

Topological effects in nanomagnetism: from perpendicular recording to monopoles

Hans-Benjamin Braun



ETH zürich

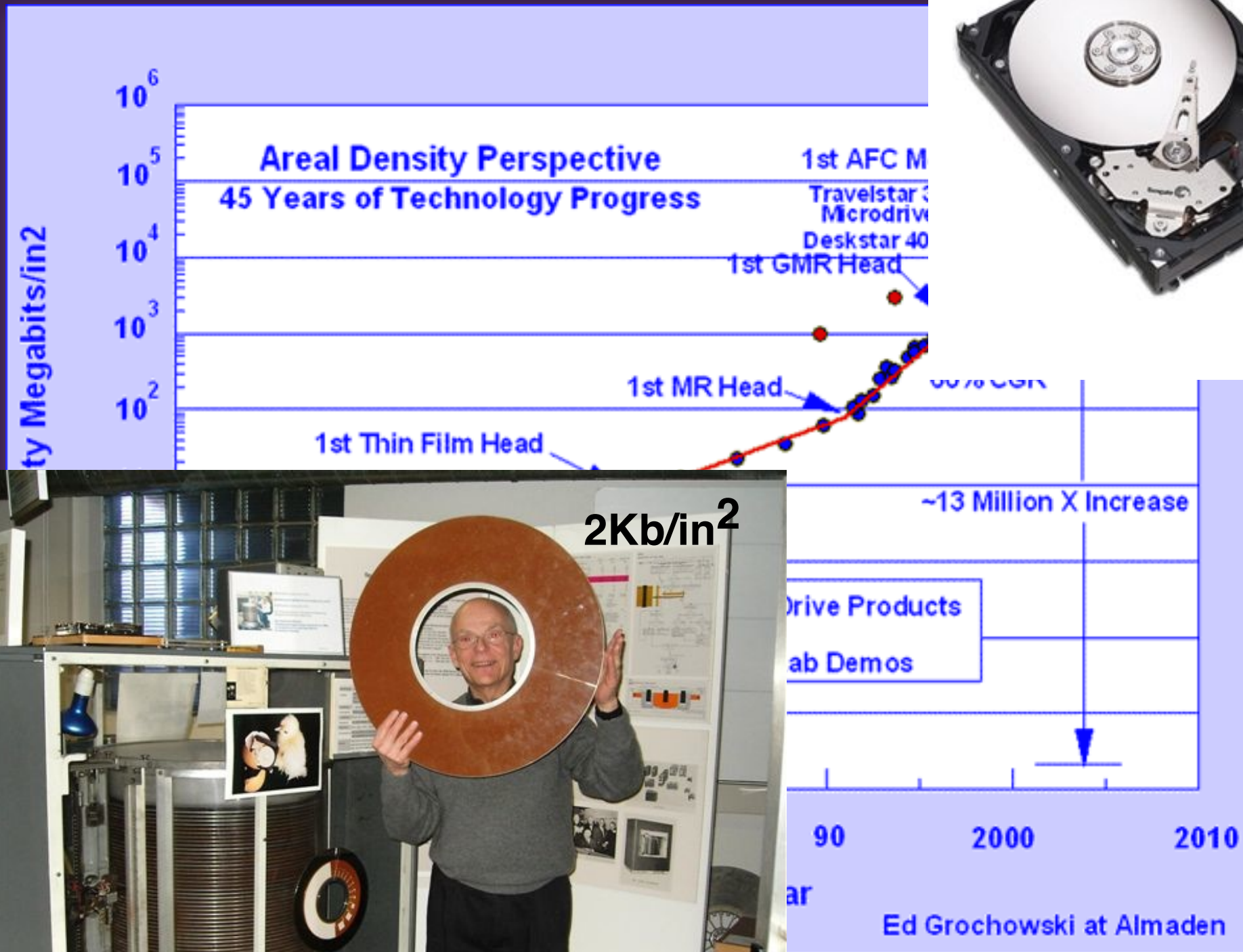
PAULI CENTER

for Theoretical Studies



IEEE Summer School, CBPF, Rio de Janeiro, Brazil, August 10, 2014

The past 50 years of magnetic data storage



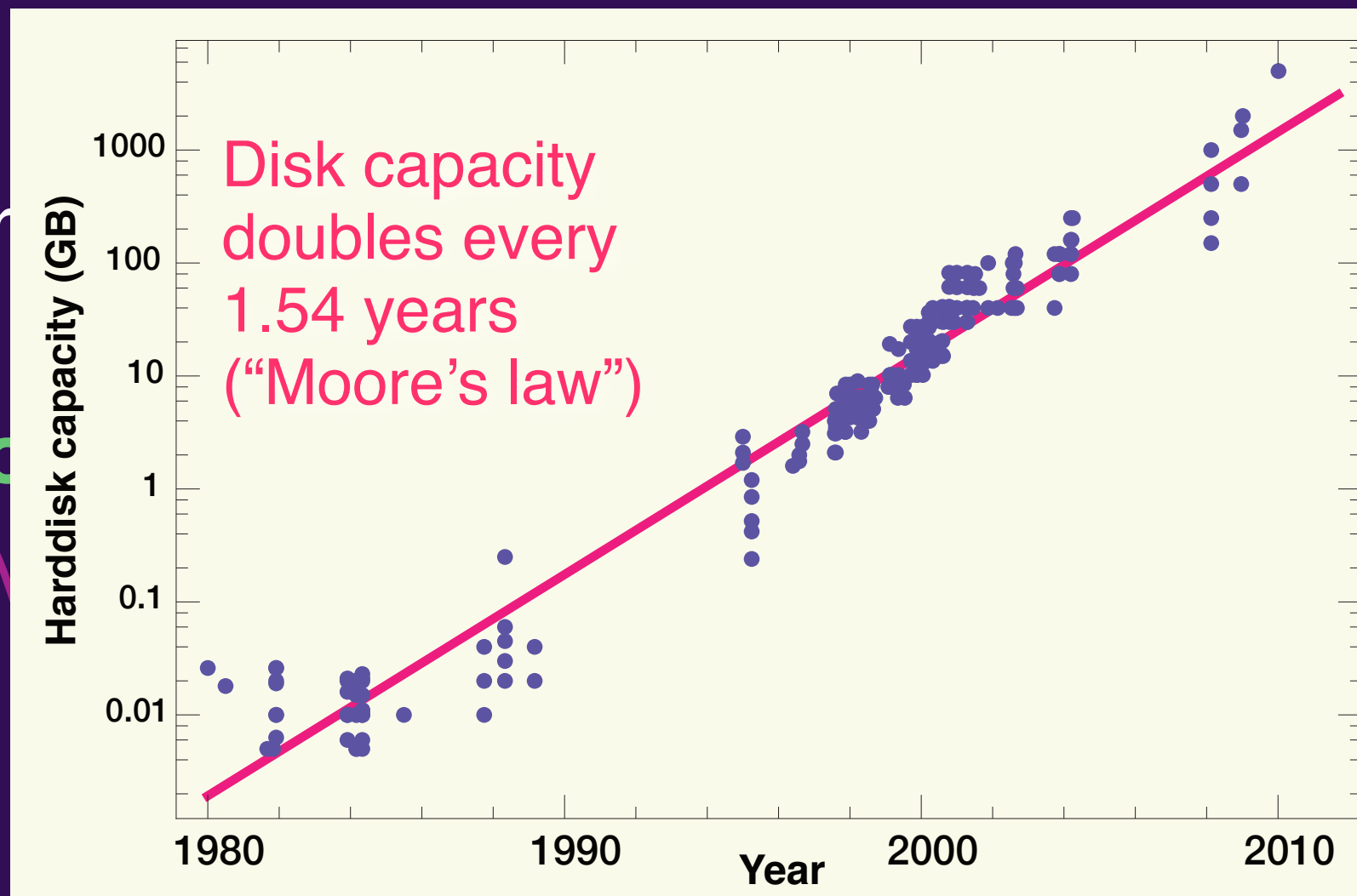
Evolution of storage density

2014: 1.0 Tb/in²

bit area (25 nm)²

2025: 0.15 Pb/in²

bit area (2 nm)² !!!



Avoid super
const. =
anisotro

total energy

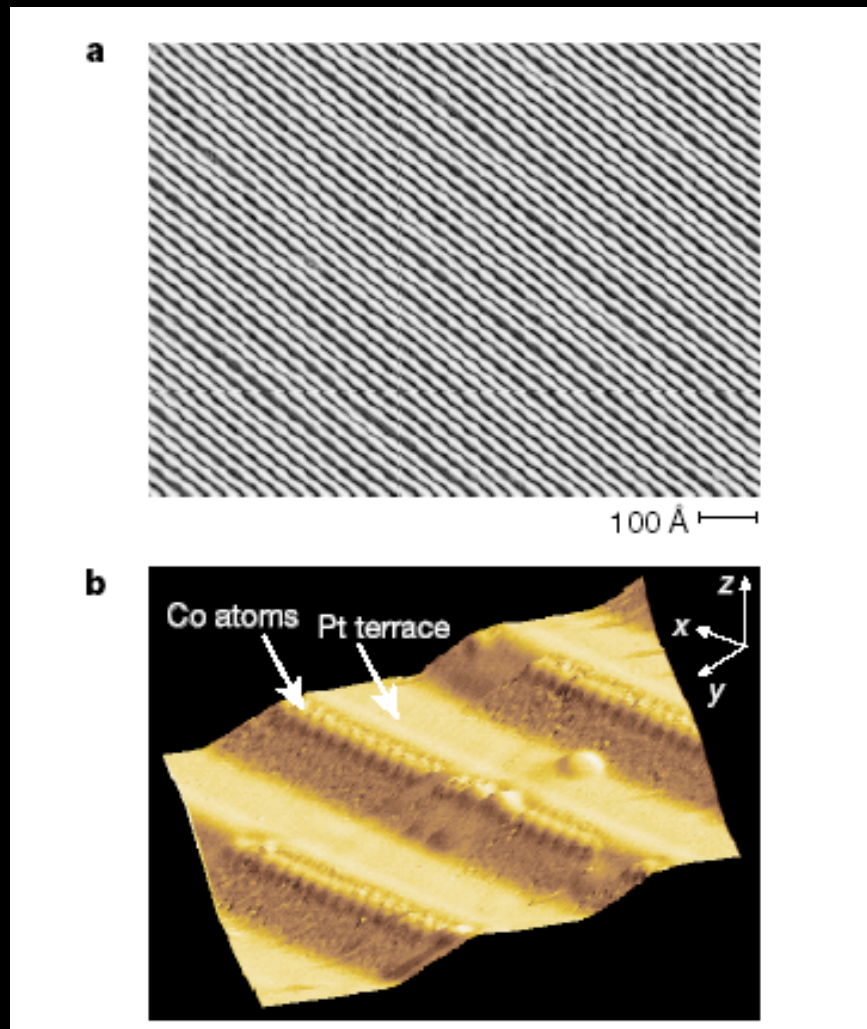


“0”

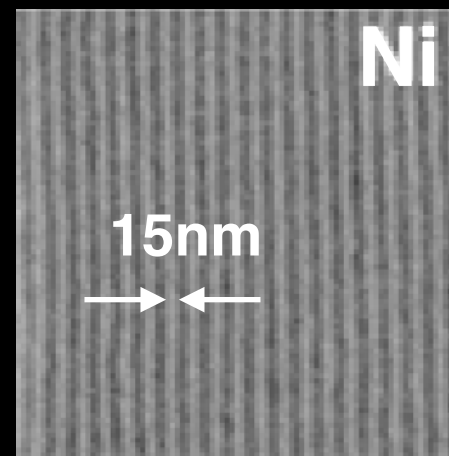


“1”

