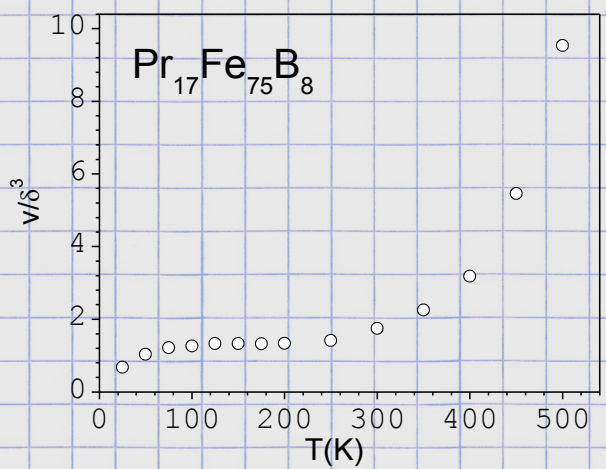
Also called: nucleation volume

Can be used for:

- ⇒ Estimating  $H_c(T)$
- ⇒ Estimating long-time relaxation
- ⇒ Determination of dimensionality



Note: of the order of domain wall width  $\delta$



Courtesy D. Givord

More detailed models:

D. Givord et al., JMMM258, 1 (2003)

## 1/cos<sup>n</sup> law

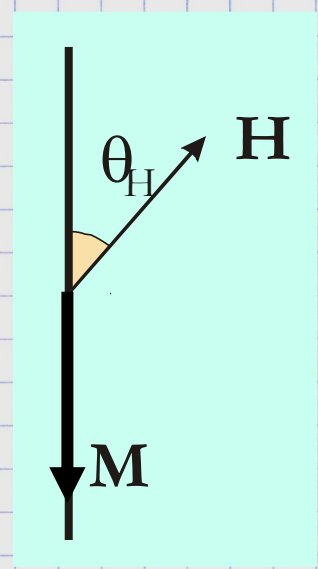
E. J. Kondorsky, J. Exp. Theor. Fiz. 10, 420 (1940)

Hypothesis:

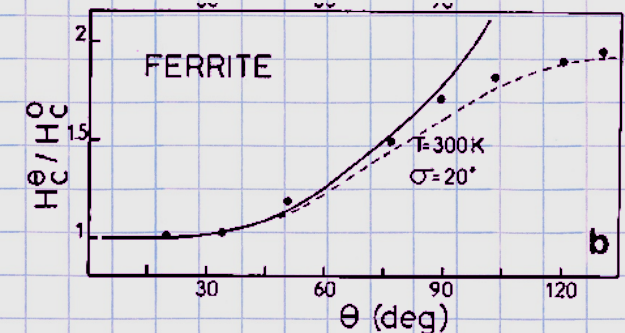
⇒ Based on nucleation volume

⇒  $H_c \ll H_a$

Energy barrier  $E_0$  overcome by gain in Zeeman energy plus thermal energy



$$E_0 = -\mu_0 M_s v_a H \cos(\theta_H) + 25k_B T$$



D. Givord et al., JMMM72, 247 (1988)



## Nucleation of new reversed domains

Fatuzzo/Labrune/Raquet model

$$dN = (N_0 - N)Rdt$$

$N$ : number of nucleated centers at time  $t$

$$\rightarrow N = N_0[1 - \exp(-Rt)]$$

$N_0$ : total number of possible nucleation centers

$R$ : rate of nucleation

## Radial expansion of existing domains

$$\sigma_n = \sigma - \sigma_c = (v_0^2/T)[t_0 + t]^2 - \pi r_c^2/T$$

$r_c$ : radius of critical nucleus

$T$ : total area of sample

$V_0$ : speed of propagation of domain wall

$$A = \int_0^t \left( \frac{dN}{dt} \right)_s (\sigma_n)_{t-s} ds + \frac{\pi r_c^2}{T} N(t)$$

Growth of existing nuclei

New nuclei

## Combination → Predicts area not yet reversed

$$\rightarrow B(t) = \exp\left(-2k^2\left(1 - (Rt + k^{-1}) + \frac{1}{2}(Rt + k^{-1})^2 - e^{-Rt}(1 - k^{-1}) - \frac{1}{2}k^{-2}(1 - Rt)\right)\right),$$

$$k = v_0 / (Rr_c)$$

$k$  is a measure of the importance of wall propagation versus nucleation events

E. Fatuzzo, Phys. Rev. 127, 1999 (1962)



Depending on structural defects

Depending on measurement dynamics

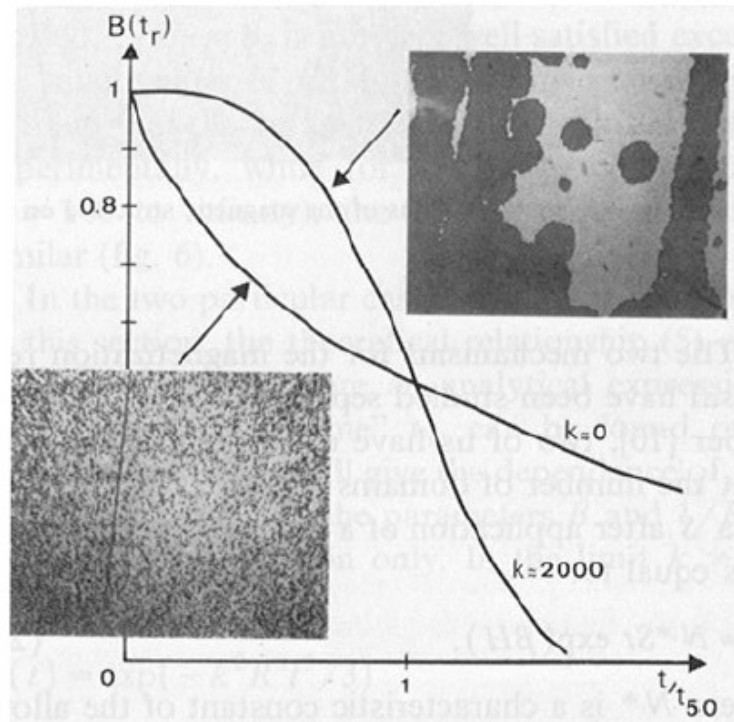
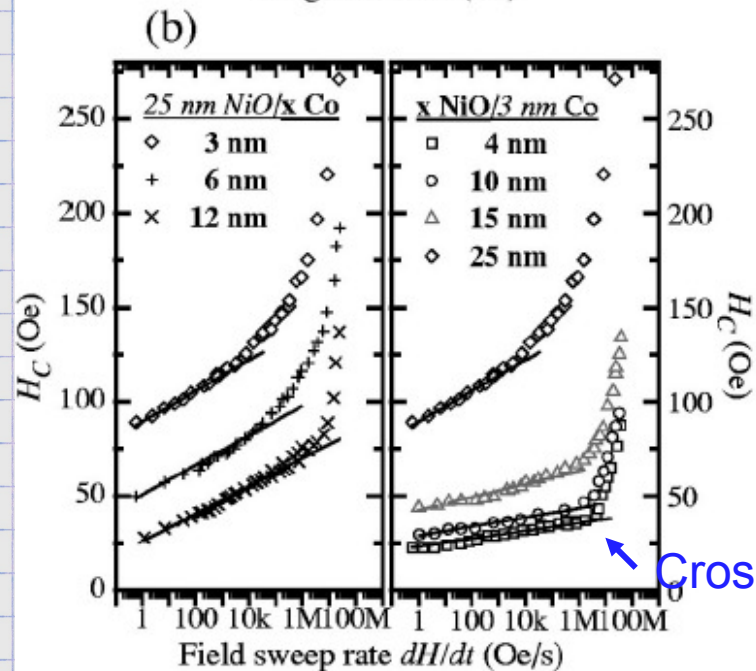
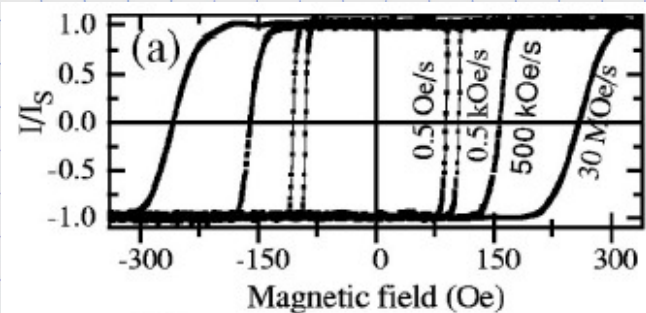


Fig. 4. Magnetization versus reduced time  $t_R$  for a GdFe sample ( $k \approx 2000$ ) and a TbCo one ( $k \approx 0$ ), corresponding domain structure observed by Kerr effect.

M. Labrune et al.,  
 J. Magn. Magn. Mater. 80, 211 (1989)



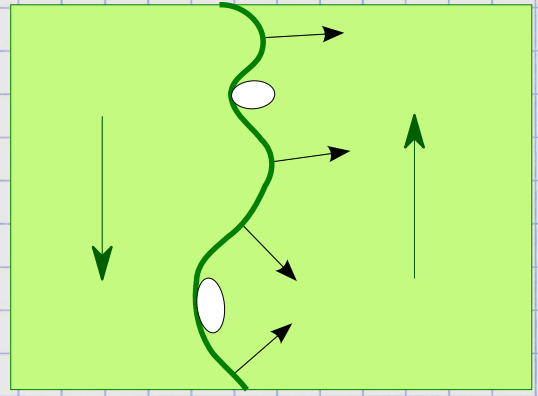
J. Camarero et al., PRB64, 172402 (2001)

Note also for fast propagation of domain walls: breakdown of propagation speed (Walker)



**Theory**

⇒ Physics : rope in a 2D medium with static disorder



⇒ Energy barriers scale like  $(1/H)^u$

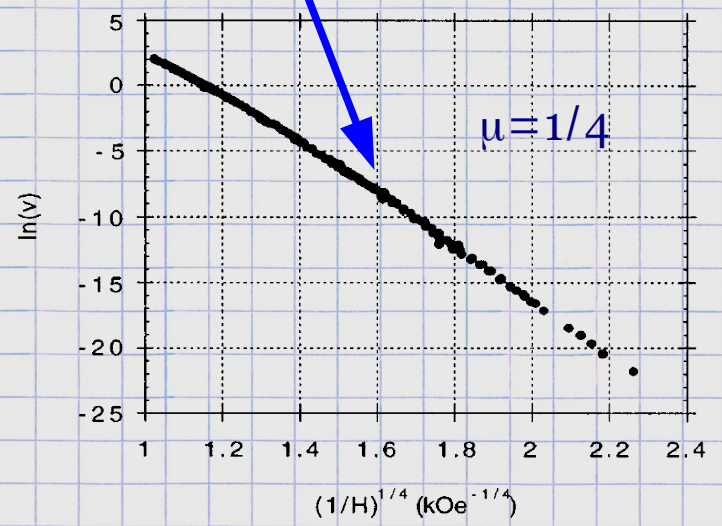
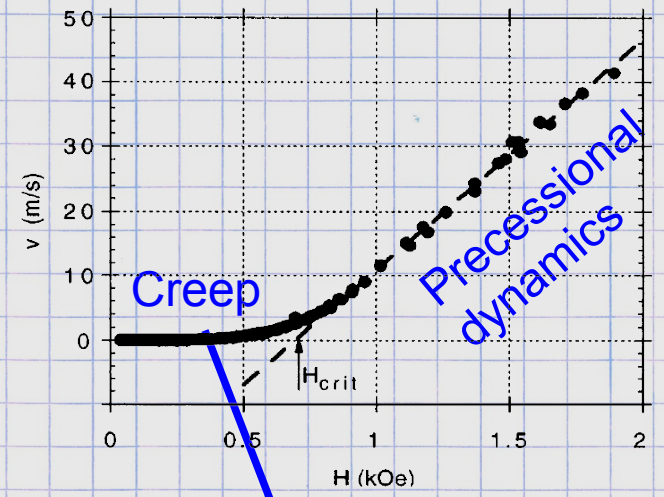
⇒ Domain wall speed determined by Arrhenius law

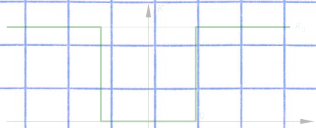
$$v(H) \sim \exp \left[ -\beta U_c \left( \frac{H_{crit}}{H} \right)^u \right]$$

S. Lemerle et al., PRB80, 849 (1998)

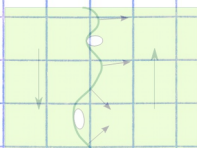
**Experiment**

Pt/Co/Pt film, perpendicular anisotropy

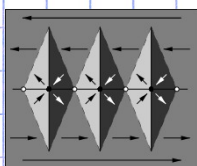




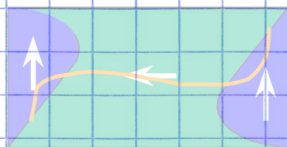
⇒ Brown paradox



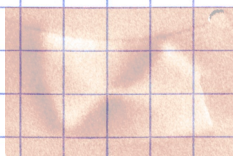
⇒ Nucleation and propagation



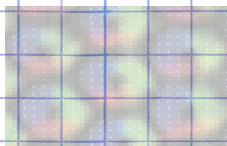
⇒ Walls and domains in films and nanostructures



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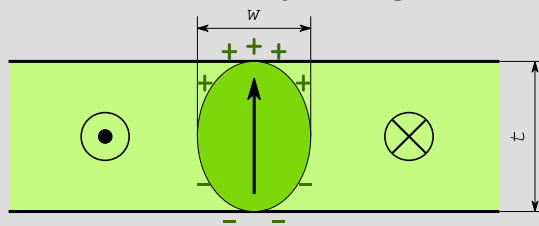
⇒ Skyrmions



## Bloch versus Néel wall

Crude model: wall is a uniformly-magnetized cylinder with an ellipsoid base

Bloch wall

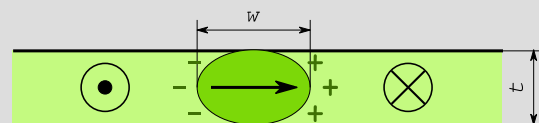


$$E_d \approx K_d \frac{W}{2t}$$

Thickness  $t$

Wall width  $W$

Néel wall



$$E_d \approx K_d \frac{t}{2W}$$

L. Néel, *Énergie des parois de Bloch dans les couches minces*,  
C. R. Acad. Sci. 241, 533-536 (1955)

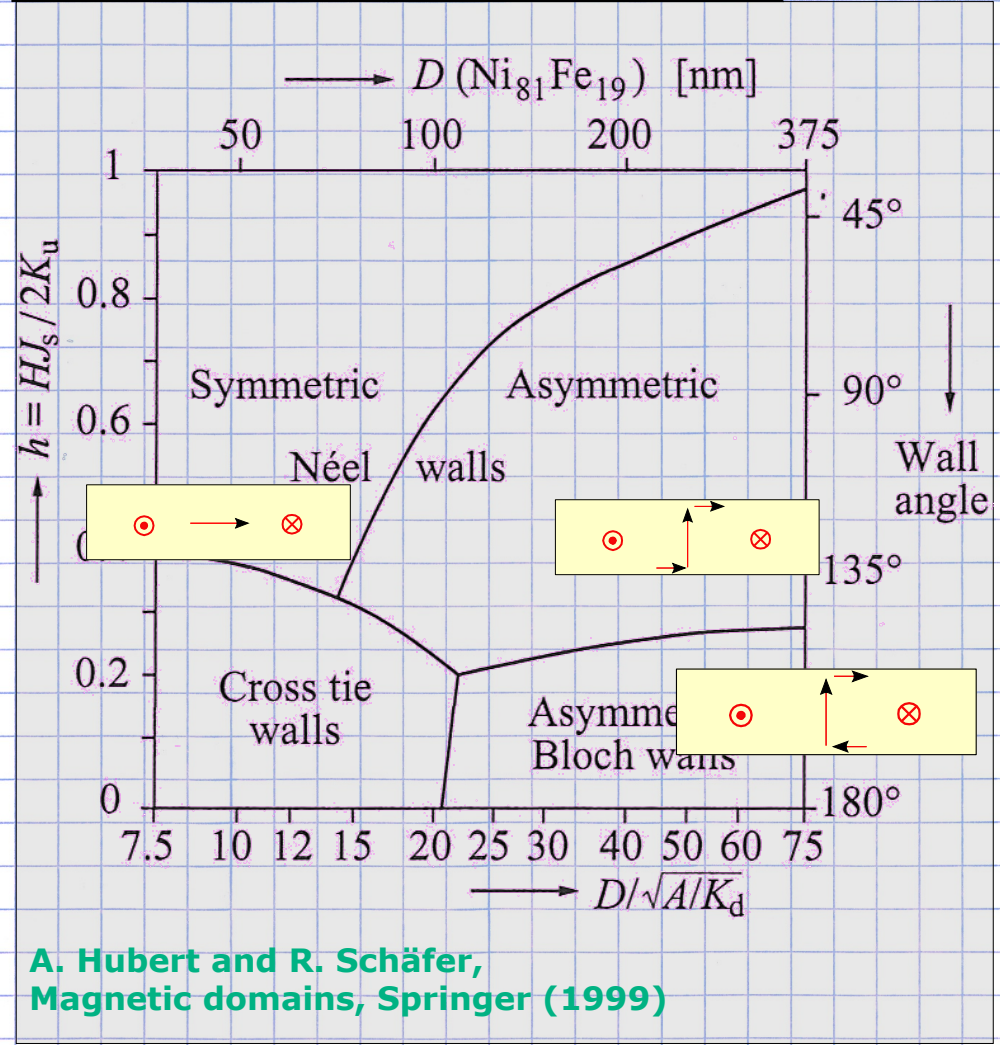
### Take-away messages

- ↪ At low thickness (roughly  $t \approx W$ ) Bloch domain walls are expected to turn their magnetization in-plane > Néel wall
- ↪ Model needs to be refined
- ↪ Domain walls not changed for films with perpendicular magnetization