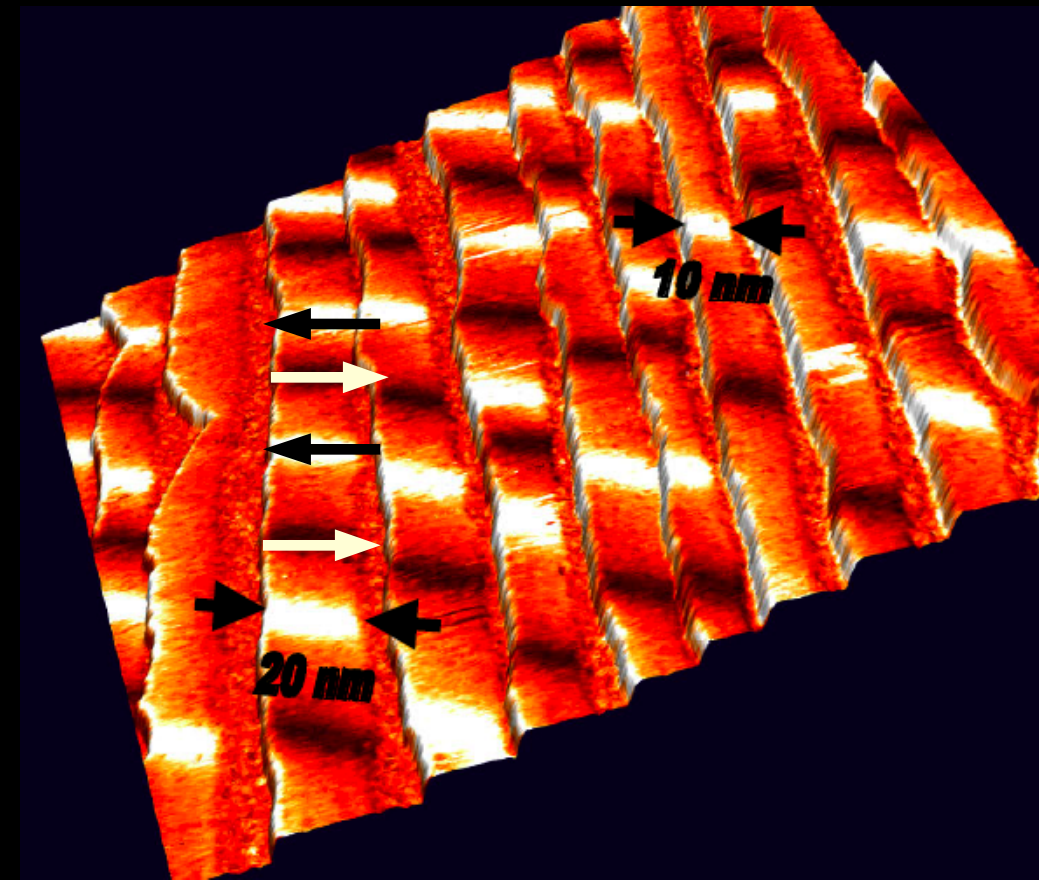
Magnetic nanostructures: quantum vs classical/thermal behaviour



P. Gambardella et al.
Nature ('02)



Y.S. Jung et al.
Nano Lett. ('10)



Kubetzka et al.
PRB ('03)

quantum

classical

“...It’s in this no-man’s land between quantum and classical physics that a wide array of “emergent” phenomena reveal themselves...”

D.F. Service, Science ('10)

Theoretical descriptions of magnetism

thermal fluctuations

'superparamagnetism'
perpendicular
recording

semiclassical
quantization

spin currents
chirality

classical
magnetism

"micromagnetics"
($T=0$)

nanoscale
experiments

$T>0$ magnetism

"statistical mechanics"

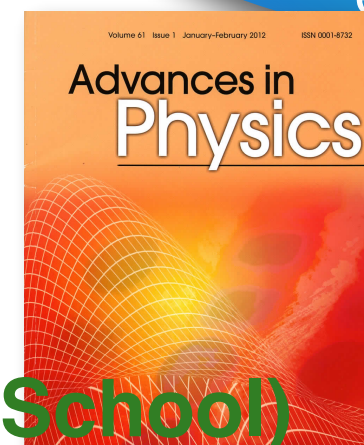
quantum
magnetism

spin-chains
strongly correlated
electrons

Topological concepts as unifying principle

HBB, Adv. Phys. **61**, 1-116 (2012)

cf. O. Fruchart's & D. Bürgler's lecture (this School)



Overview

- I. Topological defects in magnetism (domain walls, vortices, skyrmions, merons, hedgehogs)
- II. Superparamagnetism and limits of magnetic data storage
- III. Quantization of micromagnetics: emergent chirality and spin currents in quantum spin chains
- IV. Dipolar interactions in nanomagnetic arrays - emergent Dirac monopoles and Dirac strings

Why Topology?

- Within the framework of ‘micromagnetics’, one considers a continuous magnetization field $\mathbf{M}(\mathbf{x},t)$
- Magnetic data storage: Are there magnetization configurations that are particularly stable?
- May two magnetization configurations be easily transformed into each other (bit stability)?

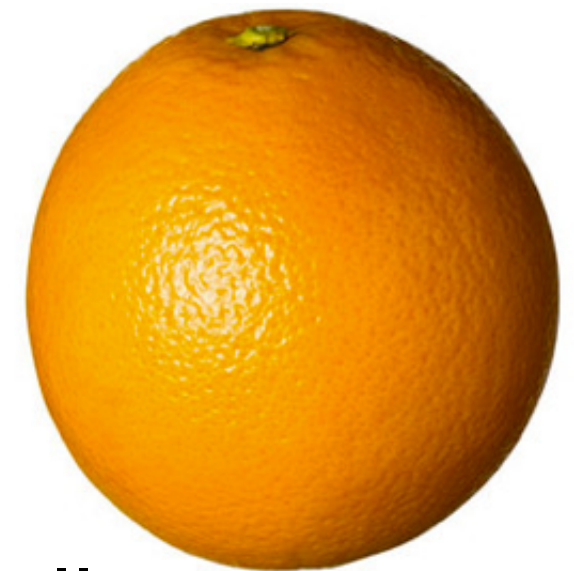
What is homotopy about?



\sim



$\not\sim$



homotopically
equivalent

homotopically
inequivalent

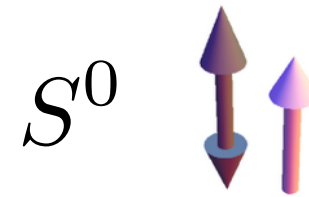
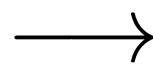
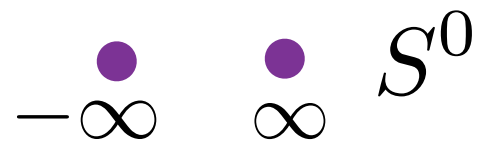
ὁμός same
τόπος place

source: Wikipedia

Topology - nontrivial mappings

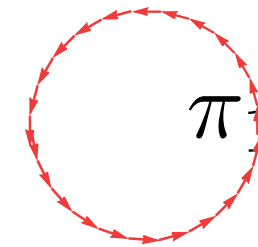
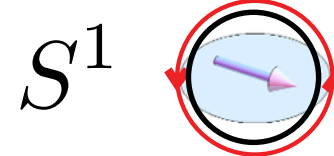
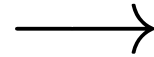
real space

spin space

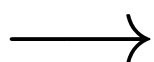


$$\pi_0(S^0) = \mathbb{Z}_2$$

mapping:



$$\pi_1(S^1) = \mathbb{Z}$$



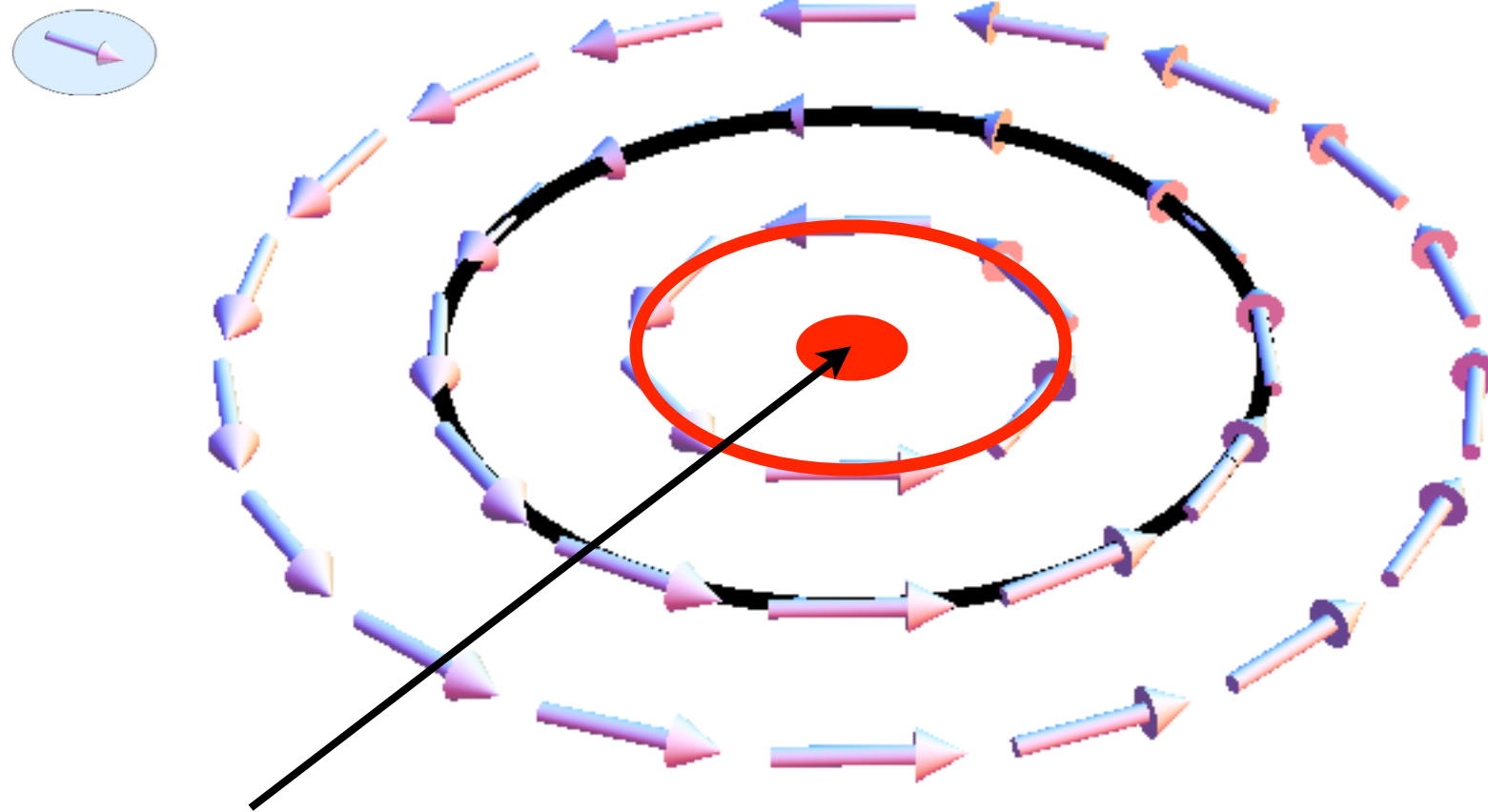
$$\pi_2(S^2) = \mathbb{Z}$$

Topologically nontrivial mappings exist between spheres of equal dimension

Winding numbers are 'fingerprints' of equivalence classes of configurations which are deformable into each other

Topological singular point defects

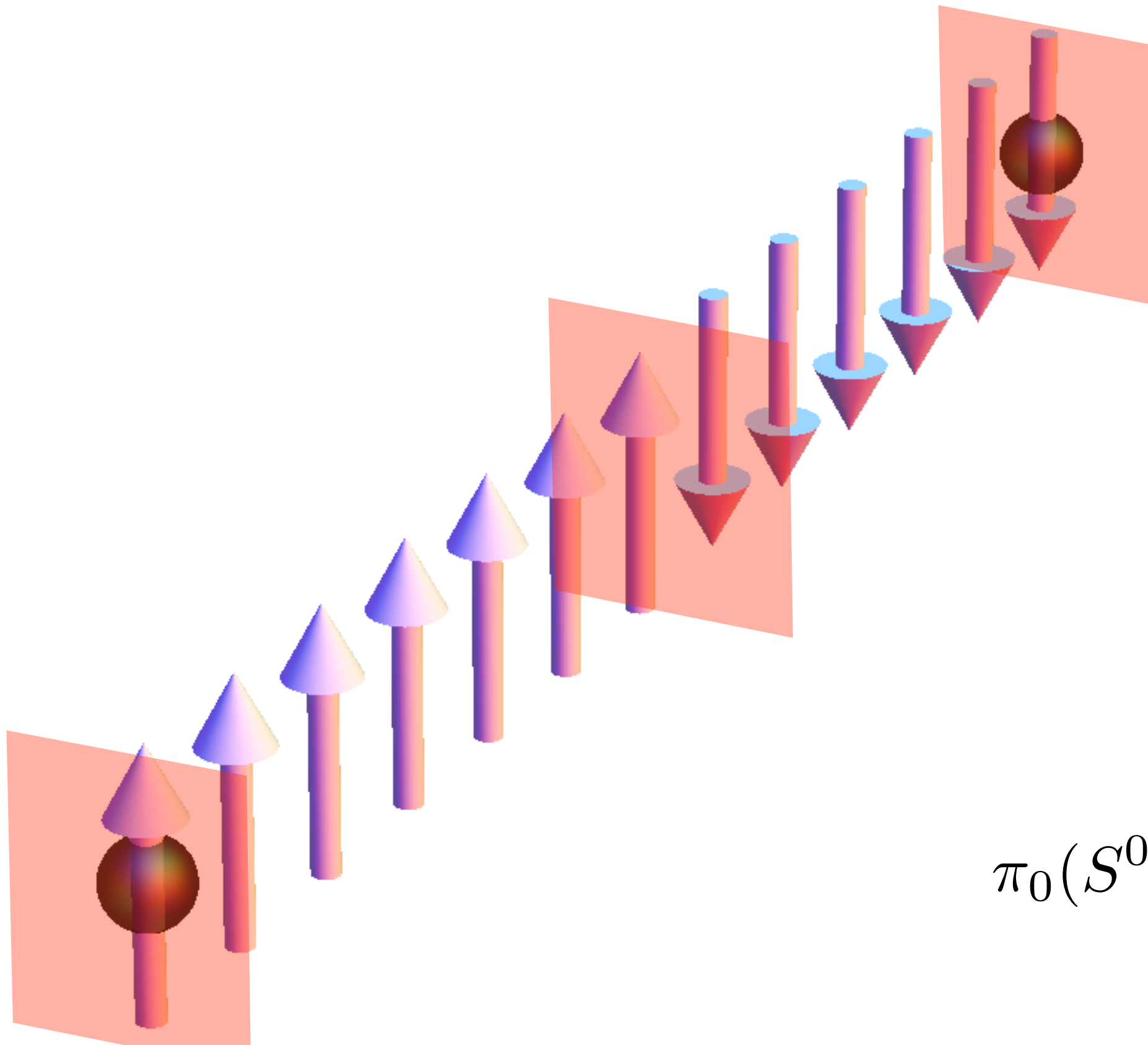
(vs. soliton type topological defects)



singular point defect

$$\pi_1(S^1) = \mathbb{Z}$$

Topological point defect - domain wall



$$\pi_0(S^0) = \mathbb{Z}_2$$

'Zooology' of singular topological defects

$$d = d' + m + 1 \quad \text{Toulouse/Kléman}$$

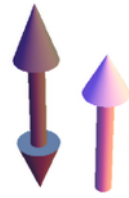
space dim.

d=1

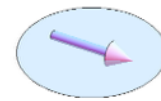
d=2

d=3

m=0



m=1



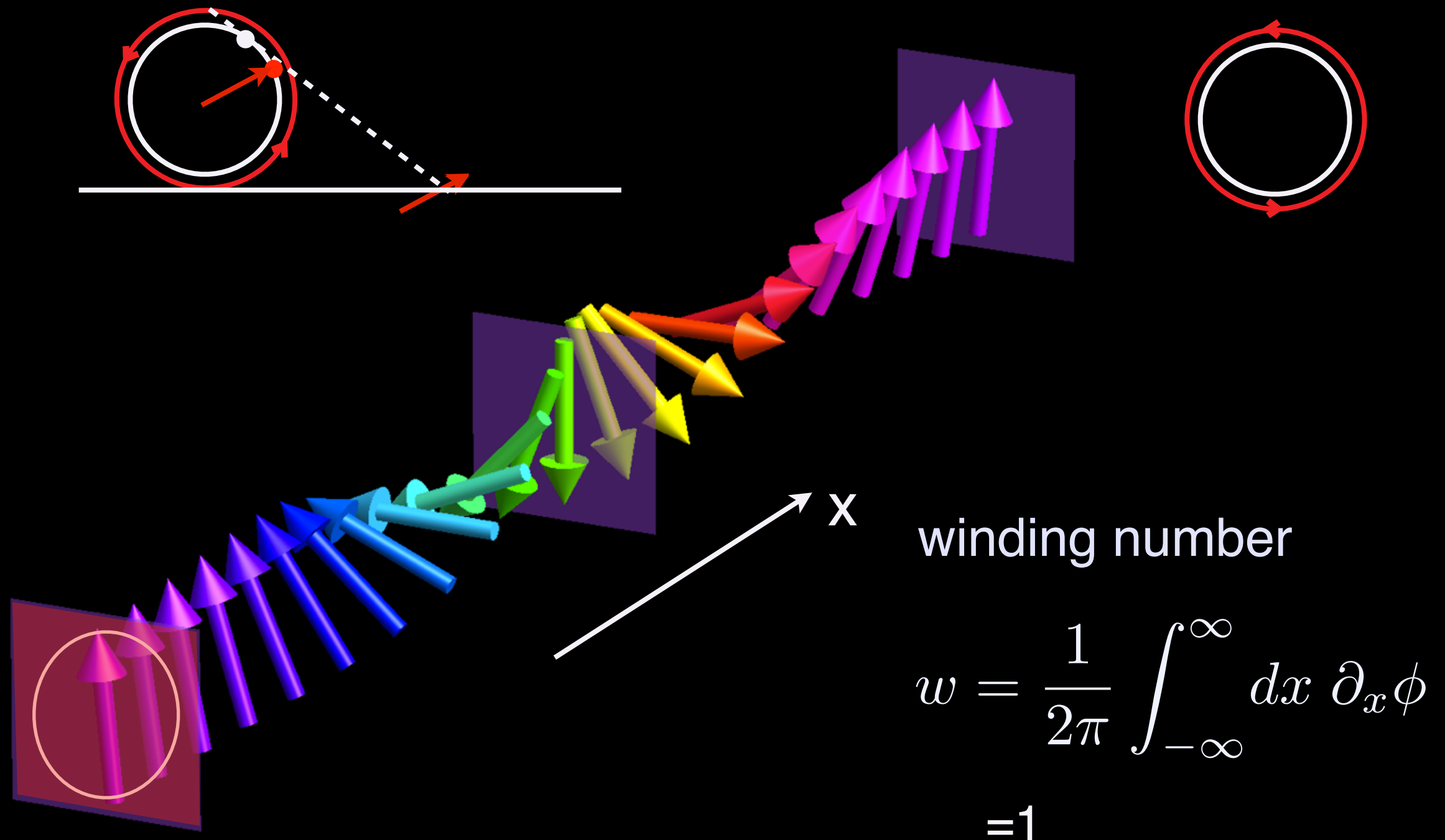
m=2



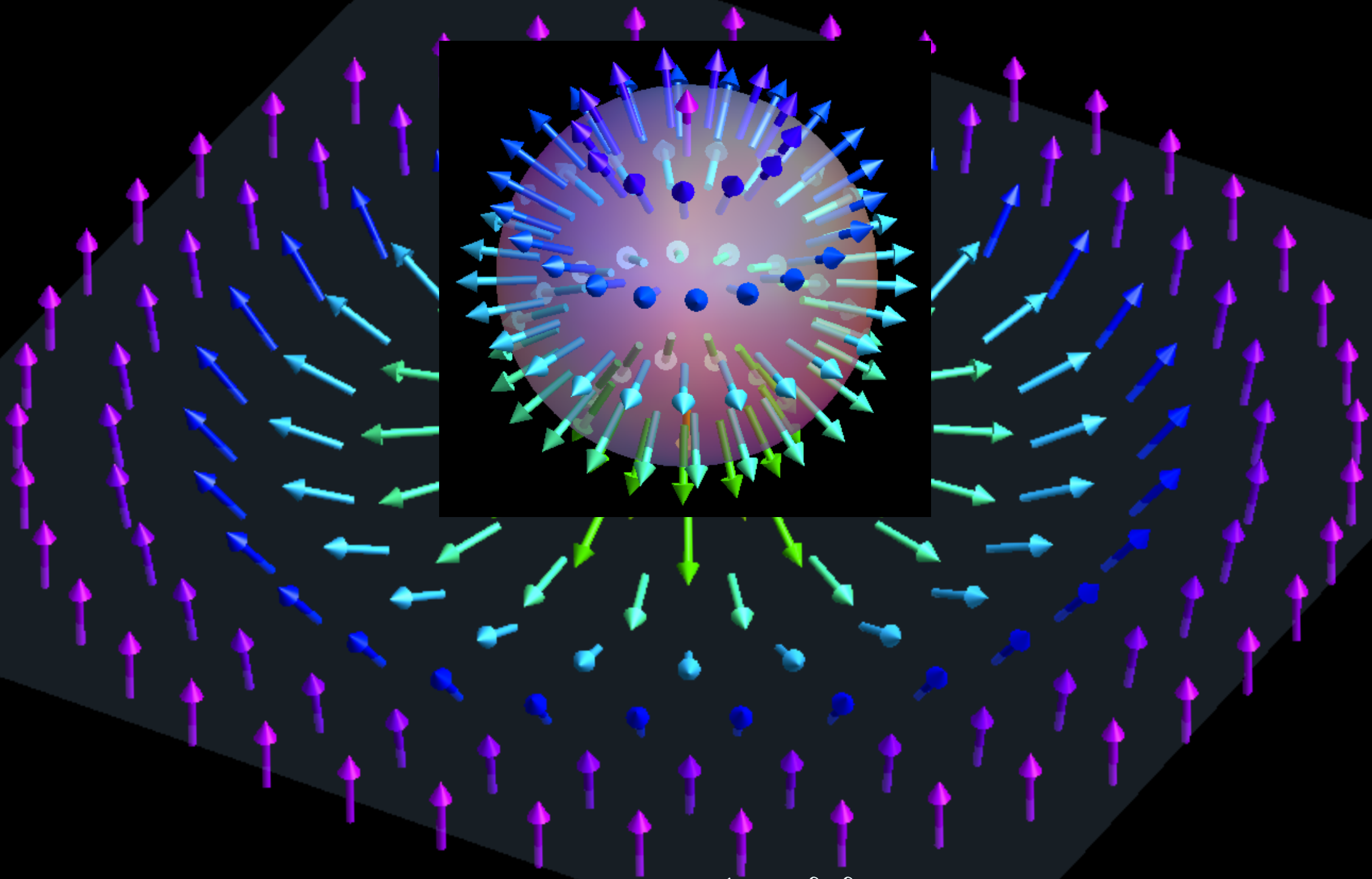
<p>$d'=0$</p>	X	X
<p>defect dimension</p> <p>$d'=1$</p>	<p>$d'=0$</p>	X
<p>$d'=2$</p>	<p>vortex line</p> <p>$d'=1$</p>	<p>'hedgehog' (Bloch point)</p> <p>$d'=0$</p>