Refined phase diagram of domain walls

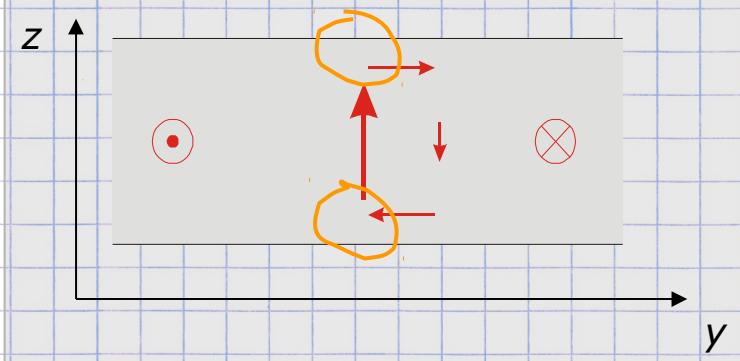


A. Hubert and R. Schäfer, *Magnetic domains*, Springer (1999)

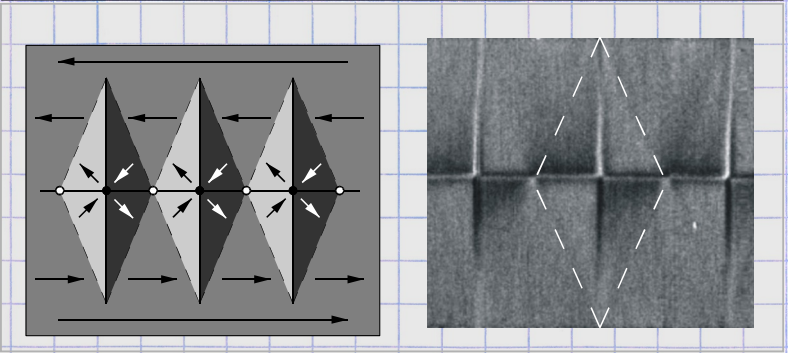
Néel caps occur atop Bloch walls to reduce surface and volume magnetic charges

$\mathbf{M} \cdot \mathbf{n} = 0$

$$-\text{div} \mathbf{M} = -\frac{\partial M_x}{\partial x} - \frac{\partial M_y}{\partial y} - \frac{\partial M_z}{\partial z}$$



From Néel walls to cross-tie walls





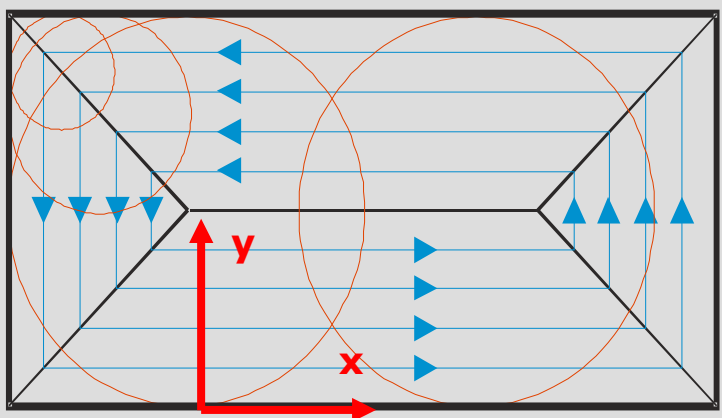
**Hypothesis** Van den Berg model

Infinitely soft material ( $K=0$ )  $e_{mc} = 0$       2D geometry (neglect thickness)  
 Zero external magnetic field  $e_z = 0$       Size  $\gg$  all magnetic length scales (wall width)  
 $e_{ex} \longrightarrow 0$

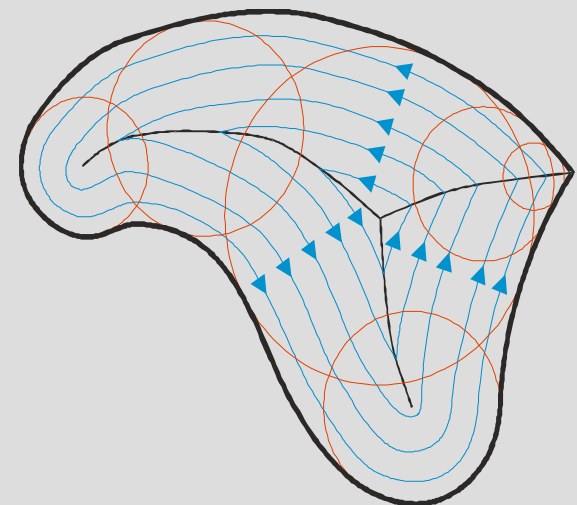
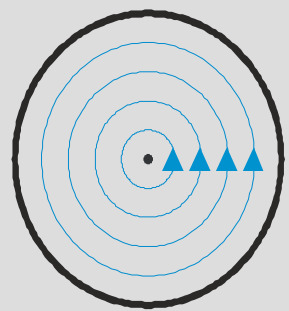
**Solution**

Looking for a solution with :  $e_d = 0$

$-\text{div}\mathbf{M} = 0$  (no volume charges)      « Flux closure »  
 $\mathbf{M}\cdot\mathbf{n} = 0$  (no surface charges)



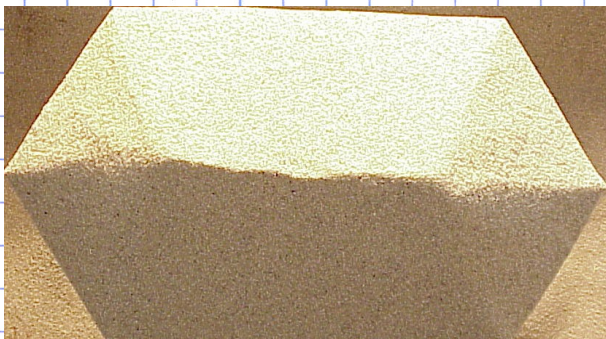
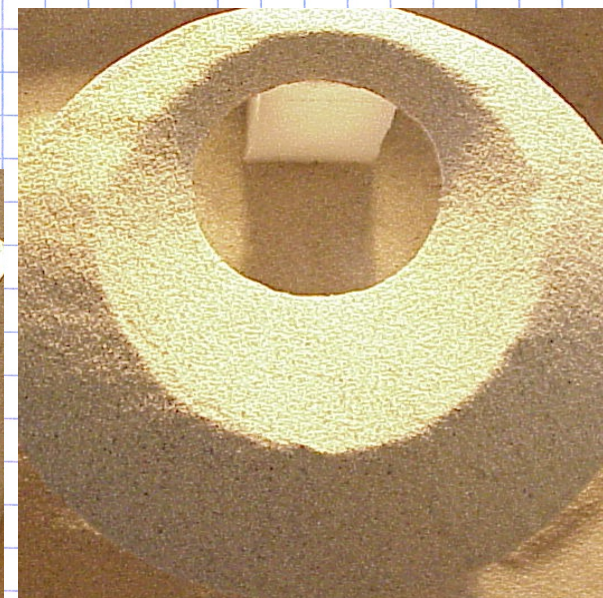
$$\text{div}\mathbf{M} = \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y}$$

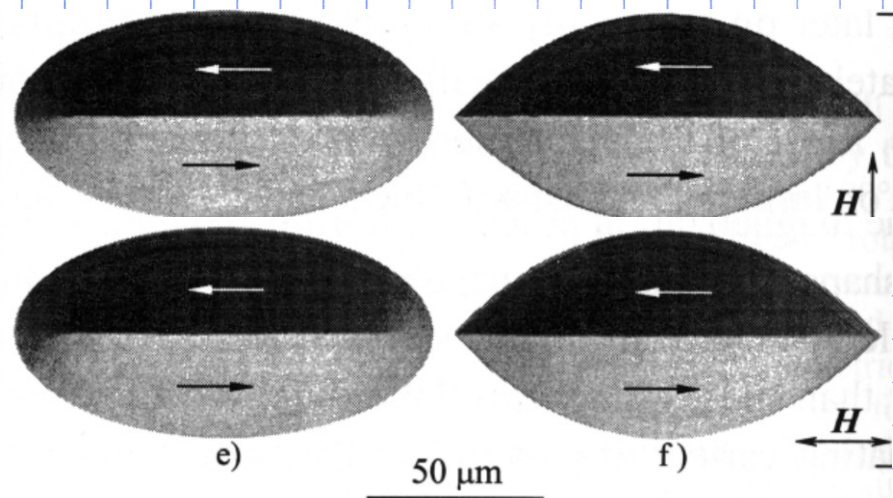
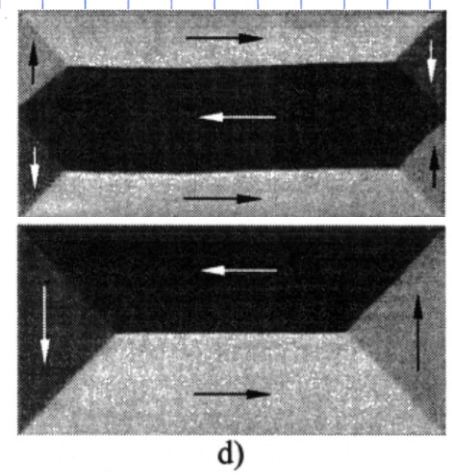


H. A. M. Van den Berg, *J. Magn. Magn. Mater.* 44, 207 (1984)

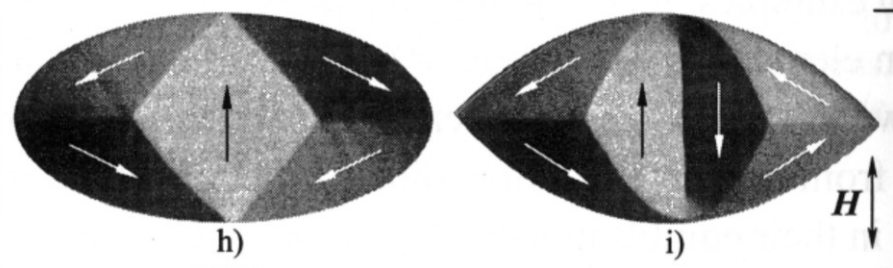
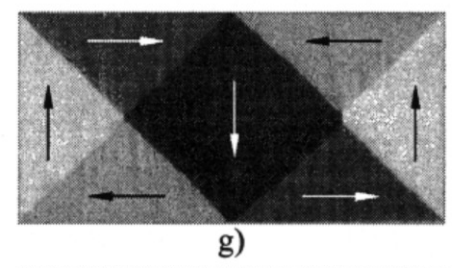


### Sandpiles for simulating flux-closure patterns

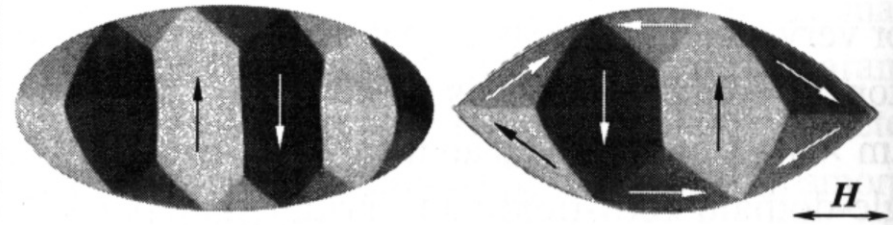
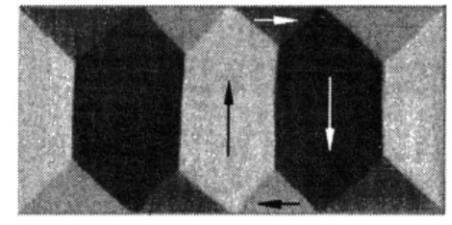




Easy axis of **weak** magnetocrystalline anisotropy



Easy axis of **weak** magnetocrystalline anisotropy



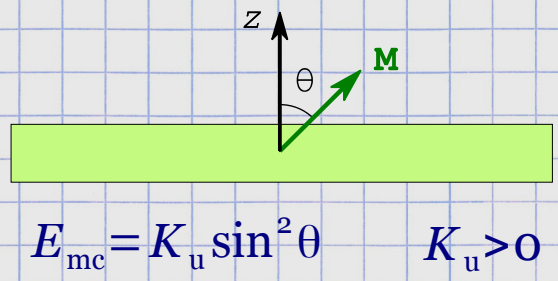
**Large dots**



- ↳ many degrees of freedom
- ↳ many possible states
- ↳ history is important
- ↳ even slight perturbations can influence the dot (anisotropy, defects, etc.).

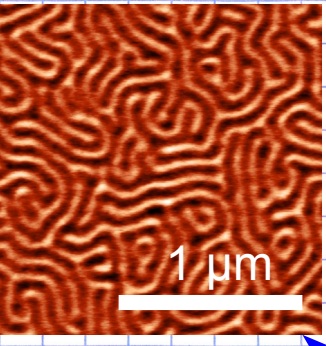


Microscopic contribution to perpendicular anisotropy



⇒ Quality factor quantifies competition between microscopic and dipolar energies

$$Q = \frac{K_u}{K_d}$$



Ge<sub>3</sub>Mn<sub>5</sub>C

$t > t_c$

Weak stripe domains

$t_c = 2\pi \Delta_u$  Second-order transition

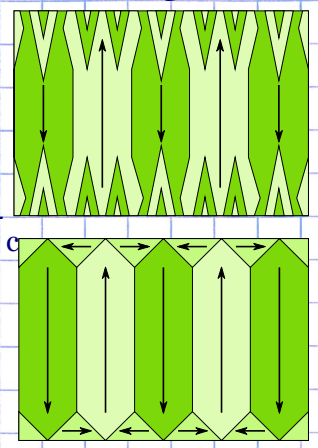
$t < t_c$

In-plane magnetized

C. Kittel, Rev. Mod. Phys. 21 (4), 541 (1949)

Y. Murayama, J. Jap. Phys. Soc. 21, 2253 (1966)

$Q > 1$



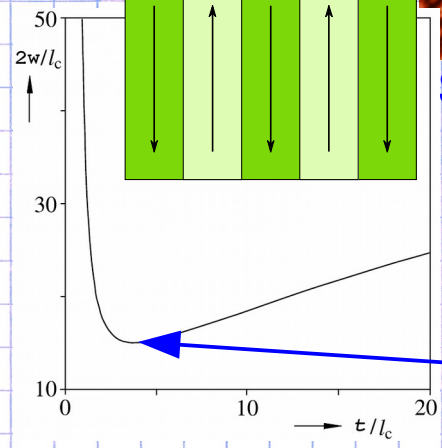
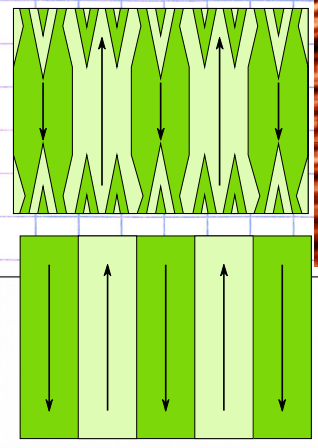
Branching

$$W \sim t^{2/3}$$

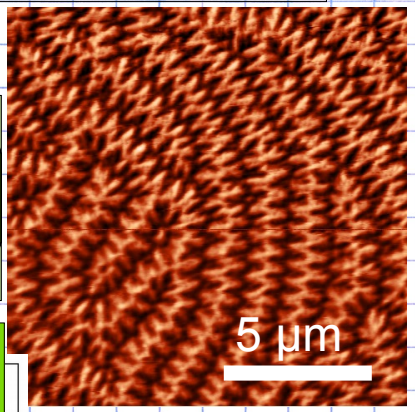
Strong stripe domains

$$W \sim t^{1/2}$$

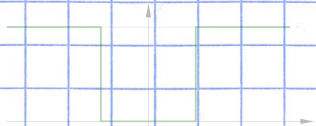
$Q < 1$



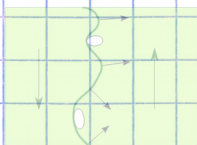
$$t_c \approx \frac{15Q}{2} \Delta_u$$



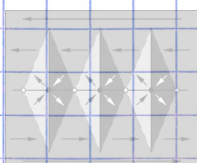
Semi-soft NdFeB



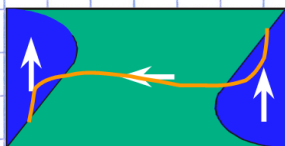
⇒ Brown paradox



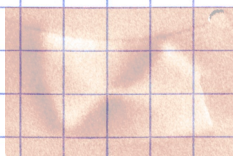
⇒ Nucleation and propagation



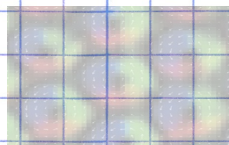
⇒ Walls and domains in films and nanostructures