$$\deg f = \frac{\int_M f^* \Omega}{\int_N \Omega}$$

Smooth solitary defect in 1D: 2 π domain wall

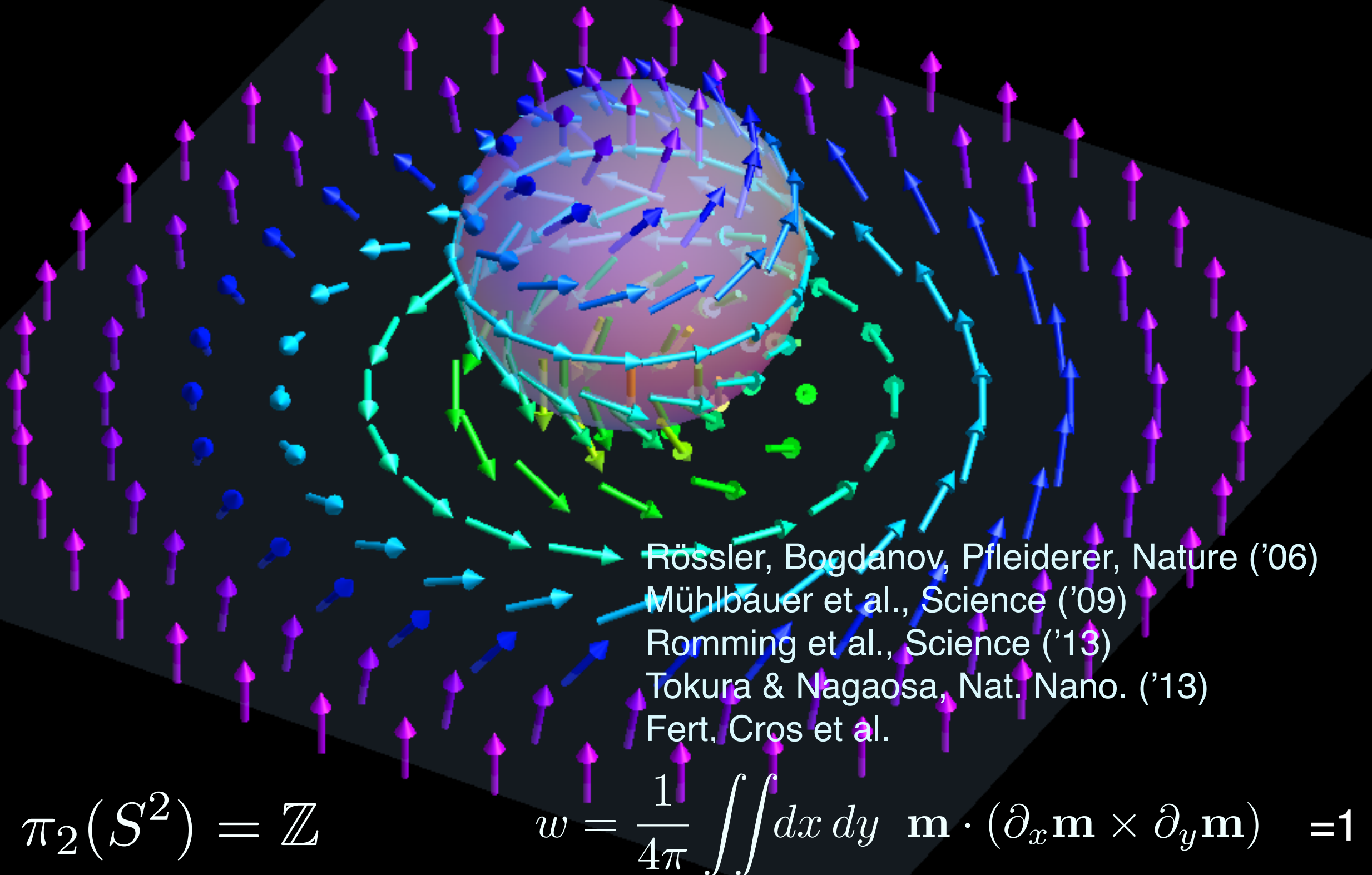


Smooth solitary defect in 2D: Skyrmion

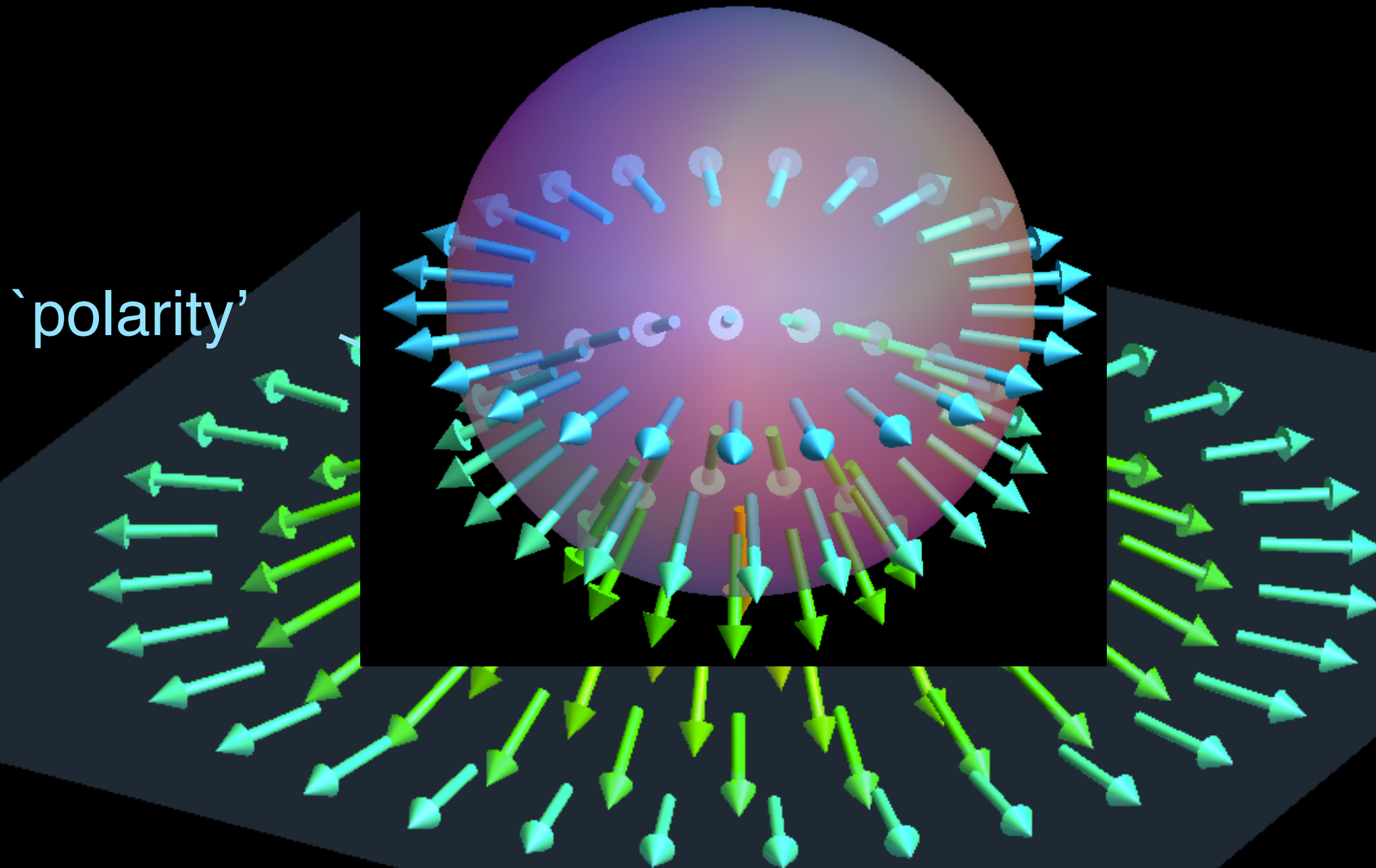


$$\pi_2(S^2) = \mathbb{Z} \quad w = \frac{1}{4\pi} \iint dx dy \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m}) = 1$$

Smooth solitary defect in 2D: Skyrmion

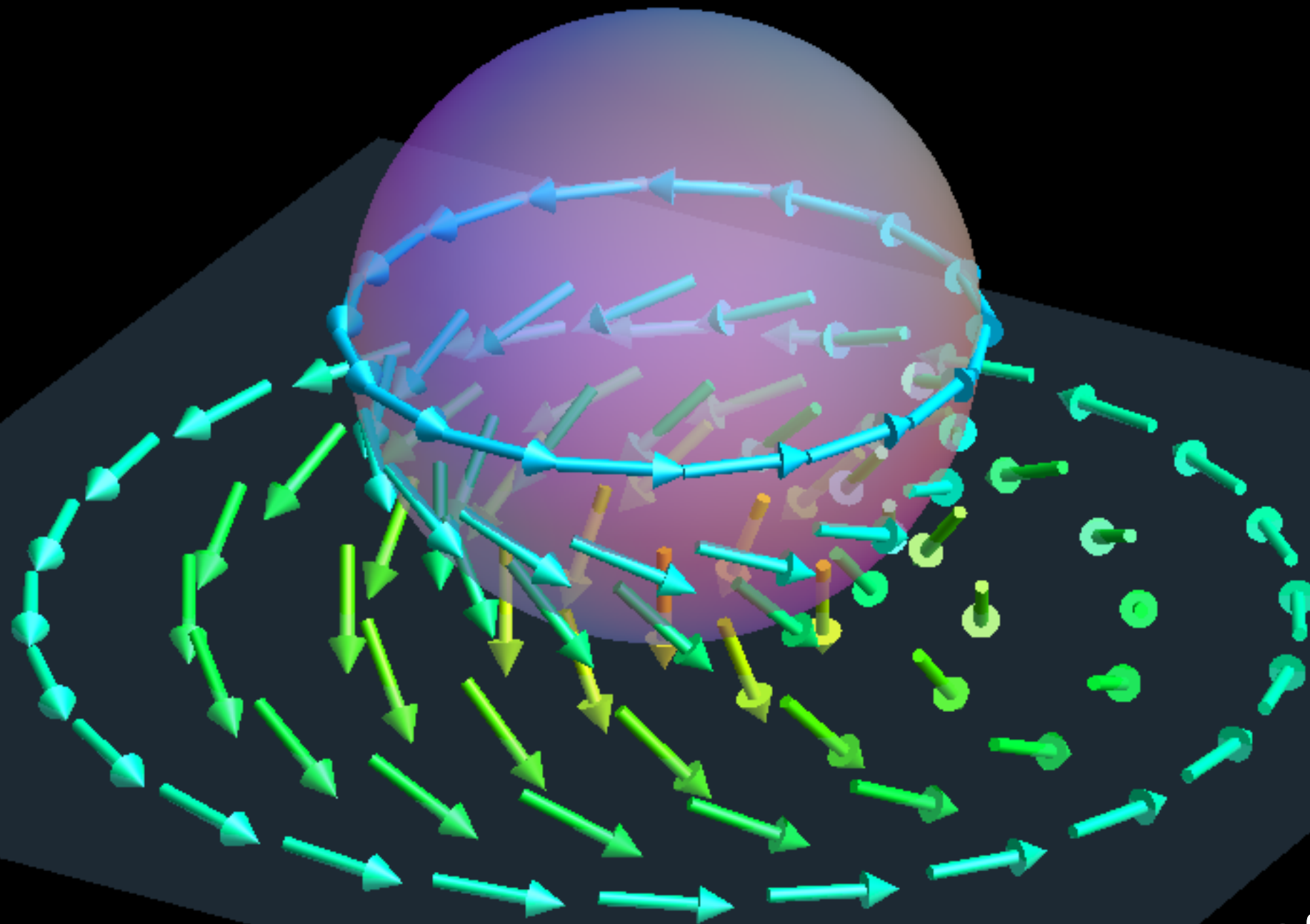


Meron (‘vortex’ with core)



‘half’ hedgehog (skyrmion)

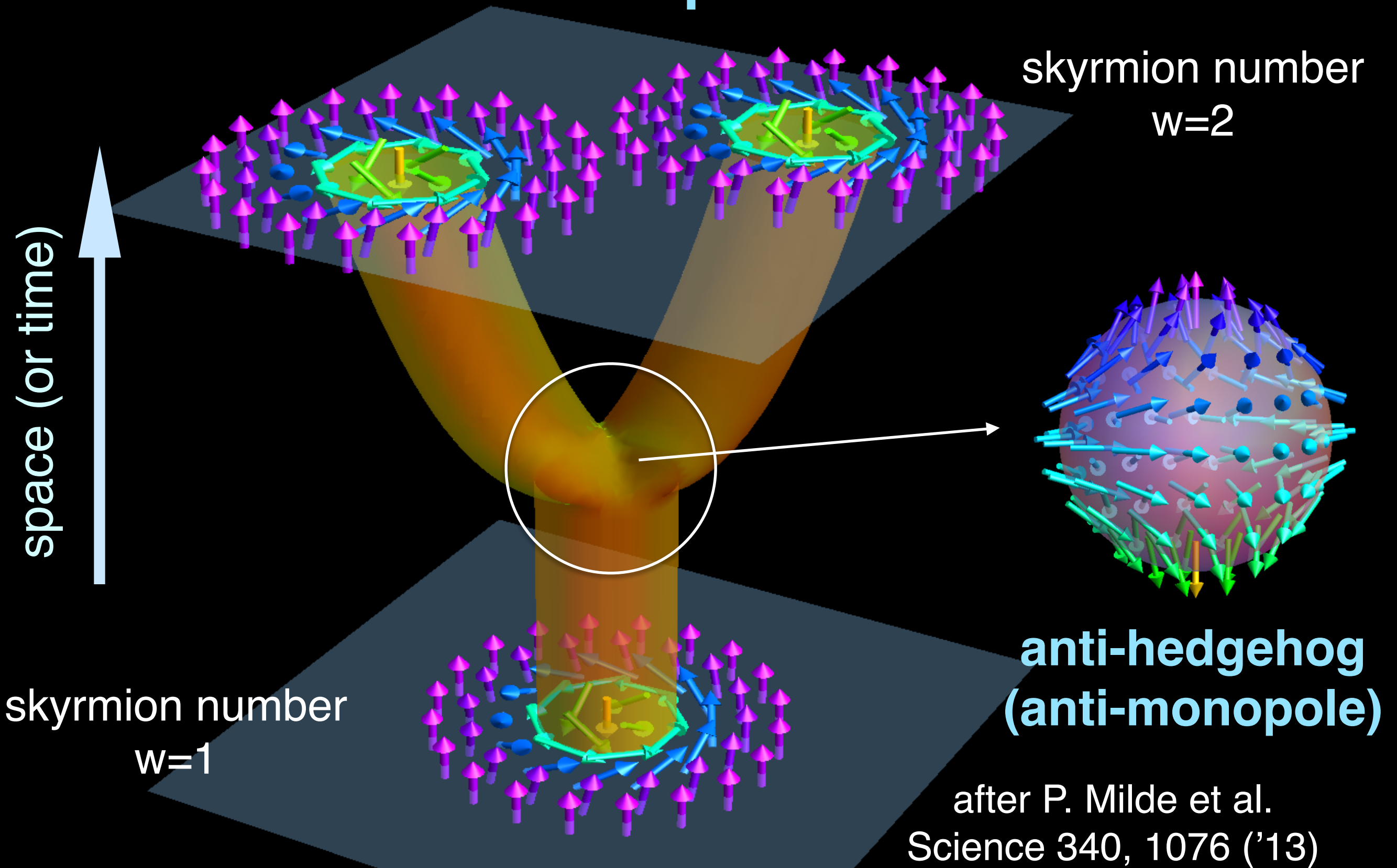
Merlon ('vortex' with core)