⇒ Near single domains



⇒ Domain walls in tracks



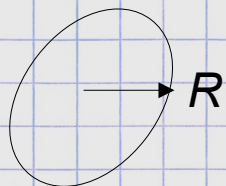
⇒ Skyrmions

## Upper bound for dipolar fields in 2D

Estimation of an upper range of dipolar field in a 2D system

$$\|\mathbf{H}_d(R)\| \leq \int_0^R \frac{2\pi r}{r^3} dr \quad \text{Integration}$$

Local dipole  $1/r^3$

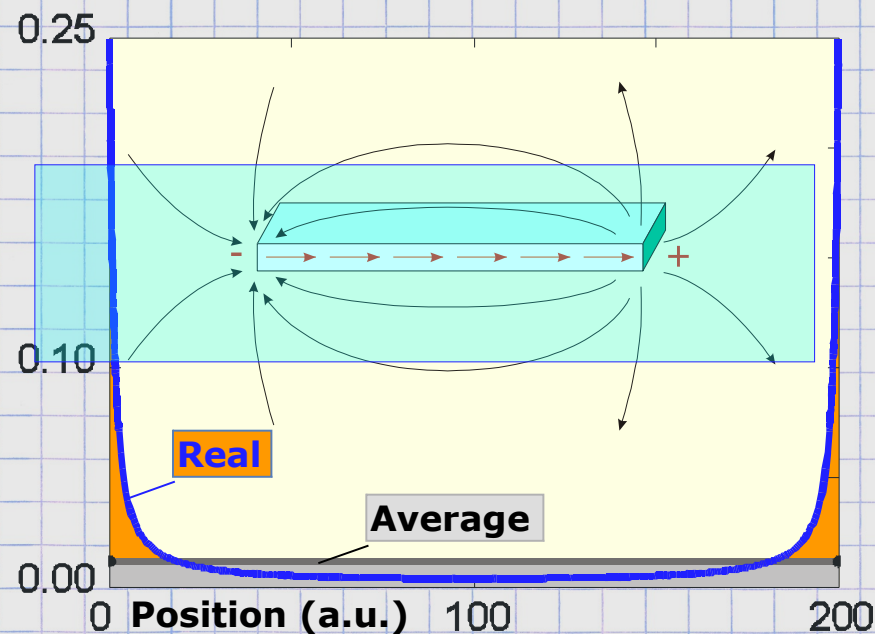


$$\|\mathbf{H}_d(R)\| \leq \text{Cte} + 1/R$$

Convergence with finite radius  
(typically thickness)

## Non-homogeneity of dipolar fields in 2D

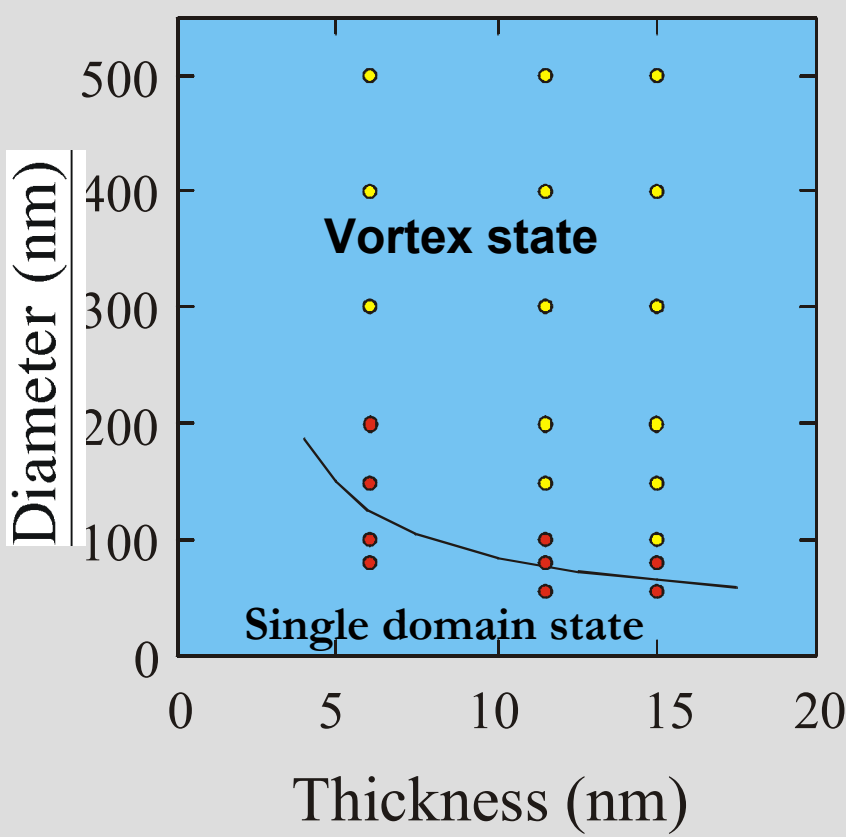
Example: flat strip with thickness/height = 0.0125



- ↪ Dipolar fields are weak and short-ranged in 2D or even lower-dimensionality systems
- ↪ Dipolar fields can be highly non-homogeneous in anisotropic systems like 2D
- ↪ Consequences on dot's non-homogenous state, magnetization reversal, collective effects etc.

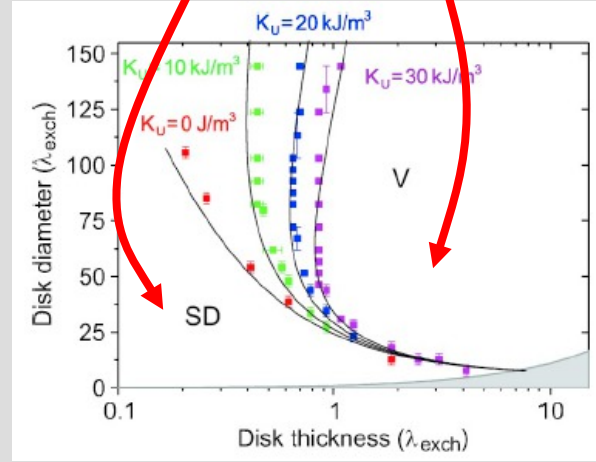
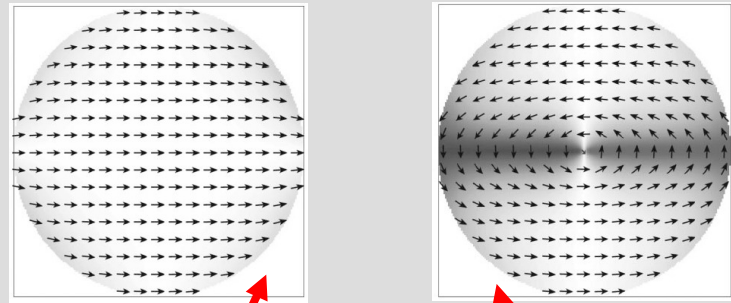


### Experiments



R.P. Cowburn, *J.Phys.D:Appl.Phys.*33, R1-R16 (2000)

### Theory / Simulation



Zero-field cross-over

$$t D \approx 20 \Delta_d^2$$



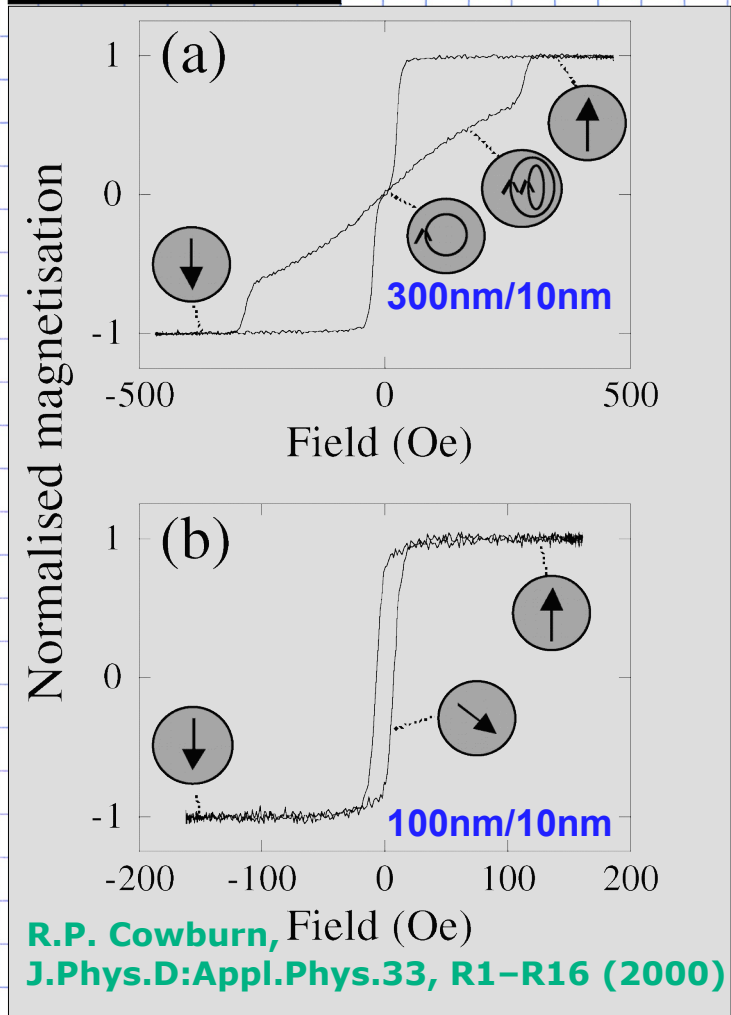
P.-O. Jubert & R. Allenspach, PRB 70, 144402/1-5 (2004)

- ↪ Vortex state (flux-closure) dominates at large thickness and/or diameter
- ↪ The size limit for single-domain is much larger than the exchange length
- ↪ Experimentally the vortex may be difficult to reach close to the transition (hysteresis)

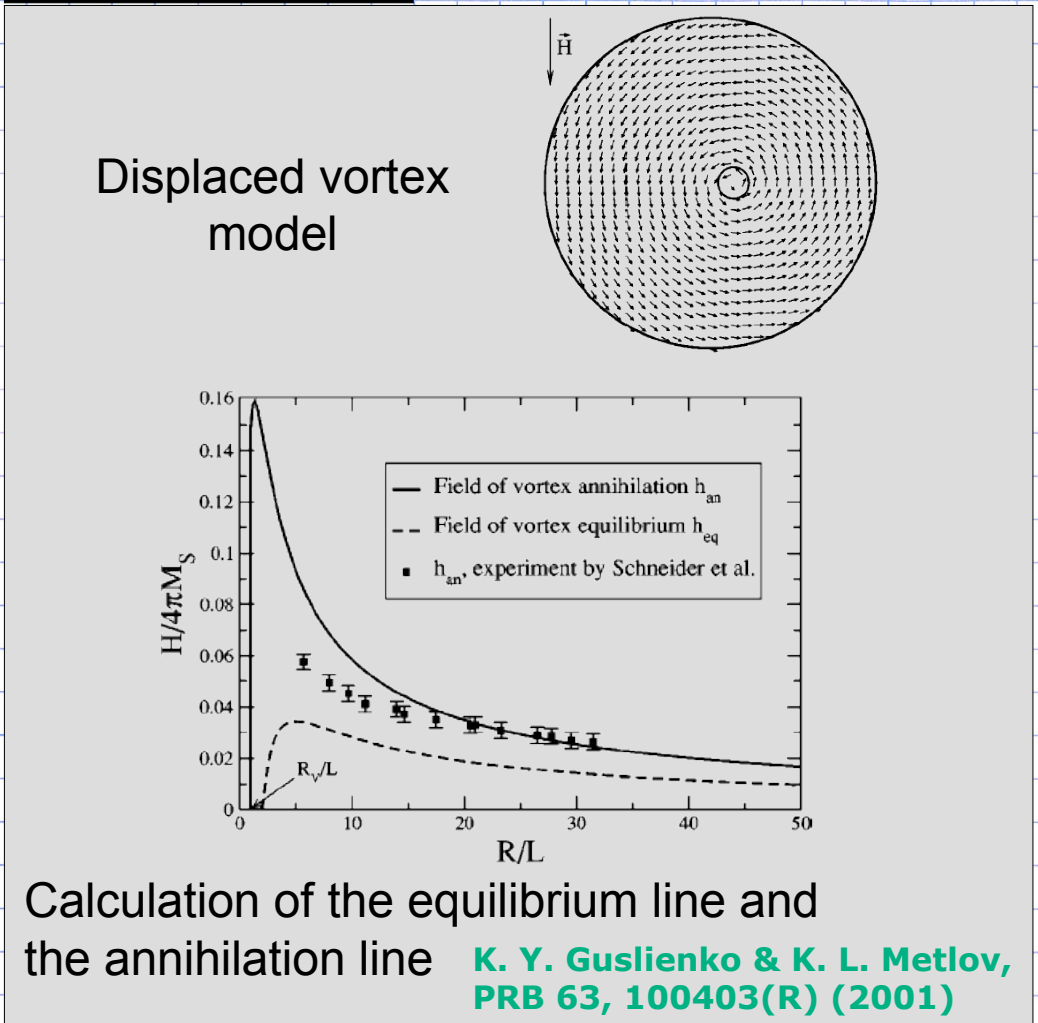




**Experiments**



**Theory / Simulations**



Typical loops for flux-closure states

Energy of the vortex state can be computed from the anhysteretic above-loop area.