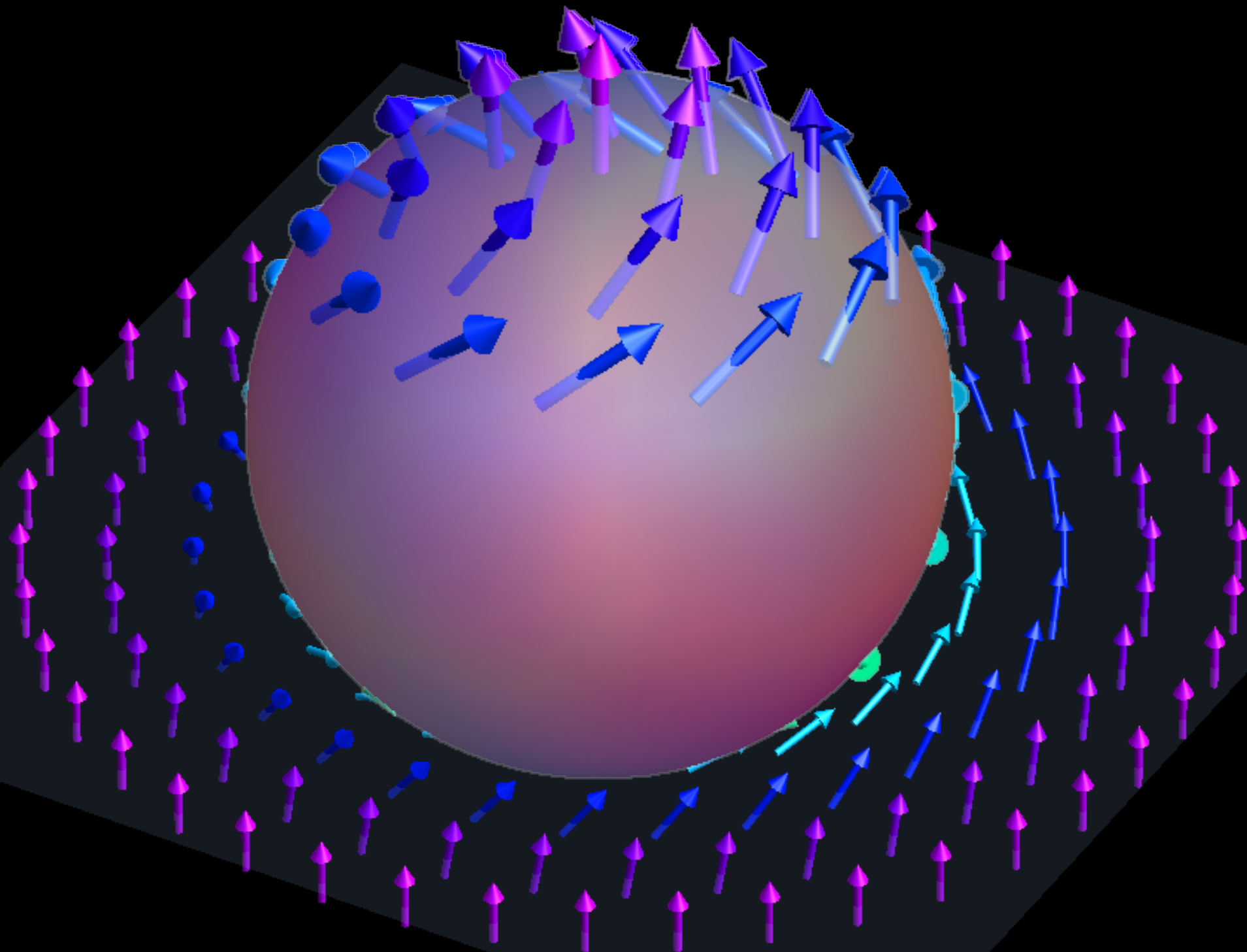
meron $w=1/2$

often simply
termed
"vortices"

Skyrmion creation via hedgehog-monopoles



How to get rid of a skyrmion



“Falling through the mesh of the lattice”

Summary

Topological defects are robust (e.g. Parkin's racetrack memory) **but** with the following `caveats`:

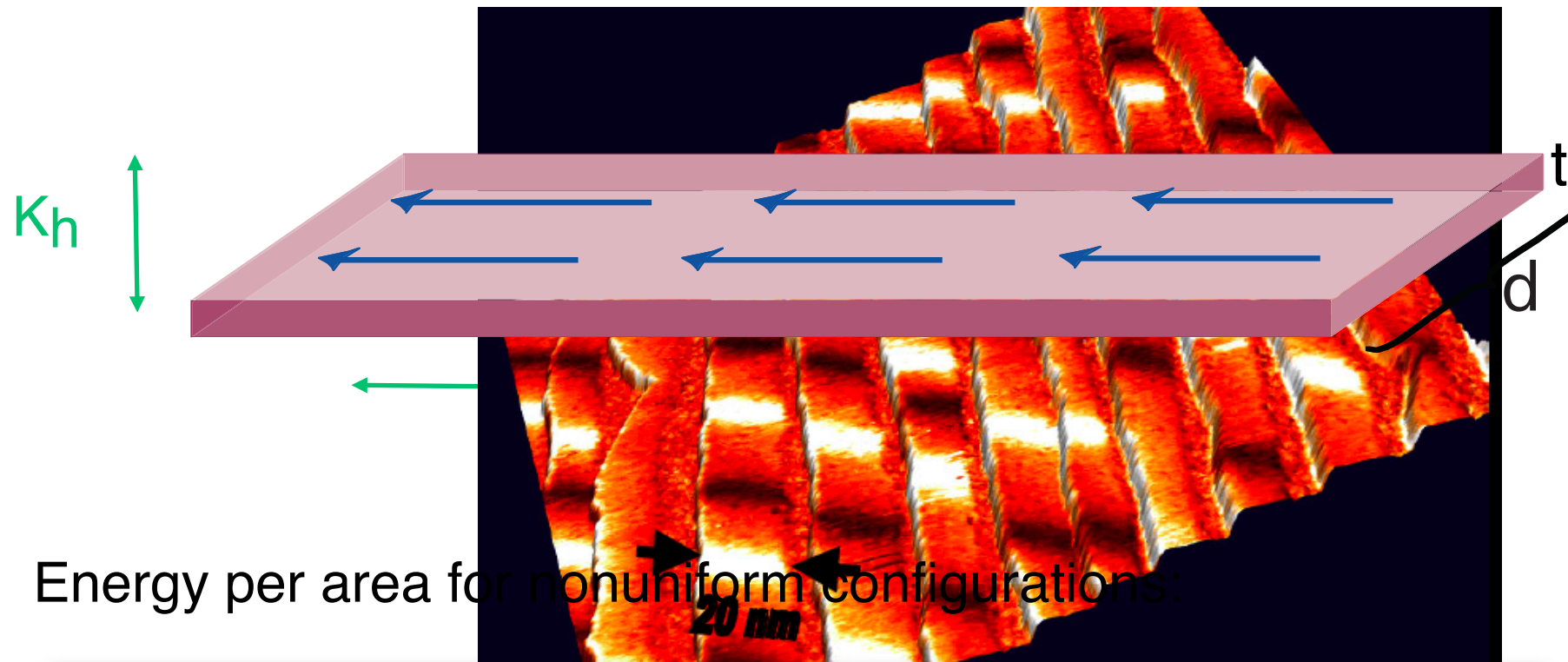
Winding ('skyrmion') number may be changed:

- i) via singular 'hedgehog' monopole topological point defects
- ii) via lattice effects (numerical work, e.g. Hertel et al., Sheka et al., Thiaville et al.)
- iii) at sample boundaries

Can quantum fluctuations restore smoothness of magnetisation field (cf. part III)?

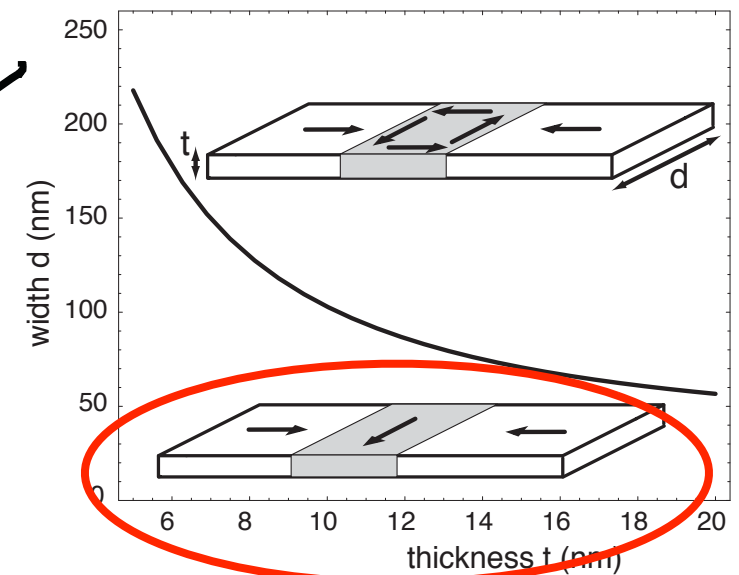
From topology back to nanomagnets

quasi 1D nanowires



Energy per area for nonuniform configurations:

$$\mathcal{E} = \int dx \left\{ A(\partial_x \mathbf{m})^2 - K_e m_x^2 + K_h m_z^2 - H M_0 m_x \right\}$$



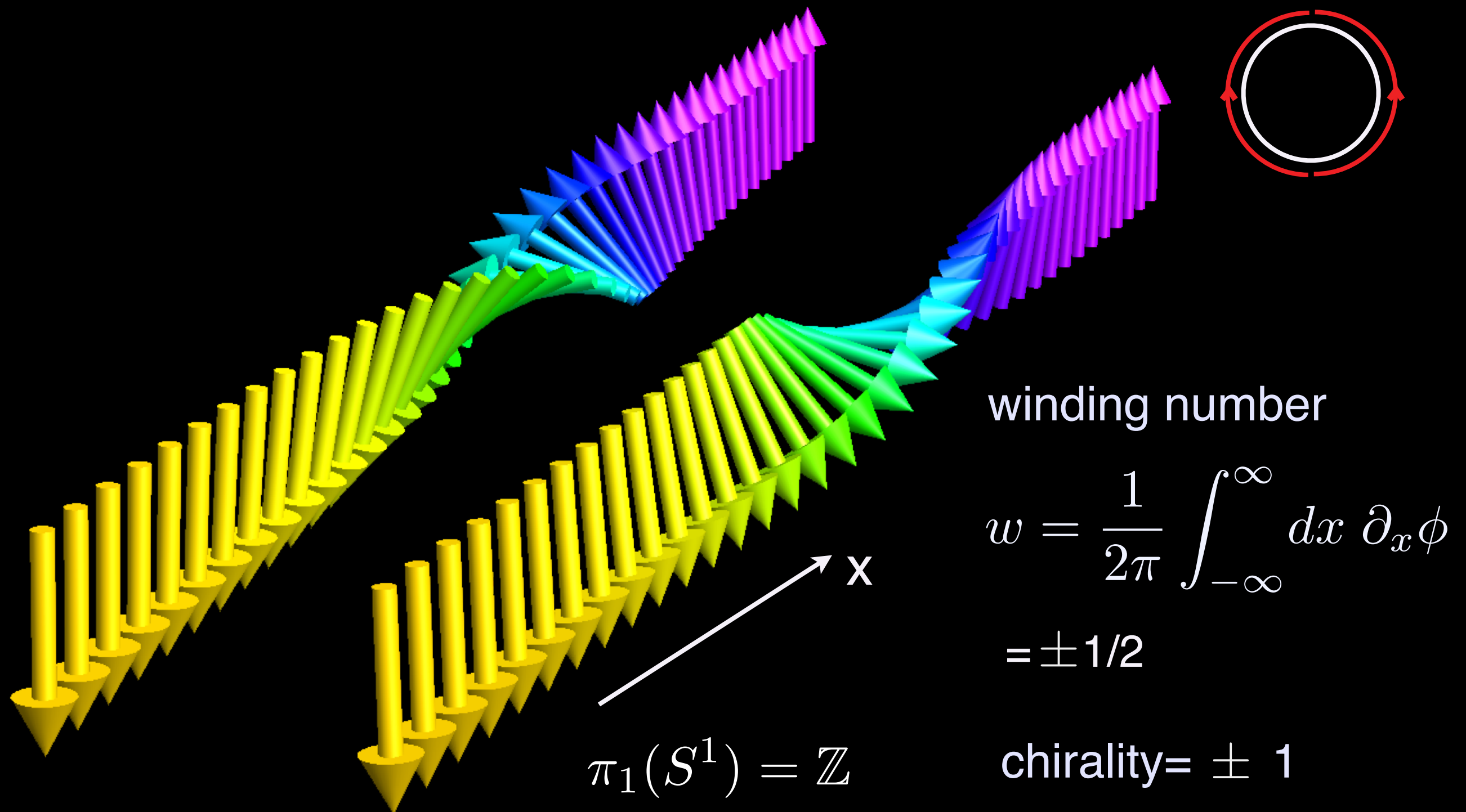
$$\mathbf{m}(x) = \mathbf{M}(x)/M_0$$

$$|\mathbf{m}| = 1$$

K_e, K_h are effective anisotropy constants
 'local approximation'
 (includes leading order demag effects)