**Configurational anisotropy: deviations from single-domain**

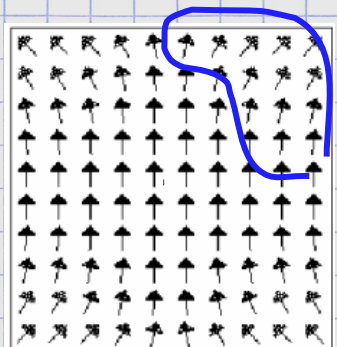
Strictly speaking, 'shape anisotropy' is of second order:

$$E_d = \frac{1}{2} \mu_0 (N_x M_x^2 + N_y M_y^2 + N_z M_z^2)$$

**2D:**  $\mathcal{E}_d = V K_d \sin^2 \theta$

In real samples magnetization is never perfectly uniform: competition between exchange and dipolar

Num.Calc. (100nm)



Flower state  
 $c/a > 2.7$



Leaf state  
 $c/a < 2.7$

**Configurational anisotropy may be used to stabilize configurations against switching**

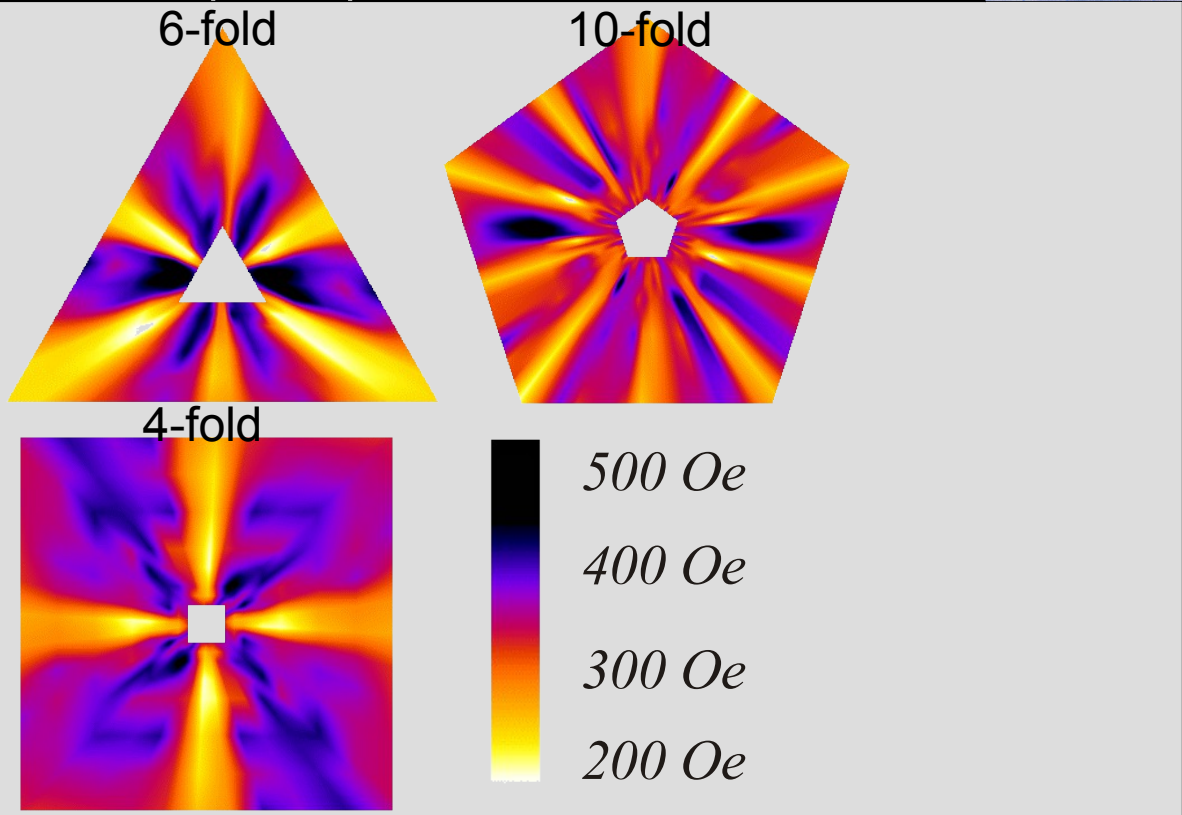
**Higher-order contributions to magnetic anisotropy**

M. A. Schabes et al., JAP 64, 1347 (1988)

R.P. Cowburn et al., APL 72, 2041 (1998)

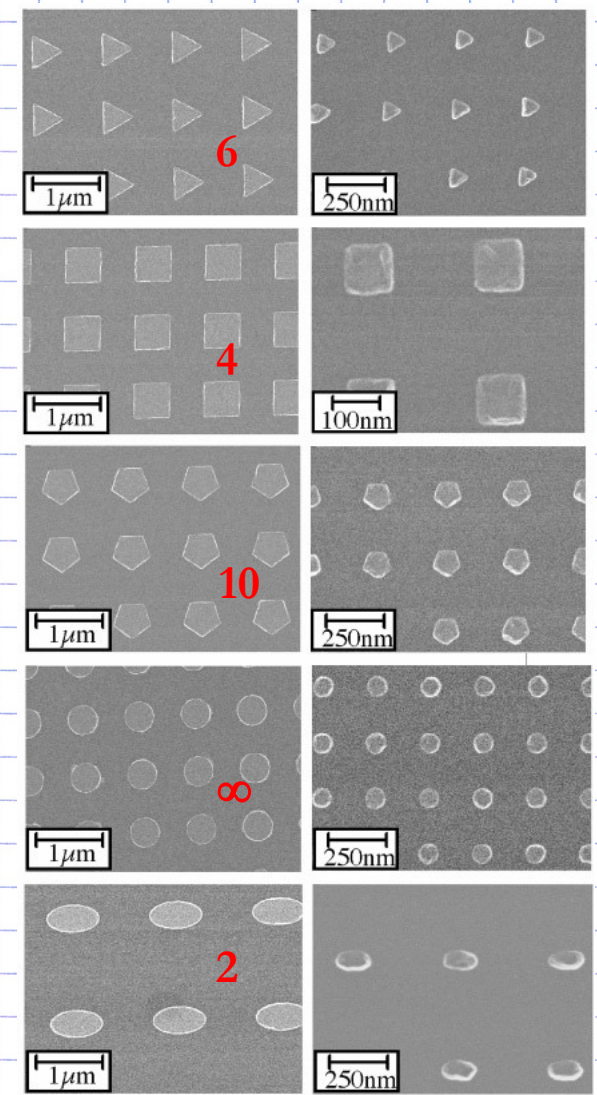


Polar plot of experimental configurational anisotropy with various symmetry



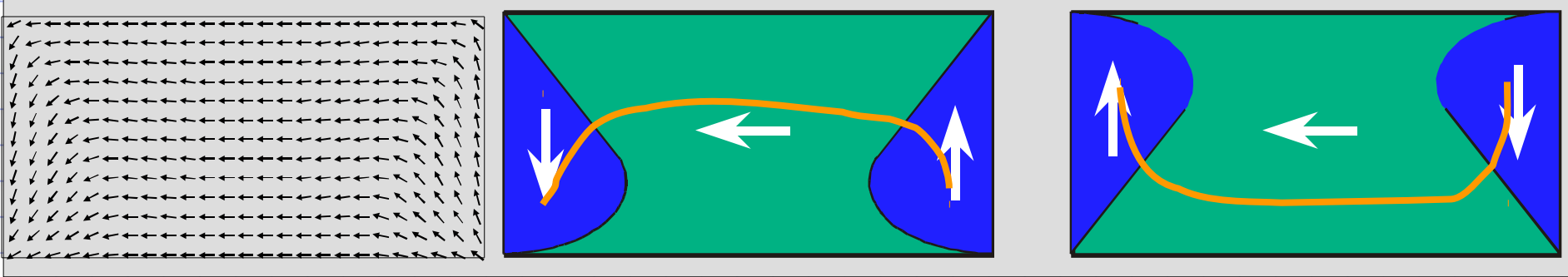
**Color code:** strength of anisotropy in a given direction  
**Radius:** size of measured pattern  
**Direction:** direction of measurement

R.P. Cowburn, J.Phys.D:Appl.Phys.33, R1-R16 (2000)

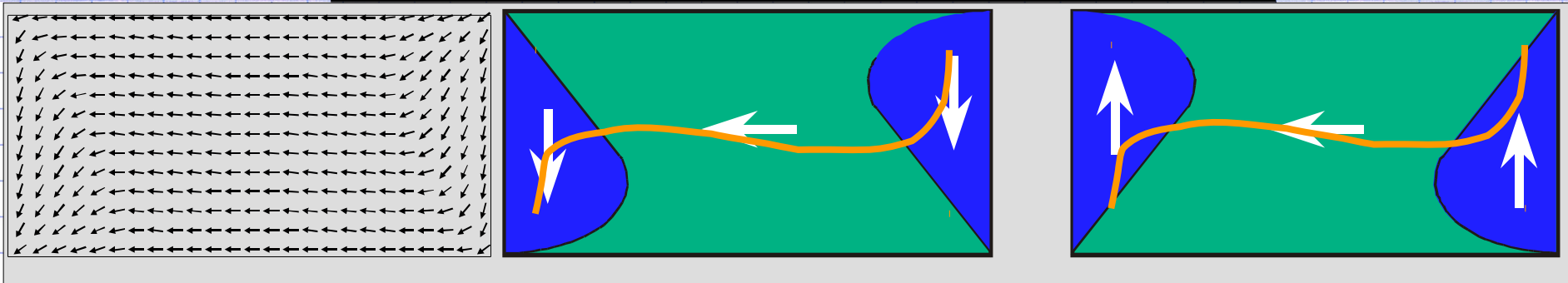




'C' state



'S' state



↪ At least 8 nearly-equivalent ground-states for a rectangular dot  
 ↪ Issue for the reproducibility of magnetization reversal

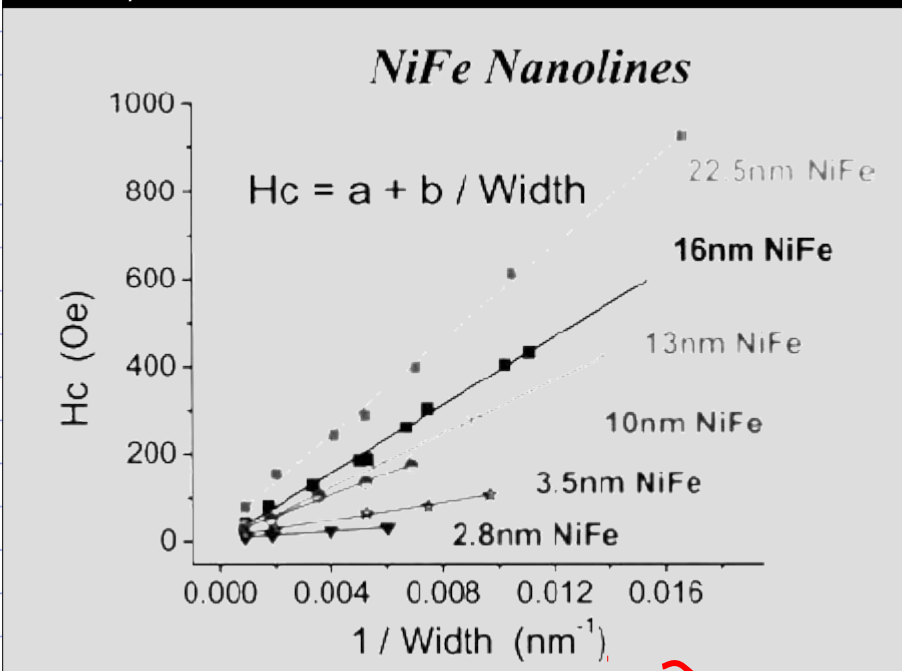


Hypotheses  $\Rightarrow$  Soft magnetic material

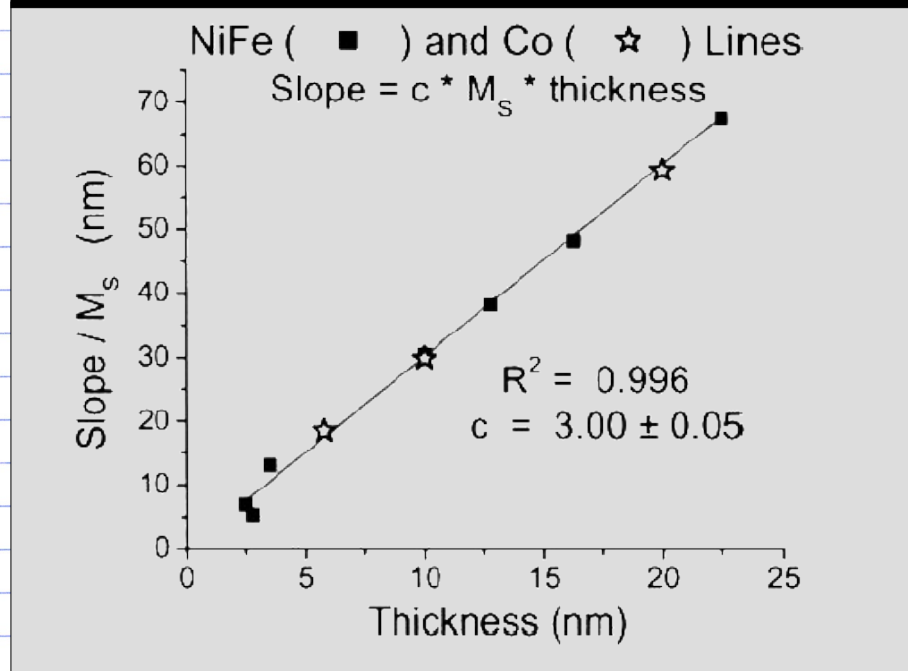
$\Rightarrow$  Not too small neither too large nanostructures



$H_c \sim 1/Width$



$H_c \sim M_s * Thickness$



$$H_c \approx a + 3M_s \frac{t}{W}$$

~Lateral demagnetizing coefficient of the strip



W. C. Uhlig & J. Shi,  
Appl. Phys. Lett. 84, 759 (2004)

## Magnetization is pinned at sharp ends

### Experiments Permalloy (soft)

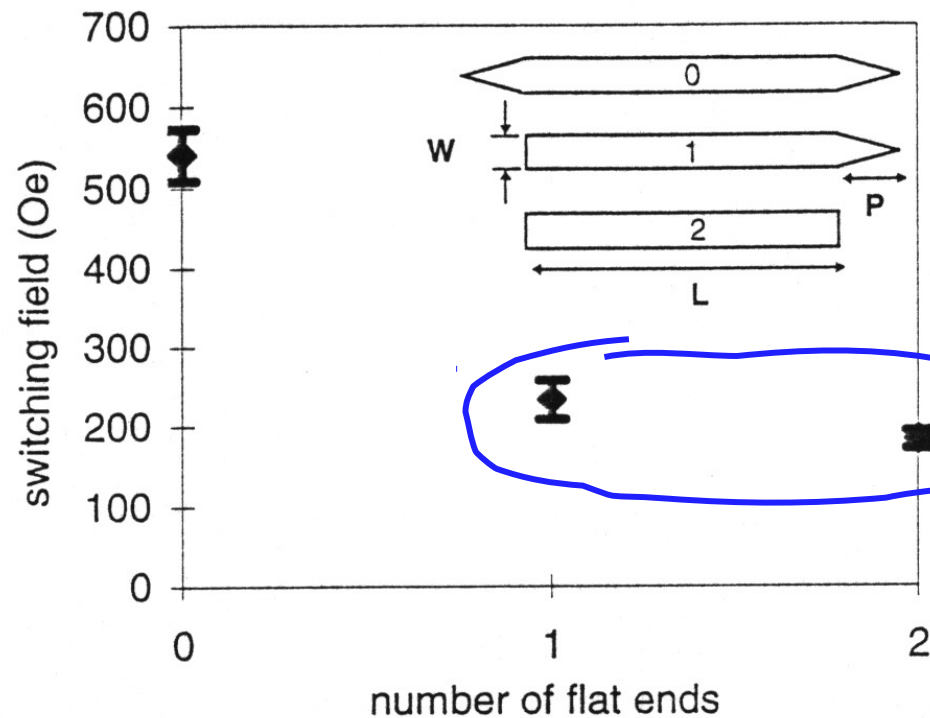
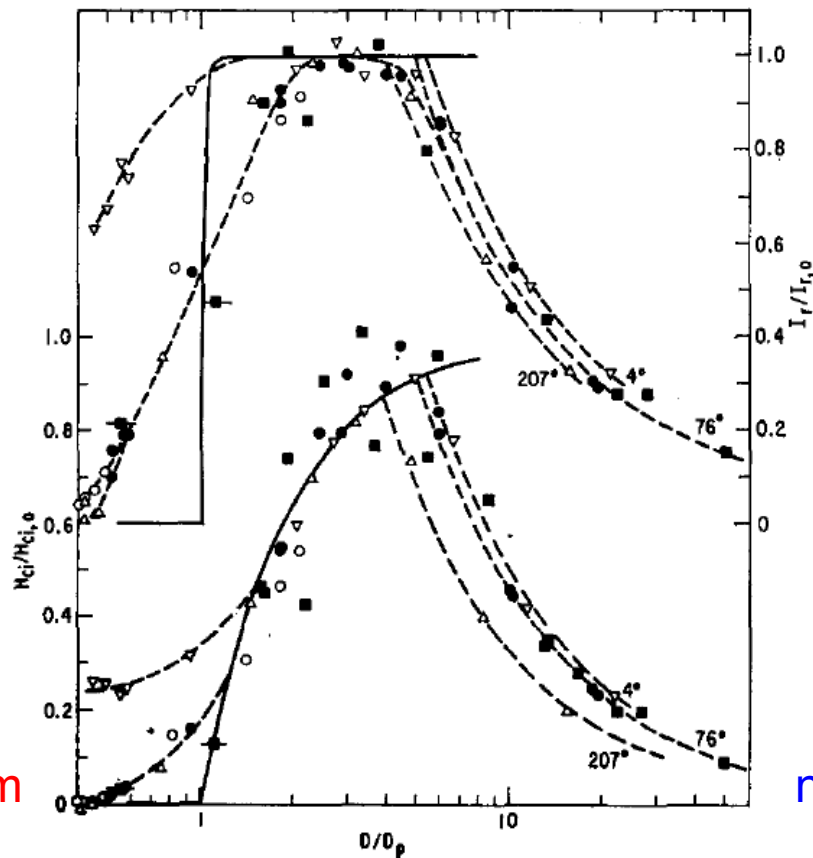


Fig. 8 Dependence of switching field of acicular elements on the number of flat ends. Element geometry is also shown:  $L=2.5\ \mu\text{m}$ ,  $W=200\ \text{nm}$ ,  $P=500\ \text{nm}$ .