HBB, PRL ('93)
~~Aharoni JAP ('96);~~
 HBB, JAP('99)
 Kohn, Slastikov ('05)

Topological stability of π domain walls - chirality

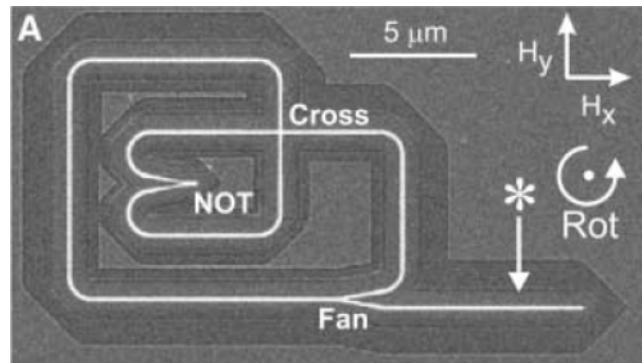


Storage & logic using domain walls

R. Cowburn

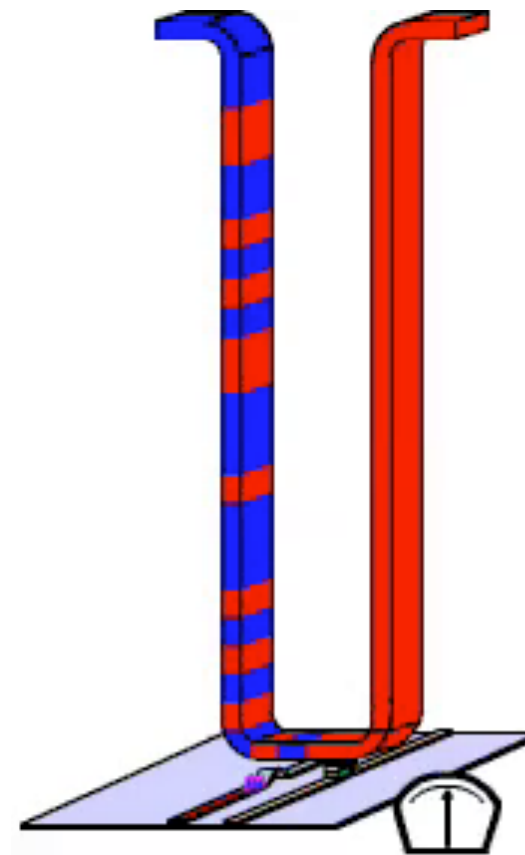
S. Parkin

Domain wall logic



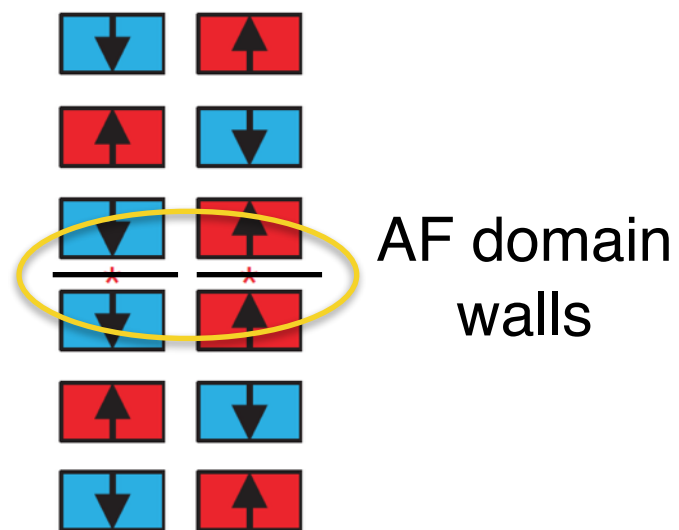
Allwood *et al.*, Science ('05)

Racetrack memory



Parkin *et al.*, Science ('08)

Magnetic ratchet

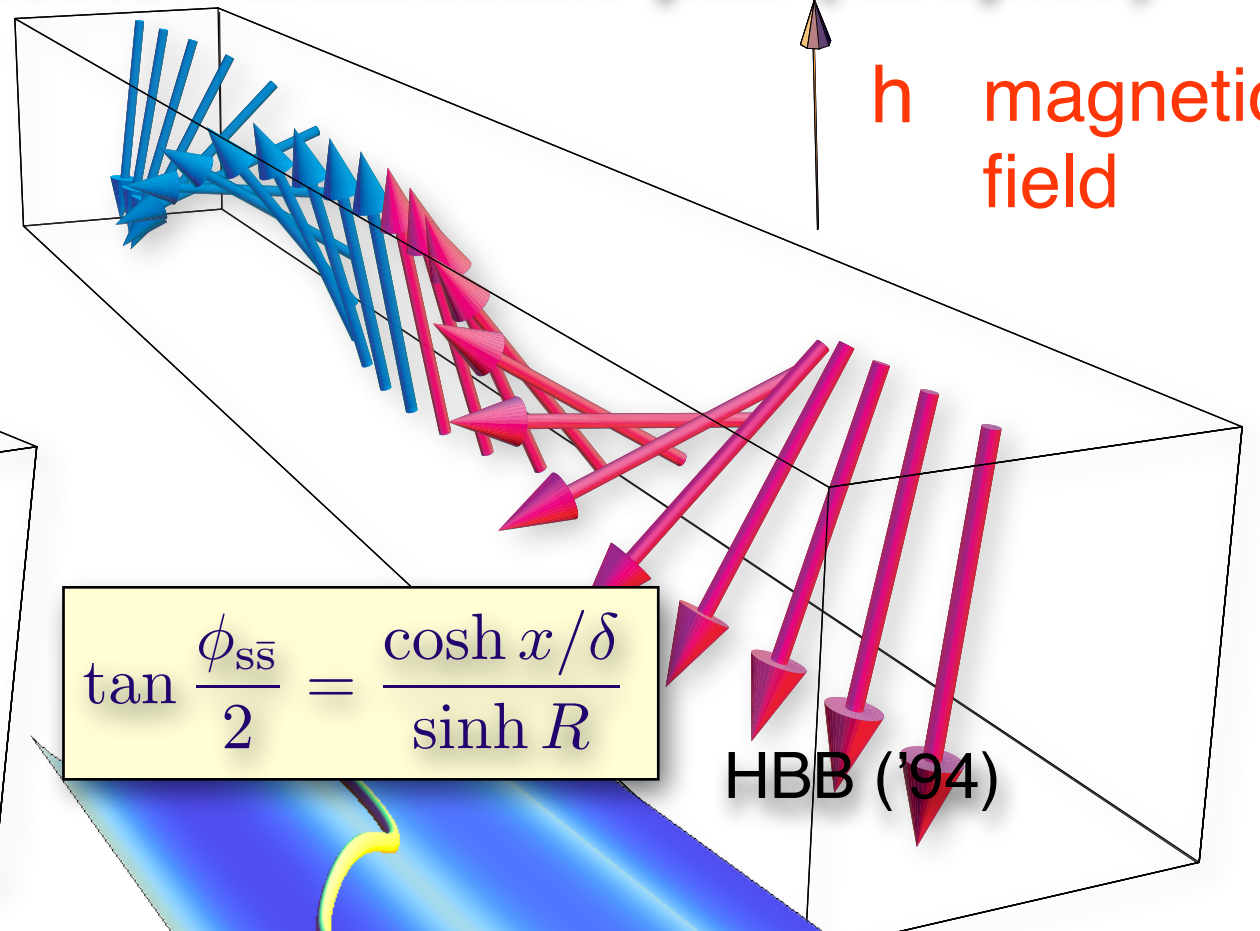
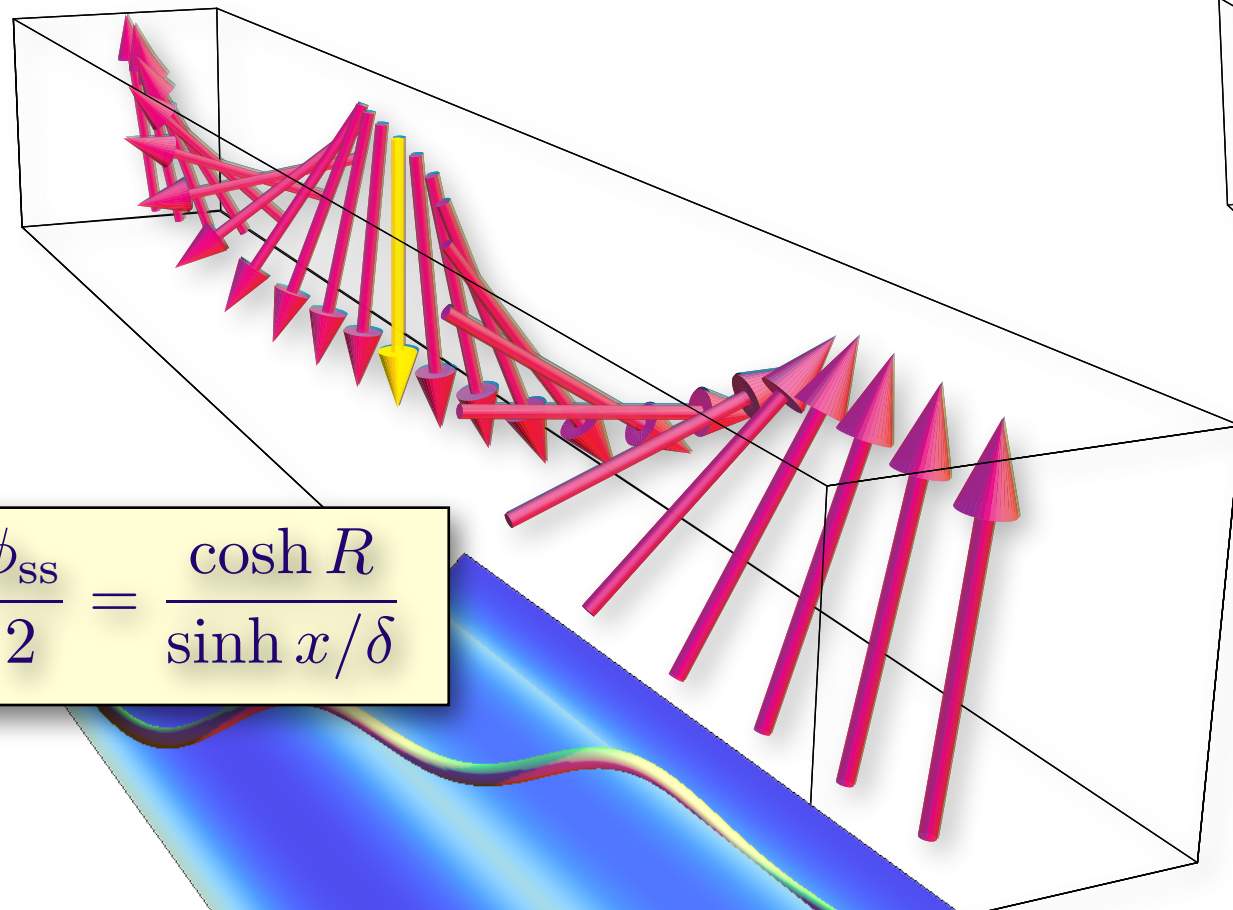


Lavrijsen, Cowburn *et al.*, Nature ('13)

Pairs of solitons

'soliton-soliton' pair (2π wall)

'soliton-antisoliton' pair ('droplet')

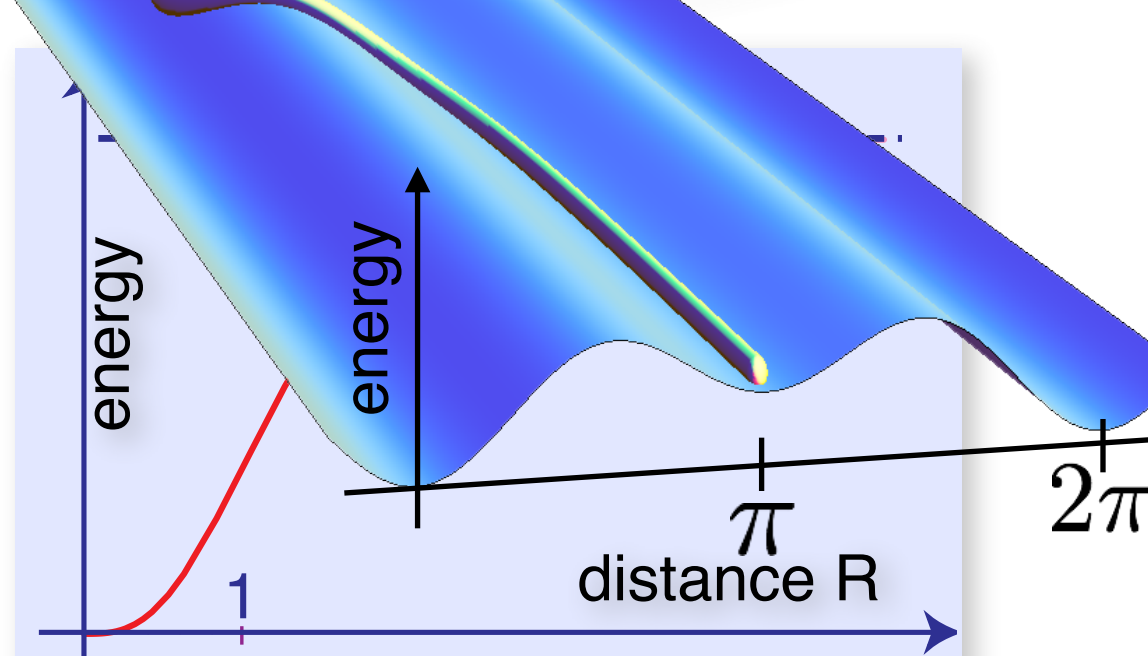
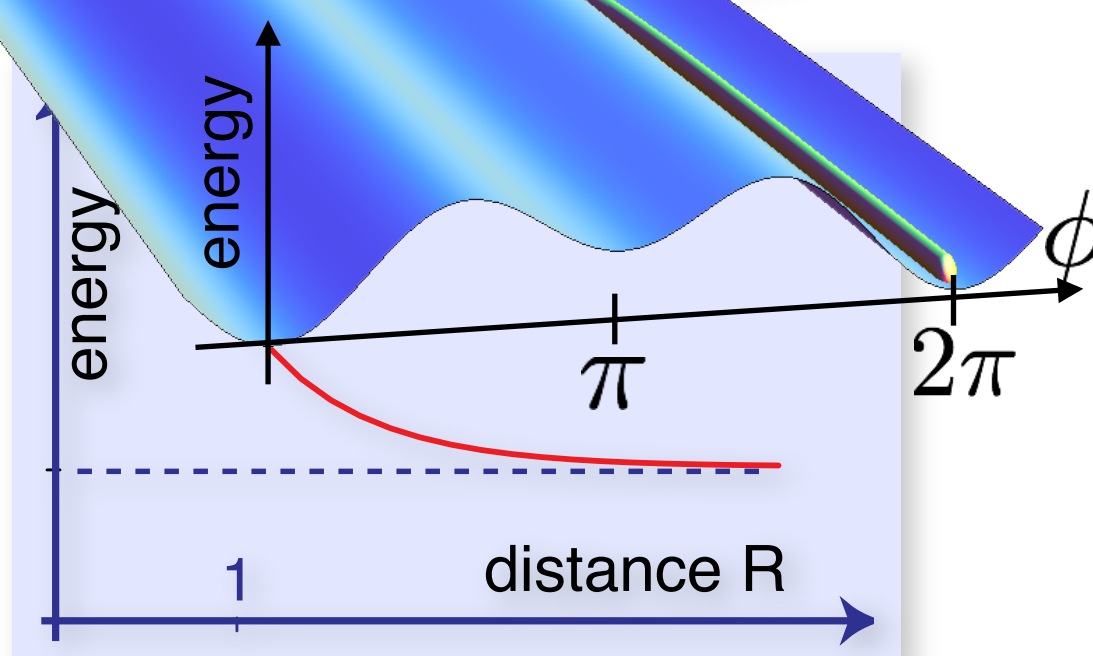


h magnetic field

$$\tan \frac{\phi_{ss}}{2} = \frac{\cosh R}{\sinh x/\delta}$$

$$\tan \frac{\phi_{s\bar{s}}}{2} = \frac{\cosh x/\delta}{\sinh R}$$

HBB ('94)



quantization

nucleation

Finite temperature generalization of micromagnetics

LL or LLG equations form basis of micromagnetism, **but** both are at variance with fluctuation-dissipation theorem (i.e. damping, but no noise!)

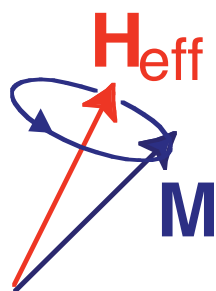
unable to describe superparamagnetism and related phenomena (important for data storage)

Remedy: introduce fluctuating fields

$$\mathbf{H}_{\text{eff}} \rightarrow \mathbf{H}_{\text{eff}} + \boldsymbol{\zeta}$$

$$\langle \zeta_i(\mathbf{x}, t) \zeta_j(\mathbf{0}, 0) \rangle = g_{ij} D_0 \delta_{ij} \delta(t) \delta(\mathbf{x})$$

$$D_0 = 2\alpha k_B T / \gamma M_0$$



$$\partial_t \mathbf{M} = -\gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + \boldsymbol{\zeta}) + \frac{\alpha}{M_0} \mathbf{M} \times \partial_t \mathbf{M}$$

$$(1 + \alpha^2) \partial_t \mathbf{M} = -\gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + \boldsymbol{\zeta}) - \frac{\alpha\gamma}{M_0} \mathbf{M} \times [\mathbf{M} \times (\mathbf{H}_{\text{eff}} + \boldsymbol{\zeta})]$$

$$\mathbf{H}_{\text{eff}} = -\delta E / \delta \mathbf{M}$$

Finite temperature generalization of micromagnetics

Consequences:

Superparamagnetism in single domain clusters (Néel-Brown);

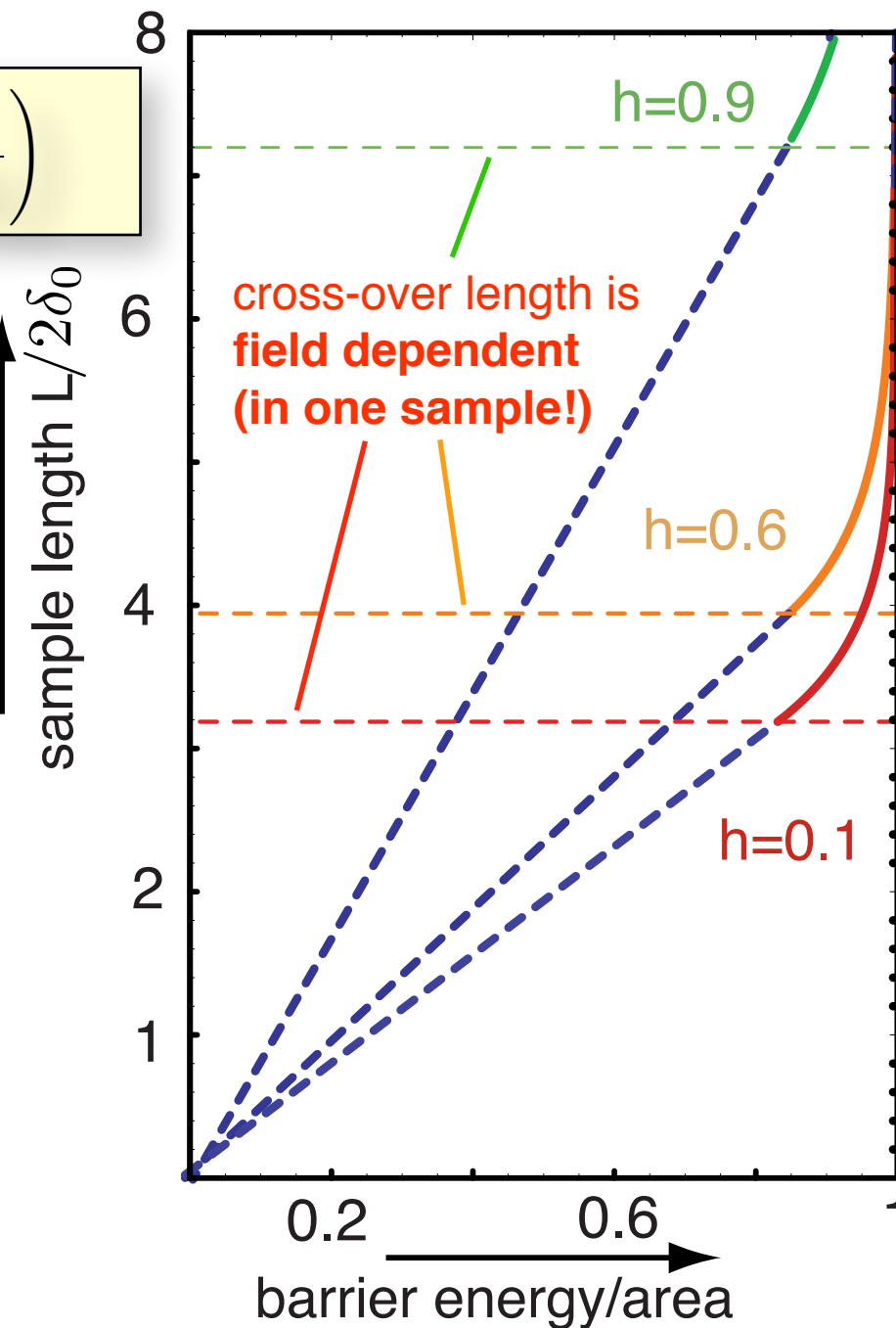
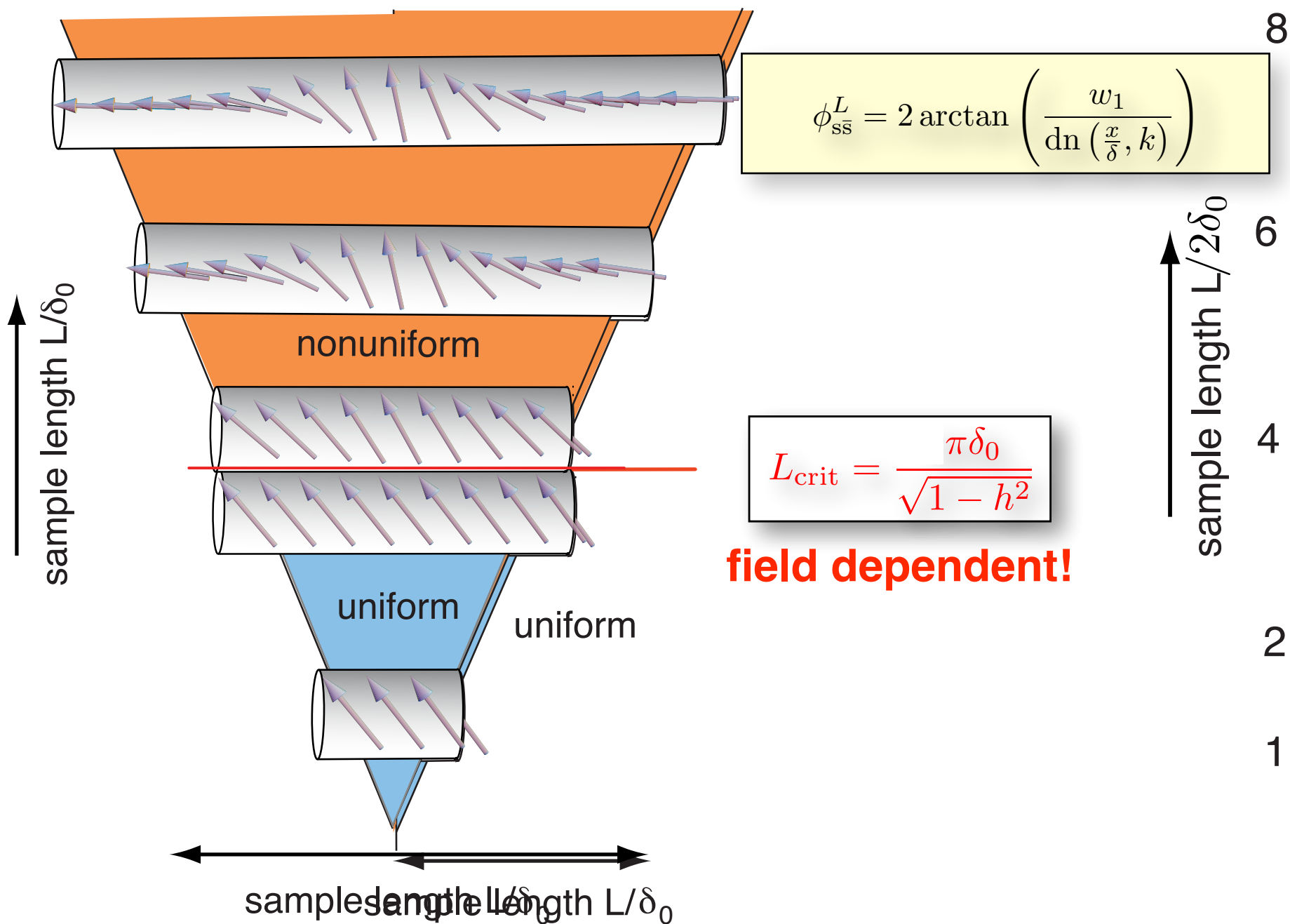
Nucleation of domain walls in nanowires (HBB '93, Adv Phys '12)

II. Superparamagnetism & limits of magnetic data storage

Nanowires: superparamagnetism via soliton-antisoliton nucleation & perpendicular magnetic recording