K.J. Kirk et al., *J. Magn. Soc. Jap.*, 21 (7), (1997)



Towards  
superparamagnetism

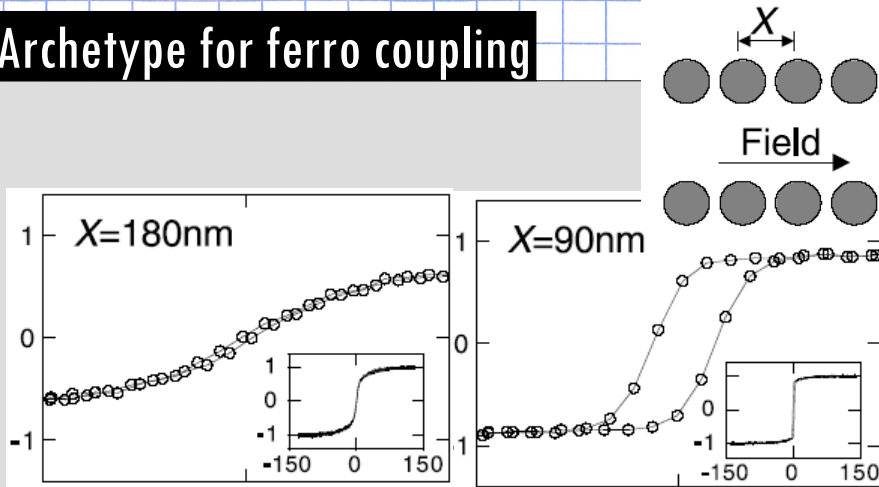
Towards  
nucleation-propagation  
and multidomain

FIG. 1. Particle size dependence of essentially spherical, randomly oriented, iron particles. Calculated curve given by solid line. Diameters  $D = \hat{d}_v$ . Data at 76°K obtained from electron microscopic examination  $\blacksquare$ , calculated from  $I_r/I_s$  vs temperature  $\circ$ , and from smoothed data of  $H_{ci}$  vs  $D$   $\bullet$ .

E. F. Kneller & F. E. Luborsky,  
*Particle size dependence of coercivity and remanence of single-domain particles,*  
J. Appl. Phys. 34, 656 (1963)

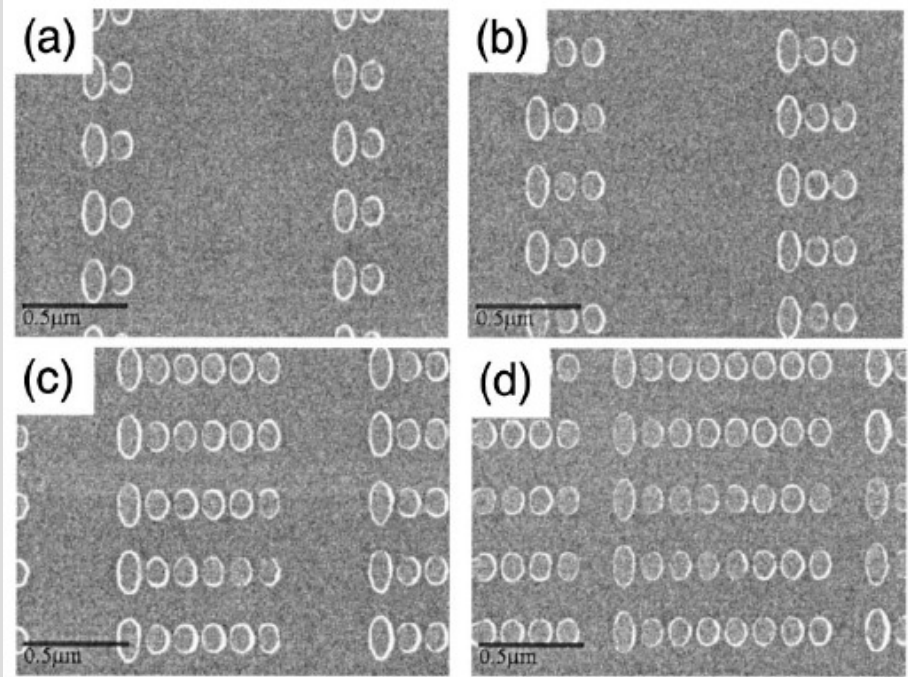


### Archetype for ferro coupling



R. P. Cowburn et al., *New J. Phys.* 1, 1-9 (1999)

### Archetype for AF coupling



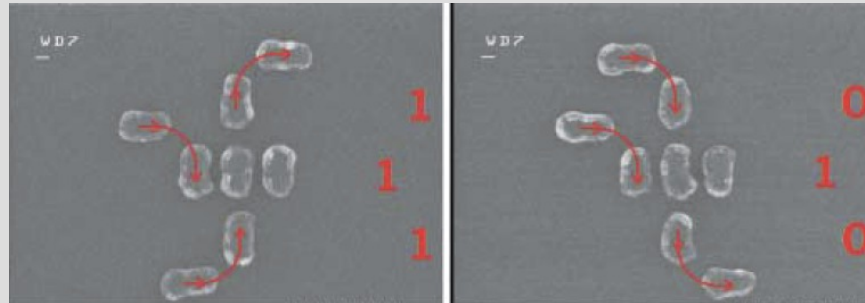
R. P. Cowburn, *PRB*65, 092409 (2002)

### Conclusion:

Interactions increase energy barriers, with F or AF interactions

### Cellular automata

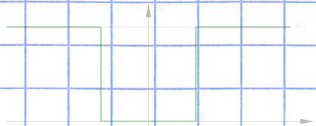
Alternative to strips and domain walls to convey and process information



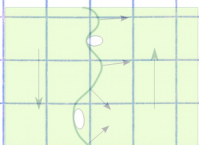
Here : majority gate

A. Imre et al., *Science* 311, 205 (2006)

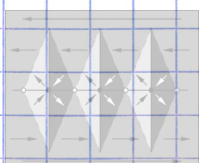




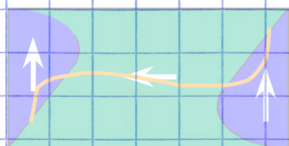
⇒ Brown paradox



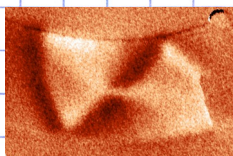
⇒ Nucleation and propagation



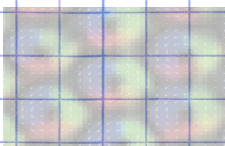
⇒ Walls and domains in films and nanostructures



⇒ Near single domains



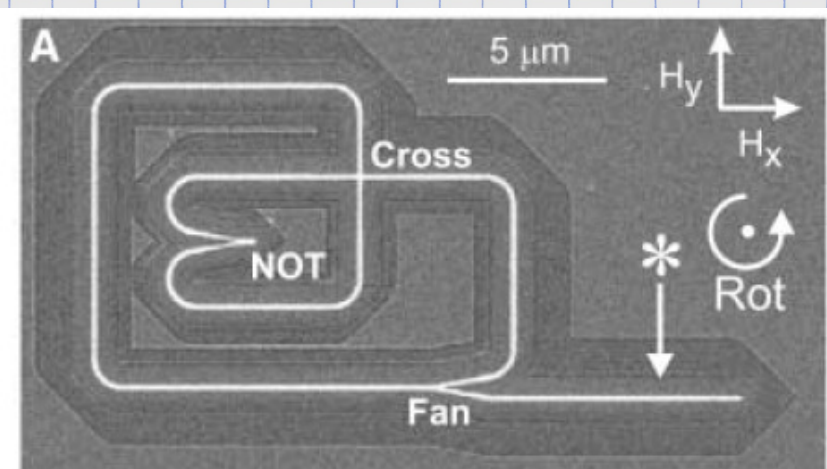
⇒ Domain walls in tracks



⇒ Skyrmions

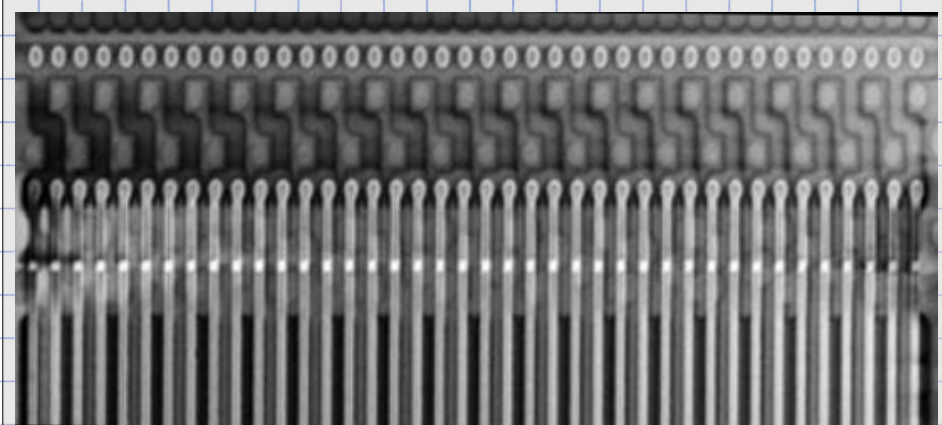


Logic (Field driven)



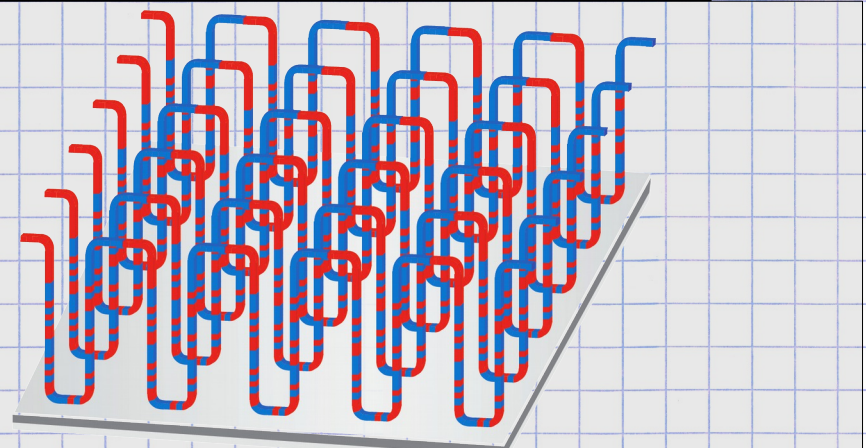
D. A. Allwood et al., Science 309, 1688 (2005)

Memory (current-driven)



L. Thomas et al., IEEE International Electron Devices meeting (2001)

Towards data 3D storage?



S. S. P. Parkin, Science 320, 190 (2008)  
Scientific American 76 (2009) + patents (IBM)

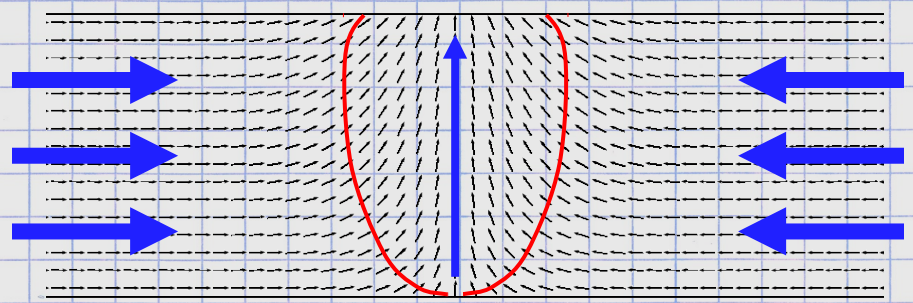
Take-away messages

- ↪ Fundamental science and device prospects
- ↪ Field-driven and later spin-torque-driven

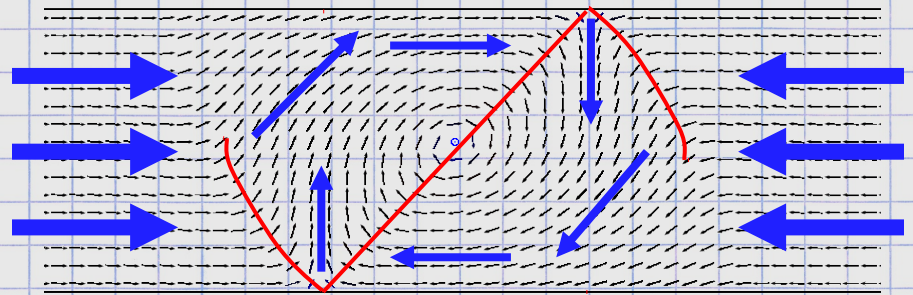


Transverse versus vortex wall (simulations)

Thin and narrow strips



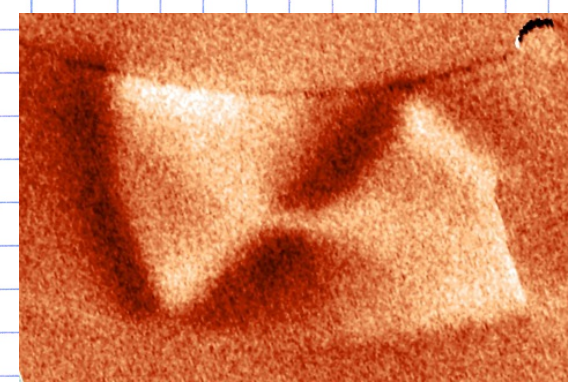
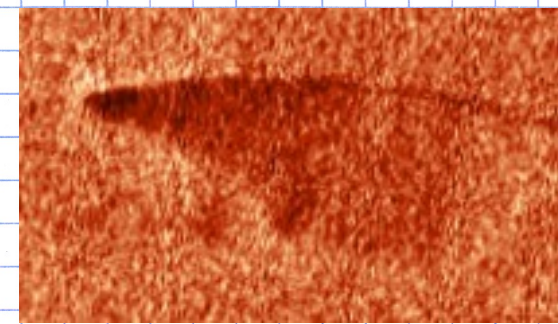
Thick and large strips



Transition for:  $tW \approx 75 \Delta_d^2$

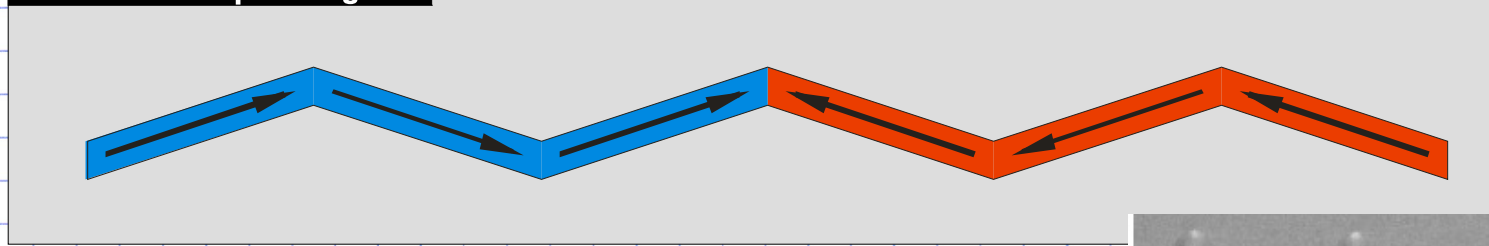
R. McMichael & M. Donahue,  
IEEE Trans. Mag. 33, 4167 (1997)

Y. Nakatani et al., J. Magn. Magn. Mater.  
290-291, 750 (2005)

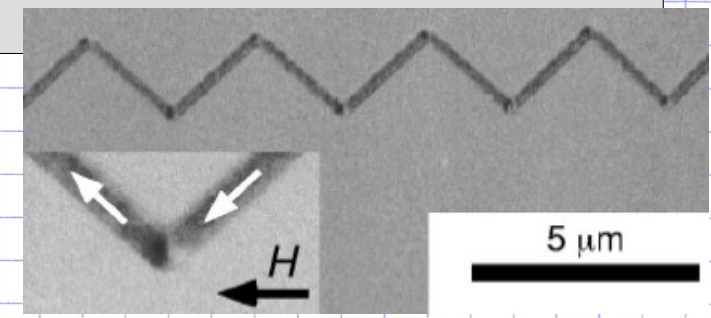
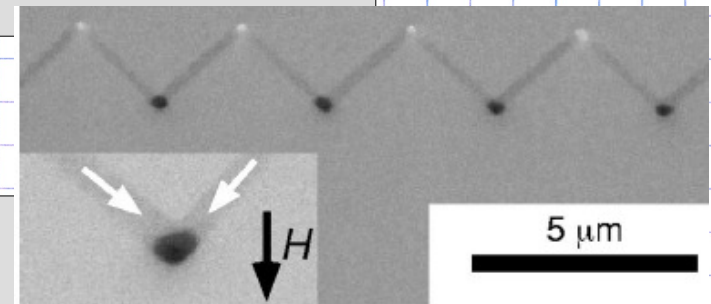
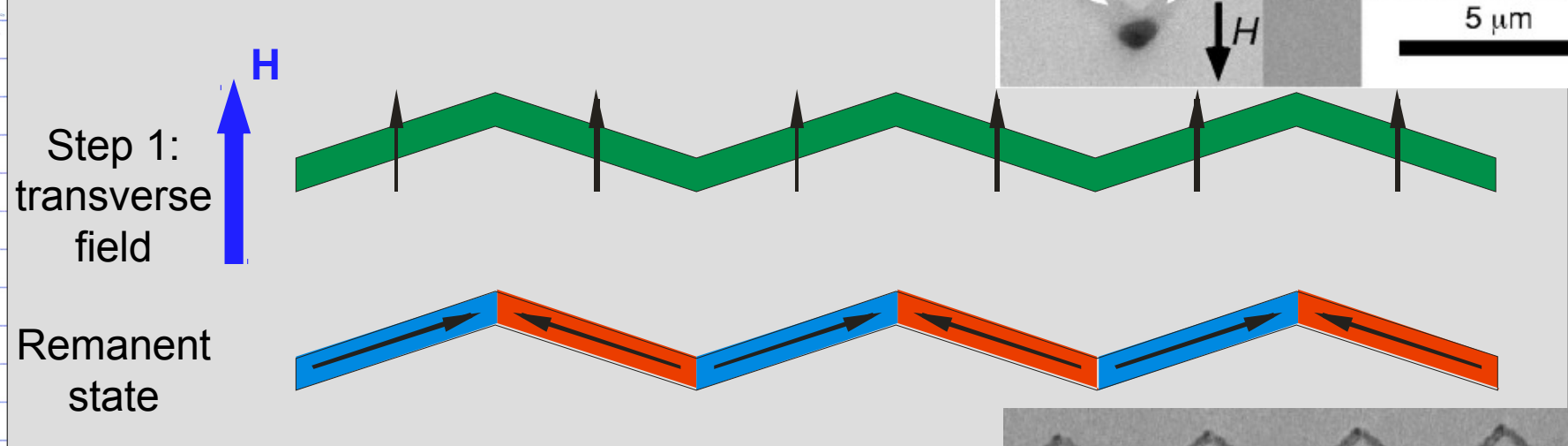




### Geometrical pinning



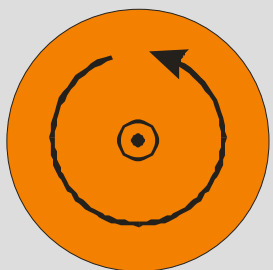
### Preparation for in-plane anisotropy



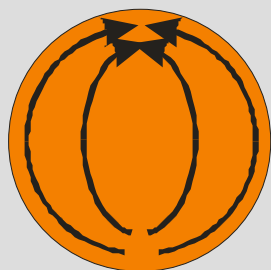
T. Taniyama et al., APL76, 613 (2000)



**Disks**



Vortex state

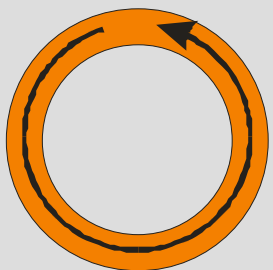


Near single-domain  
(leaf state)

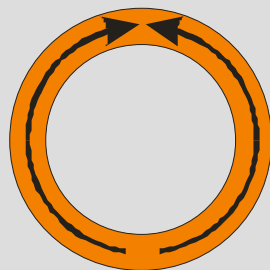
← Aspect ratio  $D/t$

← Large diameter  $D$

**Rings**



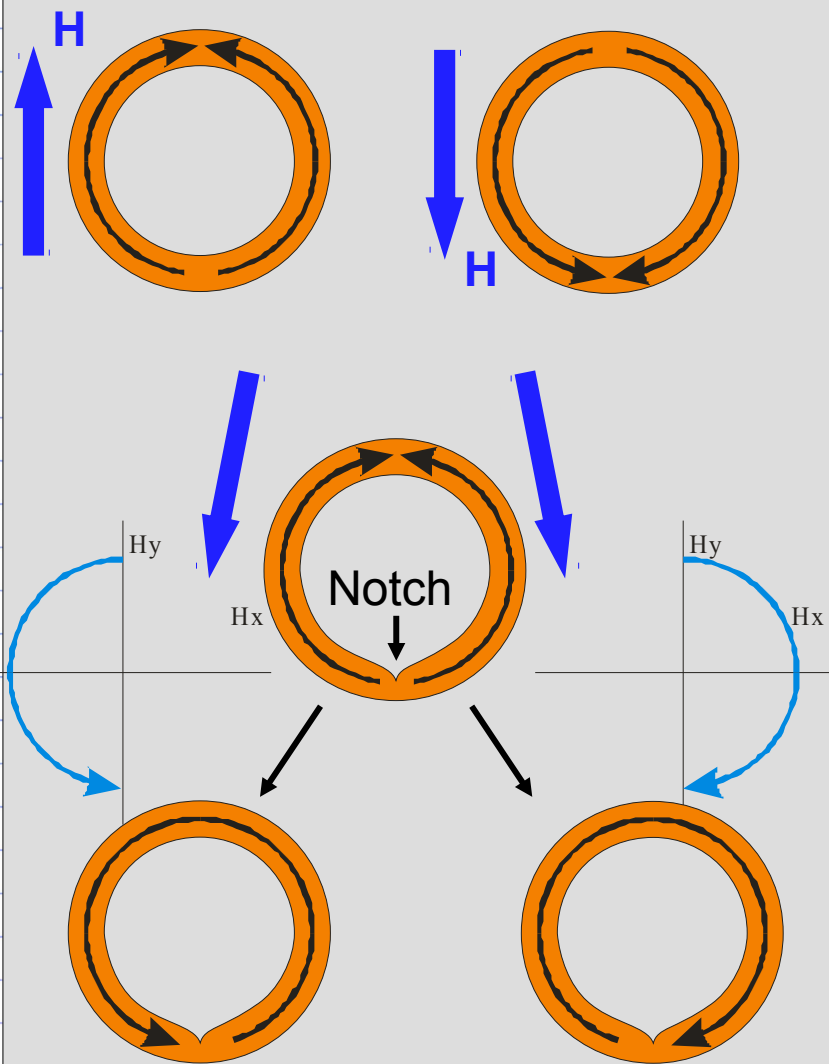
Vortex state



Onion state

Stability less dependent on geometry  
(no vortex energy)

**Control of ring states**



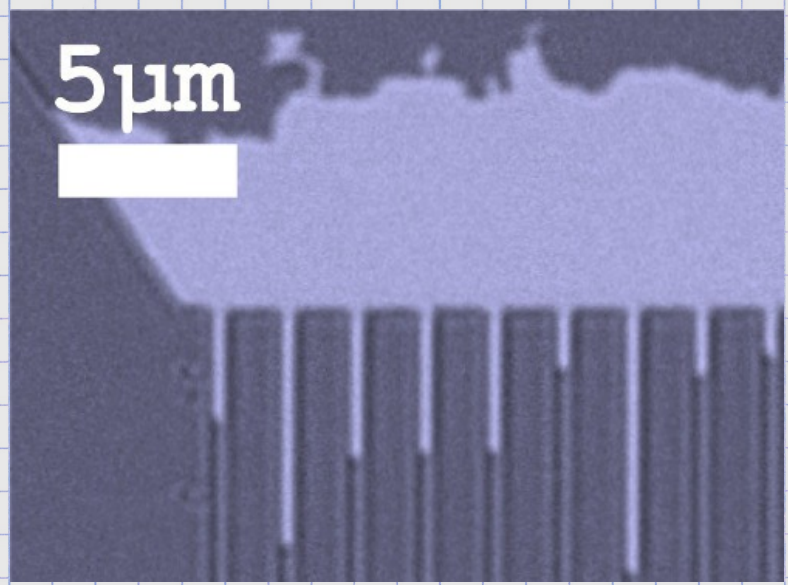
Ex: M. Kläüi et al., APL78, 3268 (2001)



### Perpendicular magnetization

#### Nucleation

Use large pads as domain reservoirs

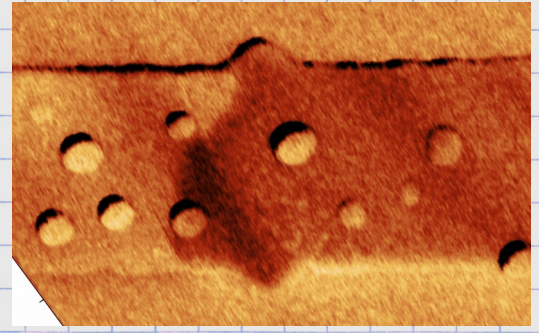


Pt\Co[0.6]\AlOx – Kerr  
Courtesy S. Pizzini (NEEL)

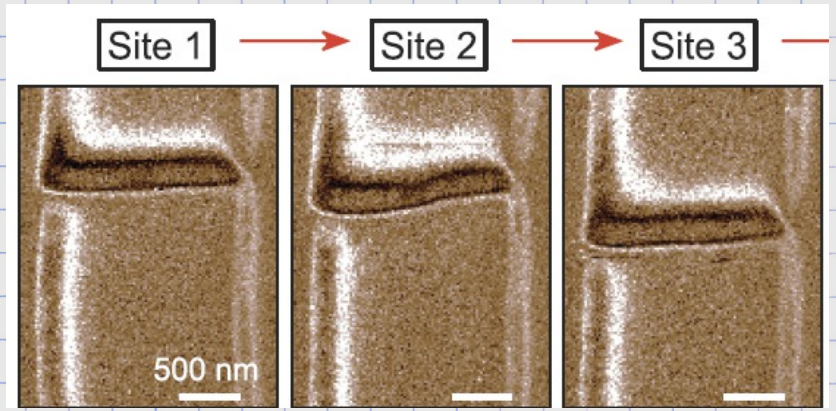
#### Magnetic imaging

Narrow domain walls (2-20nm)

⇒ Shape influenced by disorder



Pt\Co[0.6]\AlOx – MFM  
O. Fruchart, unpublished

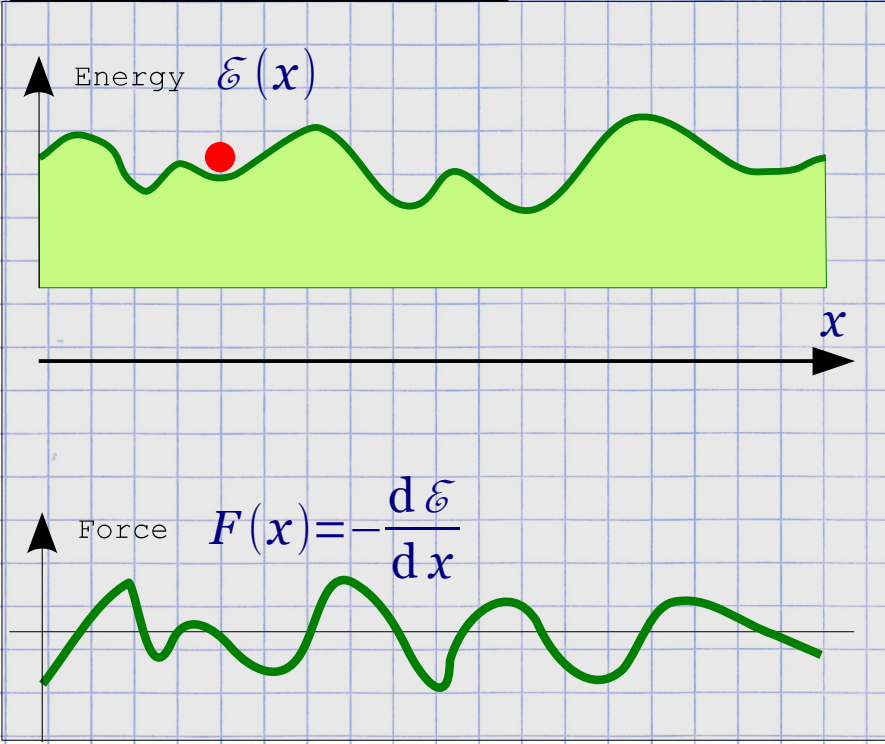


Ta\CoFeB[1]\MgO – NV center  
S.P. Tetienne t al., Science 344, 1366 (2014)

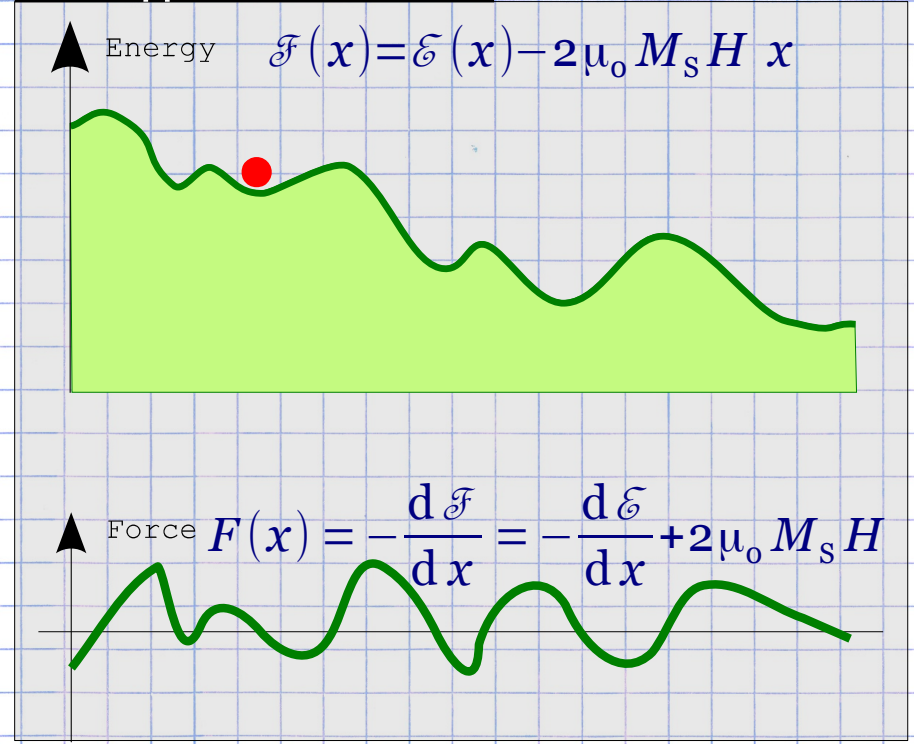


Becker-Kondorski model : domain wall to be moved along a 1d landscape

**Without applied field**



**With applied field**



E. Kondorski, On the nature of coercive force and irreversible changes in magnetisation, Phys. Z. Sowjetunion 11, 597 (1937)

**Take-away messages**

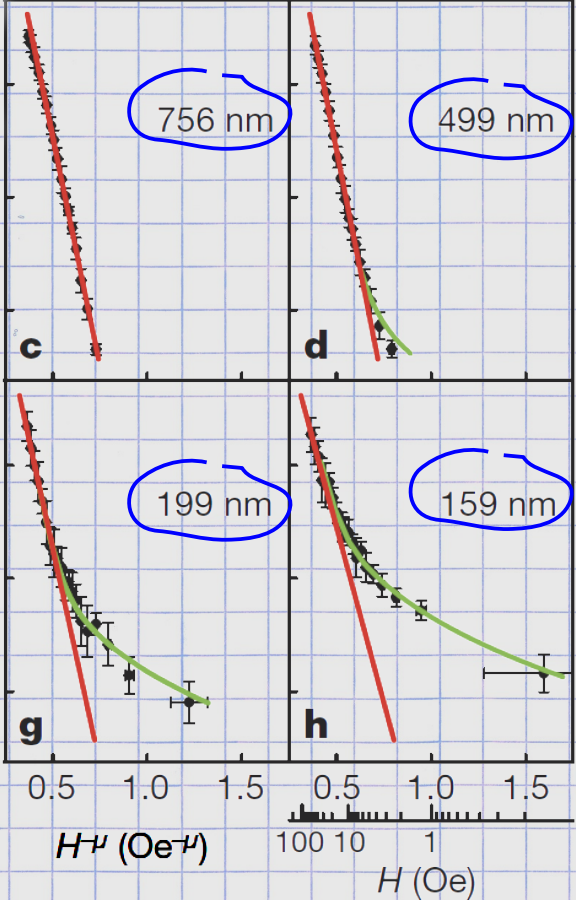
- ↪ Propagation field determined by steepest energy gradient
- ↪ Microscopic / micromagnetic model needed to build landscape
- ↪ Valid only for essentially 1d systems



From 2D to 1D scaling Creep in strips

Films :  $v(H) \sim \exp \left[ -\beta U_c \left( \frac{H_{crit}}{H} \right)^\mu \right]$   $\mu = 1/4$

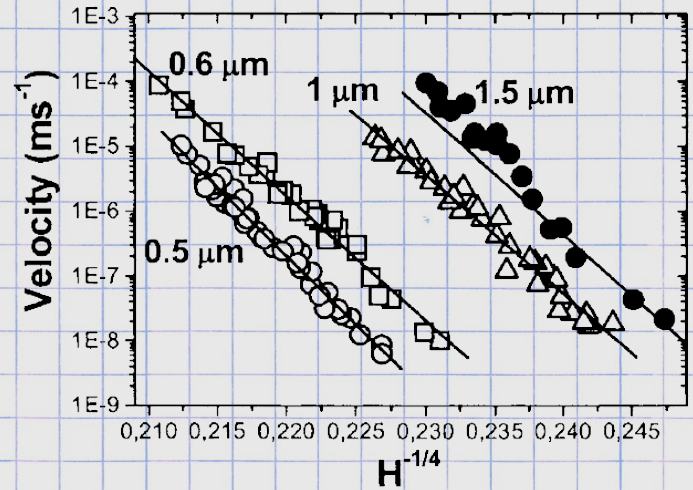
From a rope in 2D to a 1D landscape



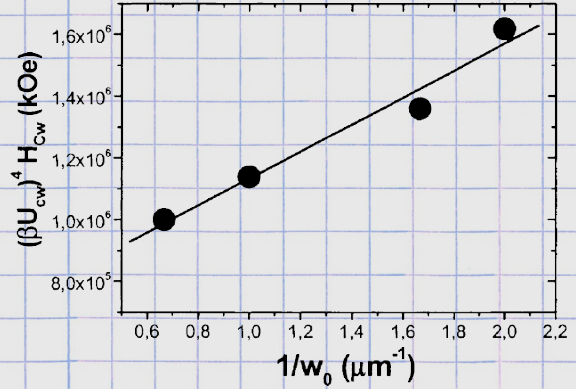
Pt/Co90Fe10 [0.3]Pt  
Decreasing strip width

K. J. Kim et al., Nature 458, 740 (2009)

Width-dependent scaling



$\mu = 1/4 \Rightarrow$  2D creep scaling



1/w scaling of critical field  
 $\Rightarrow$  Edge dominated (pseudo-1D)

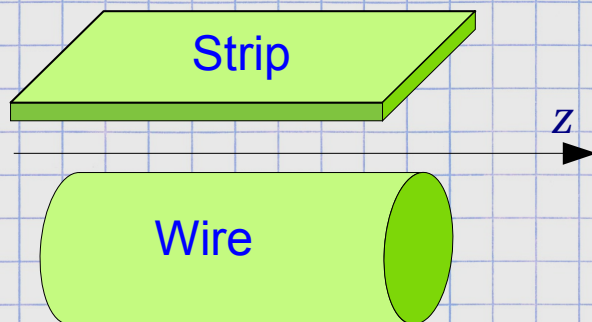
F. Cayssol et al., PRL92 (10, 107202 (2004)

Pt/Co/Pt film, perpendicular anisotropy





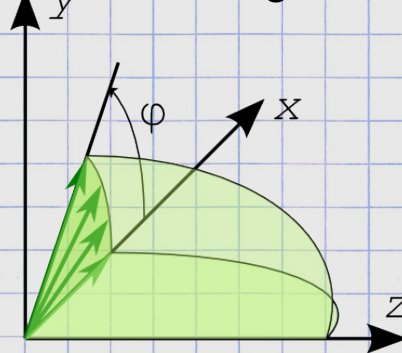
Geometry : 1D



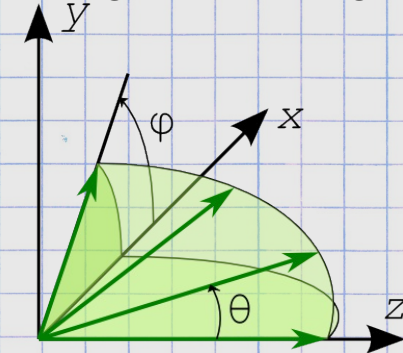
1D model :  $\mathbf{m} = \mathbf{m}(z)$

Polar coordinates for magnetization

Azimuthal angle



Longitudinal angle



Principle for naïve solving

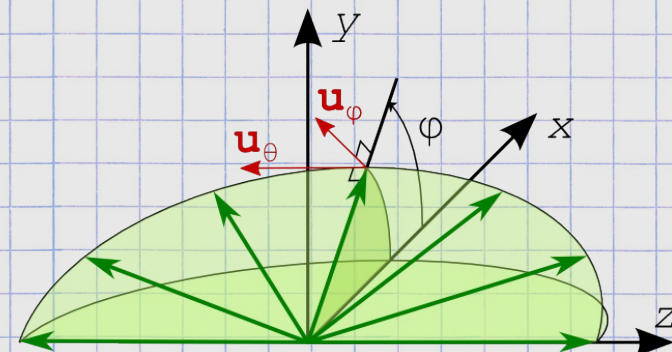
- ⇒ Assume uniform azimuth :  $\varphi = \varphi(t)$
- ⇒  $\theta = \theta(z, t)$

- ⇒ Search for steady-state motion

- ⇒ Focus on center of domain wall

- ⇒ Use particulate derivative to convert time variation into motion

$$\frac{D\mathbf{m}}{Dt} = \frac{\partial \mathbf{m}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{m} = \mathbf{0} \quad \longrightarrow \quad v = \Delta_w \left( \frac{d\mathbf{m}}{dt} \cdot \mathbf{u}_\theta \right)$$

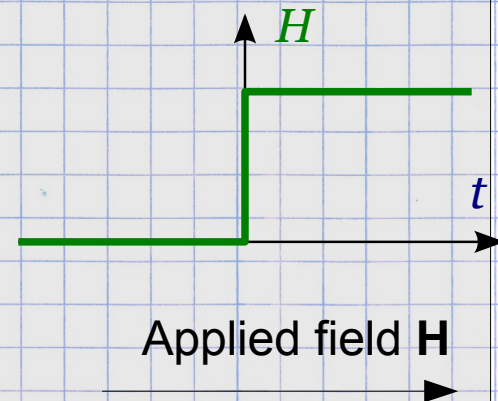
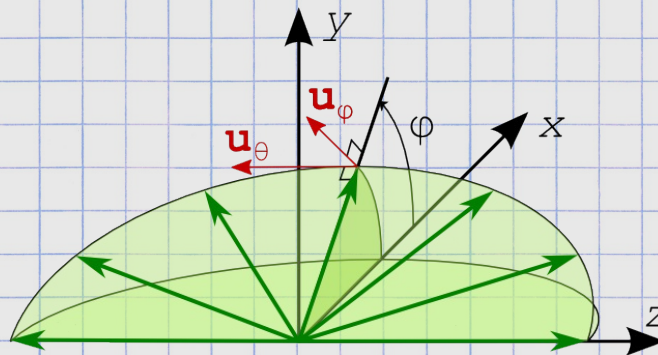
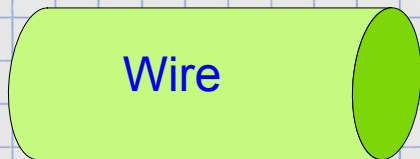


$$\frac{d\mathbf{m}}{dt} = -|\gamma_0| \mathbf{m} \times \mathbf{H} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

NB :  $\gamma_0 < 0$



Notations



$$\frac{d\mathbf{m}}{dt} = -|\gamma_o| \mathbf{m} \times \mathbf{H} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Solving

$$\left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}} = -|\gamma_o| \mathbf{m} \times \mathbf{H} = |\gamma_o| H \mathbf{u}_\varphi$$

$$\left. \frac{d\mathbf{m}}{dt} \right|_{\alpha} = \alpha \mathbf{m} \times \left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}} = -\alpha |\gamma_o| H \mathbf{u}_\theta$$

For  $\alpha \ll 1$   $\left. \frac{d\mathbf{m}}{dt} \right|_{\alpha} \approx \left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}} = |\gamma_o| H \mathbf{u}_\varphi$

Main features

Reminder :  $v = \Delta_w \left( \left. \frac{d\mathbf{m}}{dt} \right|_{\alpha} \cdot \mathbf{u}_\theta \right)$

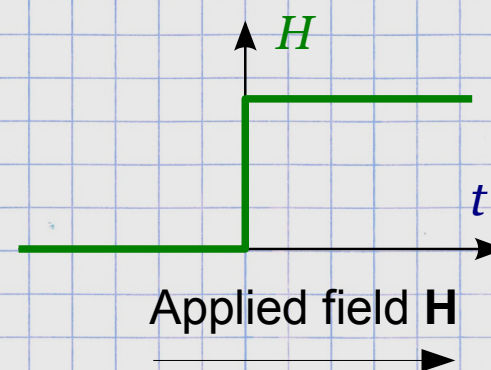
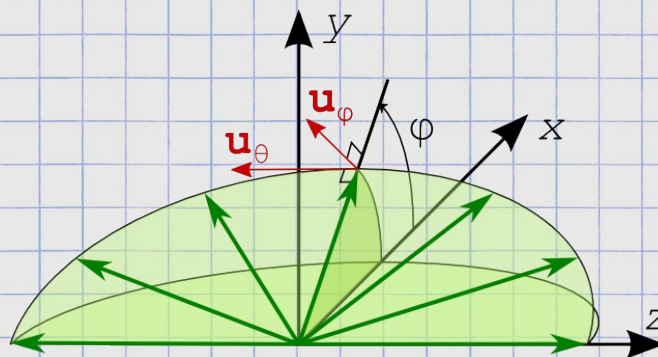
⇒ Longitudinal speed  $v = -\alpha |\gamma_o| H \Delta_w$

⇒ Azimutal precession  $\omega = |\gamma_o| H$

⇒ Fast azimutal precession, slow forward motion



Notations



$$\frac{d\mathbf{m}}{dt} = -|\gamma_0| \mathbf{m} \times \mathbf{H} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Step 1

$$\left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}} = \left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}} = -|\gamma_0| \mathbf{m} \times \mathbf{H} = |\gamma_0| H \mathbf{u}_\phi$$

⇒ Onset of azimuthal precession

⇒ Creates demag field  $\mathbf{H}_d = -(\sin \varphi) M_s \mathbf{u}_y$

⇒ Similar to precession of a macrospin dot

Later stages

$$\begin{aligned} \left. \frac{d\mathbf{m}}{dt} \right|_{\text{Field}} &= \left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}} + \left. \frac{d\mathbf{m}}{dt} \right|_{\mathbf{H}_d} \\ &= |\gamma_0| H \mathbf{u}_\phi - |\gamma_0| M_s \sin \varphi \cos \varphi \mathbf{u}_\theta \\ &\quad \text{Precession} \quad \text{Forward motion} \\ \left. \frac{d\mathbf{m}}{dt} \right|_{\alpha} &= \alpha \mathbf{m} \times \left. \frac{d\mathbf{m}}{dt} \right|_{\text{Field}} \\ &= -\alpha |\gamma_0| H \mathbf{u}_\theta - \alpha |\gamma_0| M_s \sin \varphi \cos \varphi \mathbf{u}_\phi \\ &\quad \text{Weak forward motion} \quad \text{Opposes precession} \end{aligned}$$

Balance for  $\sin 2\varphi = \frac{2H}{\alpha M_s}$

Define Walker field :  $H_w = \alpha M_s / 2$

N. L. Schryer et al., JAP45, 5406 (1974)



### Motion below the Walker field

Steady-state azimuth :  $\sin 2\varphi = \frac{2H}{\alpha M_s}$

High speed  $v = |\gamma_o| \Delta_w H / \alpha \sim 1/\alpha$



$\Delta_w$  Is a dynamic parameter and is not the DW width at rest

### Motion above the Walker field

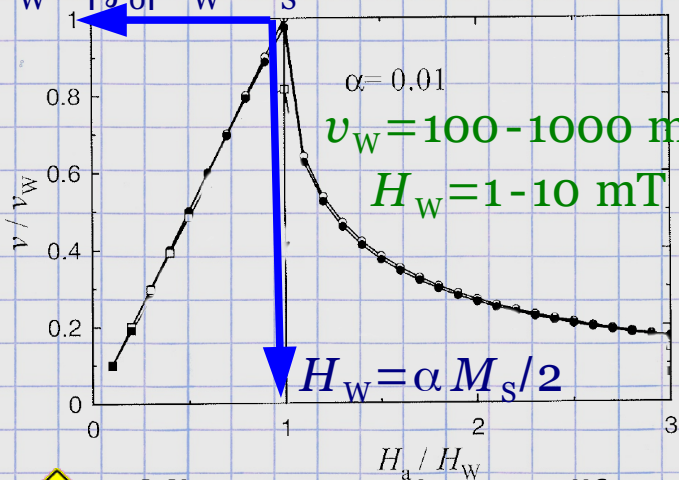
⇒ Precession with non-steady angular speed

⇒ Soon recovers speed  $v \approx \alpha |\gamma_o| H \Delta_w$

$v_w = |\gamma_o| \Delta_w M_s / 2$  Walker speed limit

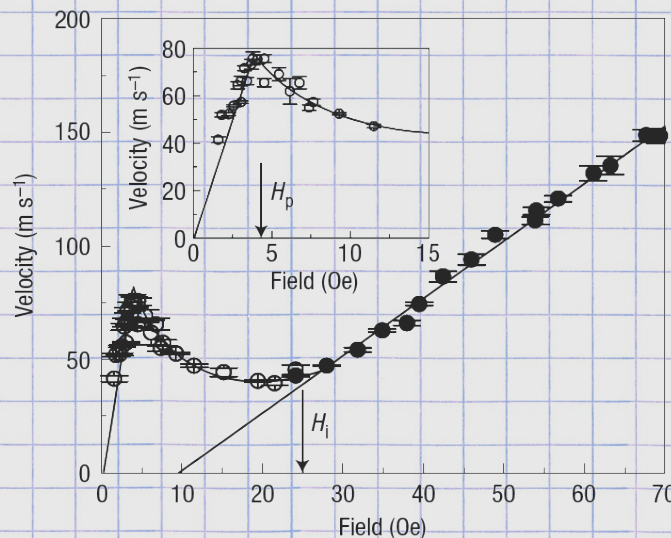
### Motion below the Walker field

$v_w = |\gamma_o| \Delta_w M_s / 2$



Micromagnetics modify these figures

### Experimental confirmation



Py(20nm) strips  
600nm wide

G. S. D. Beach et al., Nat. Mater. 4, 741 (2005)

A. Thiaville & Y. Nakatani, in Spin dynamics in confined magnetic structures III, B. Hillebrands & A. Thiaville (ed.), Springer, 101, 161-206 (2006)



Closure domains (flat)

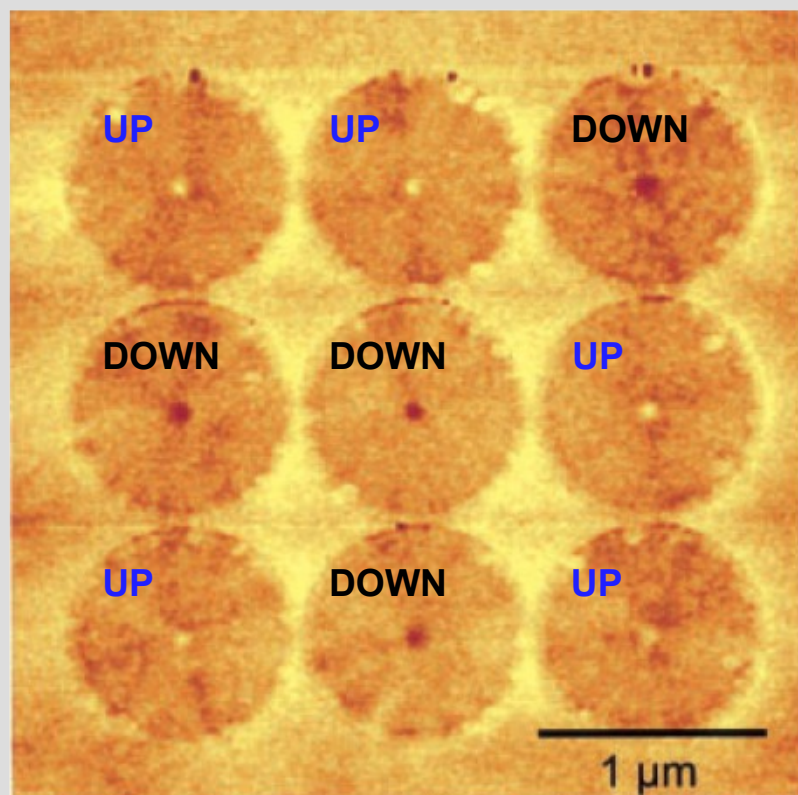


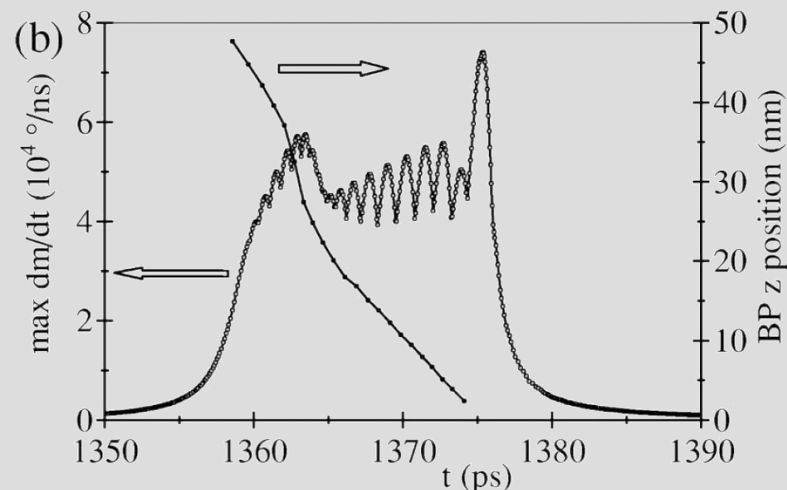
Fig. 2. MFM image of an array of permalloy dots 1 μm in diameter and 50 nm thick.

The central magnetic vortex may be magnetized up or down using a perpendicular field

T. Shinjo et al., *Science* 289, 930 (2000)  
 T. Okuno et al., *JMMM*240, 1 (2002)

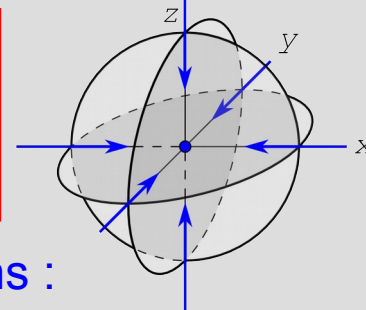
Theory and simulation

Simulation



A. Thiaville et al., *Phys. Rev. B* 67, 094410 (2003)

Requires a Bloch point  
 → Not well described  
 in micromagnetism



Multiscale simulations :

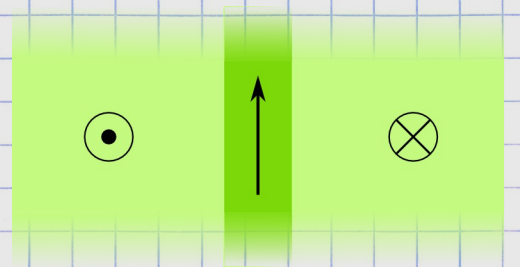
C. Andreas et al., *JMMM* 362, 7 (2014)

First theoretical insight in Bloch points :

W. Döring, *J. Appl. Phys.* 39, 1006 (1968)



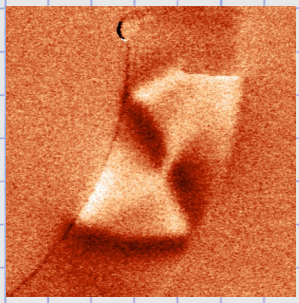
**Bloch domain wall in the bulk (2D)**



- ⇒ No magnetostatic energy
- ⇒ Width  $\Delta_u = \sqrt{A/K}$
- ⇒ Areal energy  $\gamma_w = 4\sqrt{AK}$

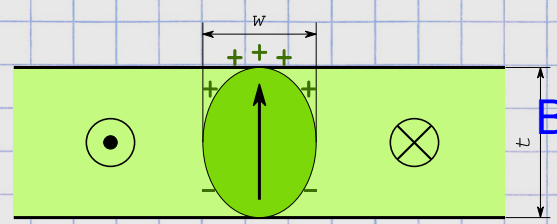
**i** Other angles & anisotropy  
 F. Bloch, *Z. Phys.* **74**, 295 (1932)

**Constrained walls (eg : in stripes)**

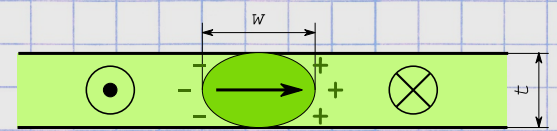
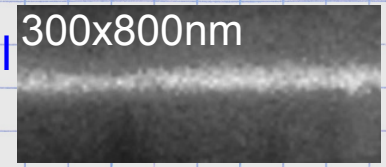


Permalloy (15nm)  
 Strip 500nm

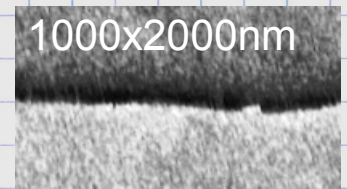
**Domain walls in thin films (2D → 1D)**



Bloch wall  
 $t \gtrsim w$



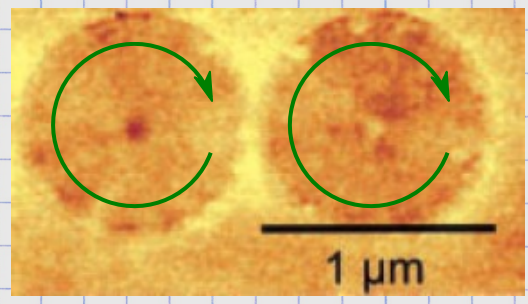
Néel wall  
 $t \lesssim w$



- ⇒ Contains magnetostatic energy
- ⇒ No exact analytics

L. Néel, *C. R. Acad. Sciences* **241**, 533 (1956)

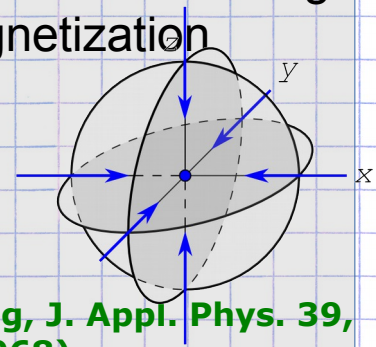
**Magnetic vortex (1D → 0D)**



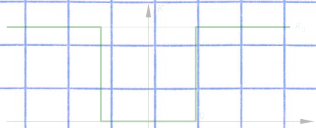
T. Shinjo et al.,  
*Science* **289**, 930 (2000)

**Bloch point (0D)**

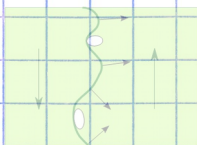
- ⇒ Point with vanishing magnetization



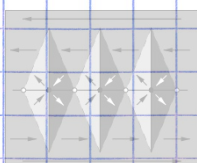
W. Döring, *J. Appl. Phys.* **39**, 1006 (1968)



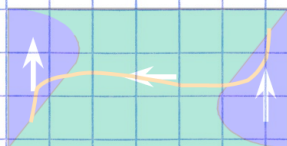
⇒ Brown paradox



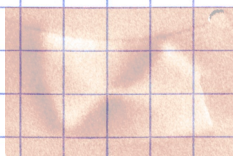
⇒ Nucleation and propagation



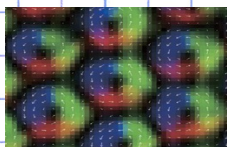
⇒ Walls and domains in films and nanostructures



⇒ Near single domains



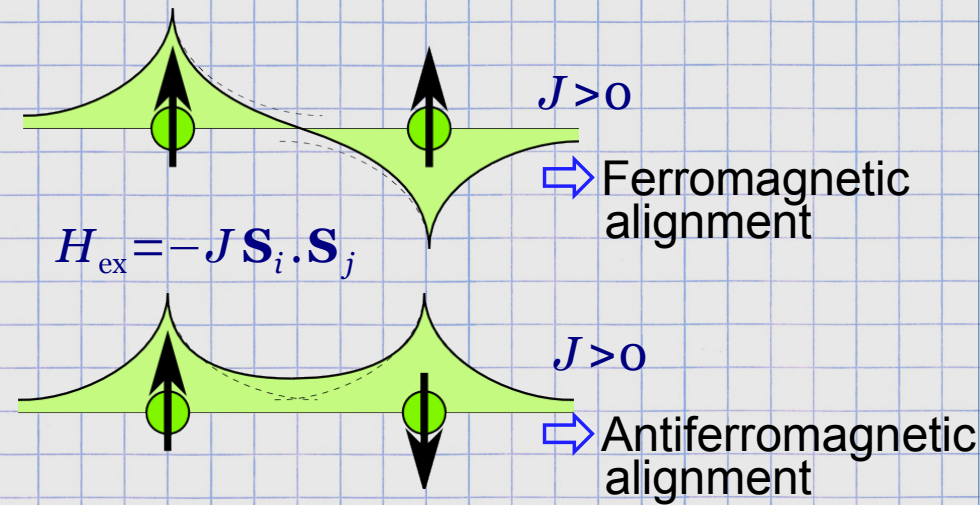
⇒ Domain walls in tracks



⇒ Skyrmions

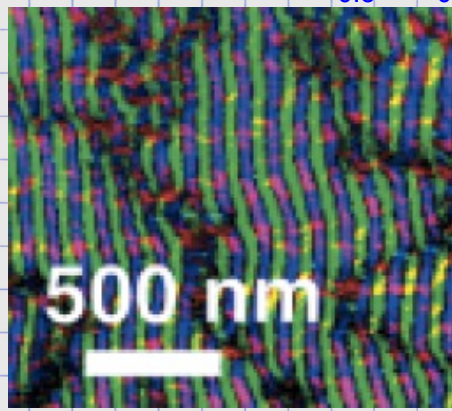


**Exchange interaction (reminder)**



**Experimental confirmations**

Thinned bulk  $Fe_{0.5}Co_{0.5}Si$ , Lorentz microscopy

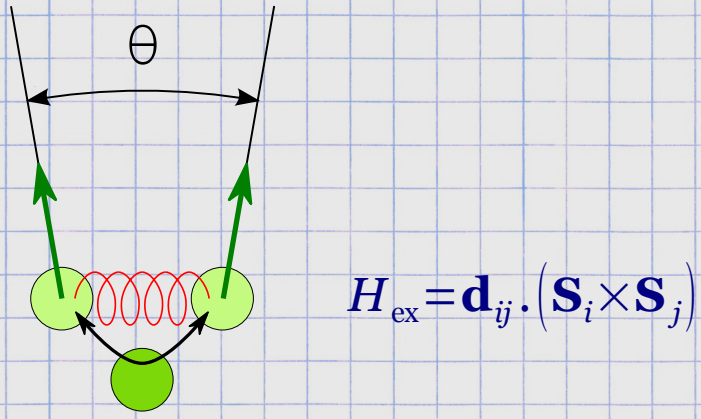


NB : requires low temperature

M. Uchida et al., Science 311, 359 (2006)

**Dzyaloshinskii-Moriya Interaction (DMI)**

- ⇒ Favors non-collinear alignment via interaction with non-magnetic atom
- ⇒ Requires a **non-centro symmetric environment** for non-cancellation



- ⇒ Favors spirals or cycloids
- ⇒  $\mathbf{d}_{ij} = -\mathbf{d}_{ji}$  selects a unique chirality

I. E. Dzyaloshinskii, Sov. Phys. JETP 5, 1259 (1957)

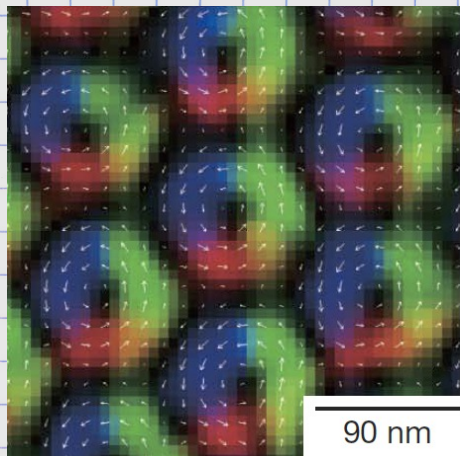
T. Moriya, Phys. Rev. 120, 91 (1960)





### Skyrmions under applied field

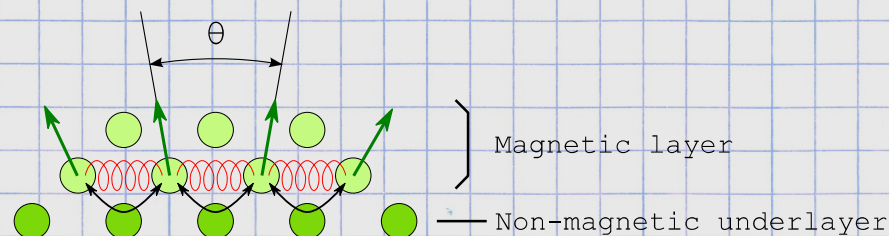
Thinned  $\text{Fe}_{0.5}\text{Co}_{0.5}\text{Si}$ , Lorentz microscopy



- ⇒ Arrays only
- ⇒ Present case : spiral order

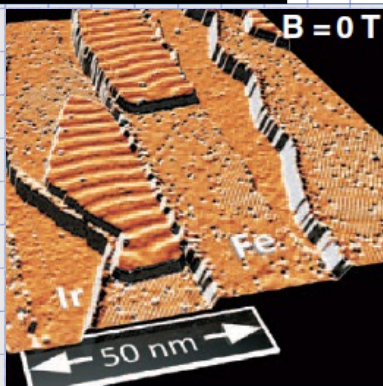
X. Z. Yu et al., *Nature* 465, 901 (2010)

### Thin films as artificial materials



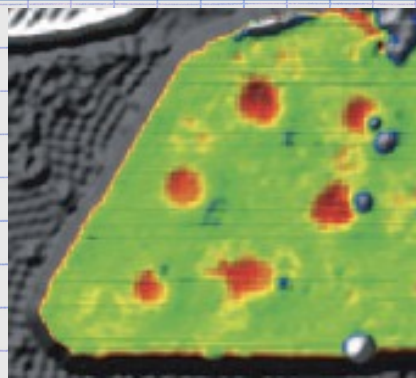
- ⇒ Should bring more versatility, and operation at room temperature
- ⇒  $\mathbf{d}_{ij} = d \mathbf{u}_{ij} \times \mathbf{n}$  favoring cycloids
- ⇒ Interfacial effect

### Thin film results $\text{Ir}(111)\backslash\text{Fe}\backslash\text{Pd}$ , sp-STM



Cycloids

N. Romming et al., *Science* 341, 636 (2013)



Single skyrmions

### Micromagnetics

- ⇒ Isolated skyrmions as metastable objects, moved with current
- S. Rohart et al., *PRB* 88, 184422 (2013)  
 A. Fert et al., *Nat. Nanotech* 8, 152 (2013)