Energy barriers and Arrhenius prefactors

Crossover between Néel-Brown mechanism and soliton nucleation



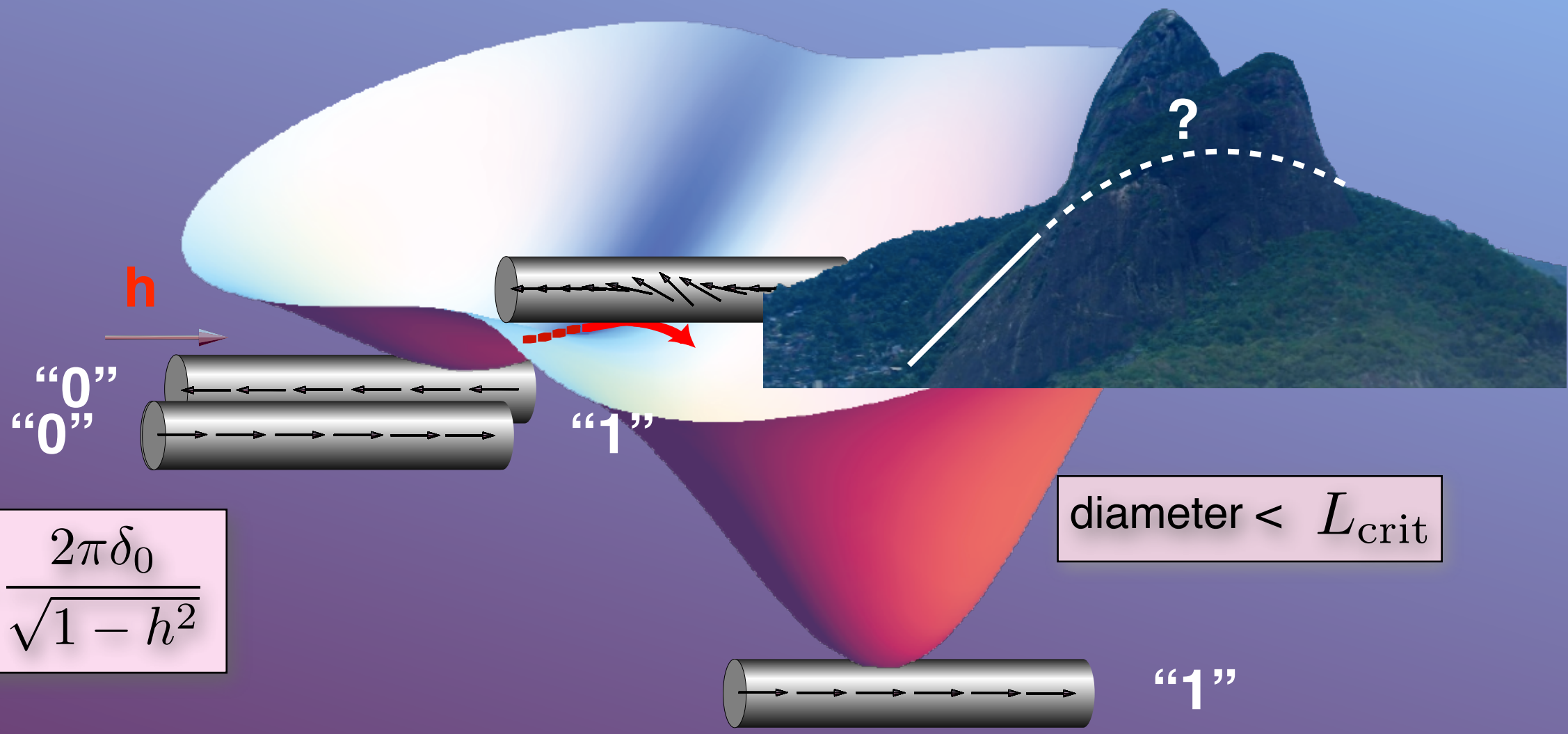
barrier energy / area:

HBB (*JAP* ('06), *Adv. Phys.* ('12))

$$\mathcal{E}_m = 8\sqrt{AK_e} \frac{\delta_0}{\delta} \left\{ E(k) + \left(1 - \frac{k^2}{\alpha^2}\right) K(k) + \left(\alpha^2 + \frac{k^2}{\alpha^2} - 2\right) \Pi(\alpha^2, k) \right\} - 2K_e L(c + h)$$

Cross-over has "critical" character

Soliton-antisoliton pairs and thermal energy barriers



$$L_{\text{crit}} = \frac{2\pi\delta_0}{\sqrt{1-h^2}}$$

$\text{diameter} < L_{\text{crit}}$

- $L < L_{\text{crit}}$

uniform (Néel-Brown)
- $L > L_{\text{crit}}$

small fields
soliton-antisoliton mechanism (HBB 93)
- $L > L_{\text{crit}}/2$

**2 domain walls
opposite chirality**
~~'activation volume'~~
 $\sim 4\text{nm (FePt) !!}$

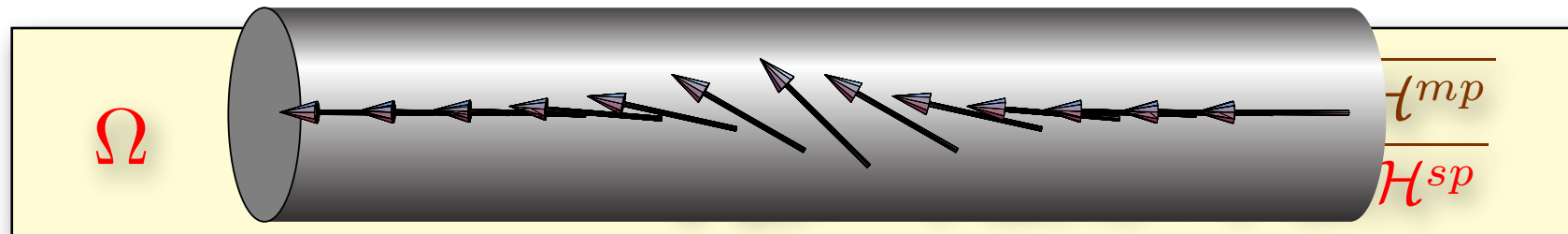
Switching rates for soliton-antisoliton nucleation

switching rate

$$\Gamma = \Omega e^{-\beta A \mathcal{E}_s}$$

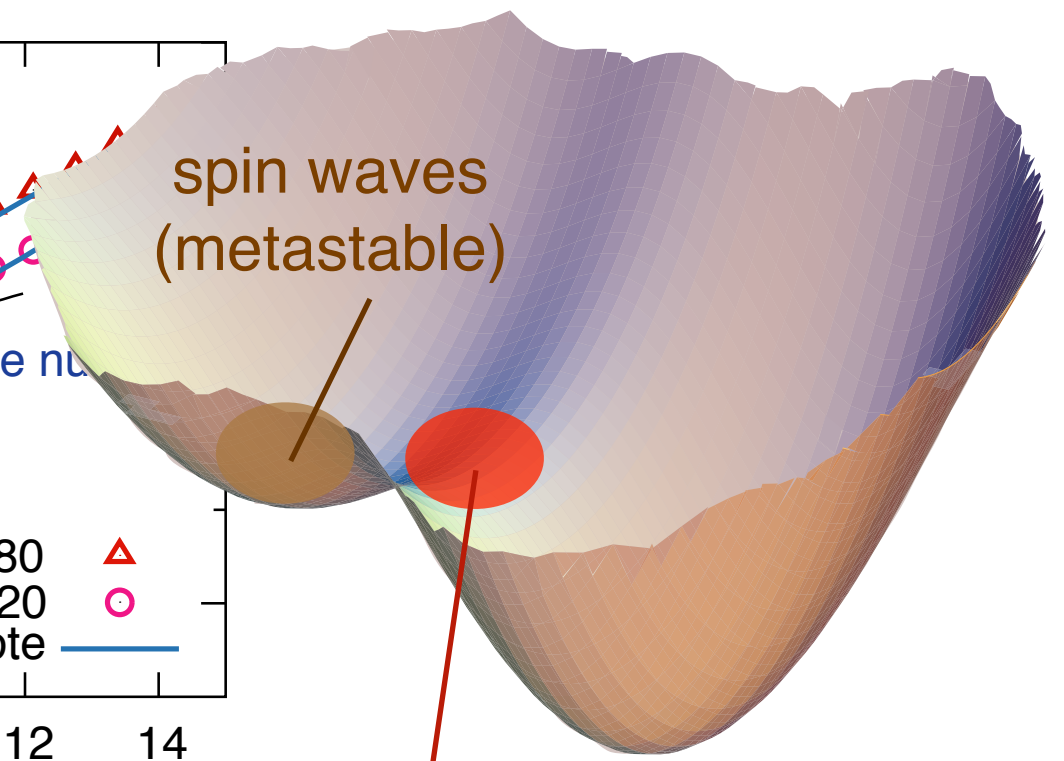
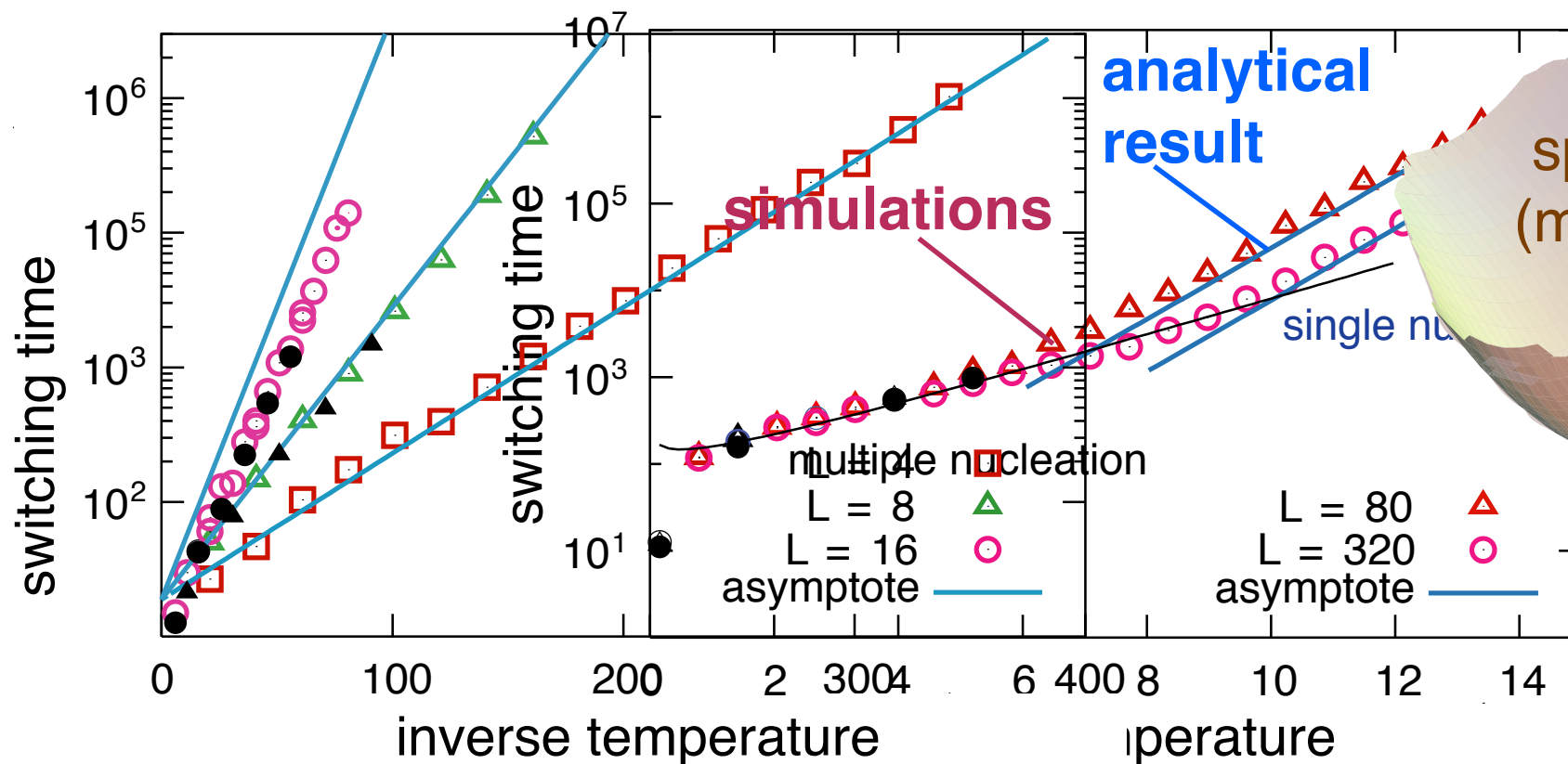
(HBB, PRL'93, PRB '94, JAP'06
Adv Phys.'12),
cf. P. Loxley, R. Stamps, '06

prefactor



$$= \text{escape frequency} \times \text{zero modes (symmetry)} \times \left(\frac{\text{curvature (M)}}{\text{curvature (S)}} \right)^{1/2}$$

uniform soliton nucleation

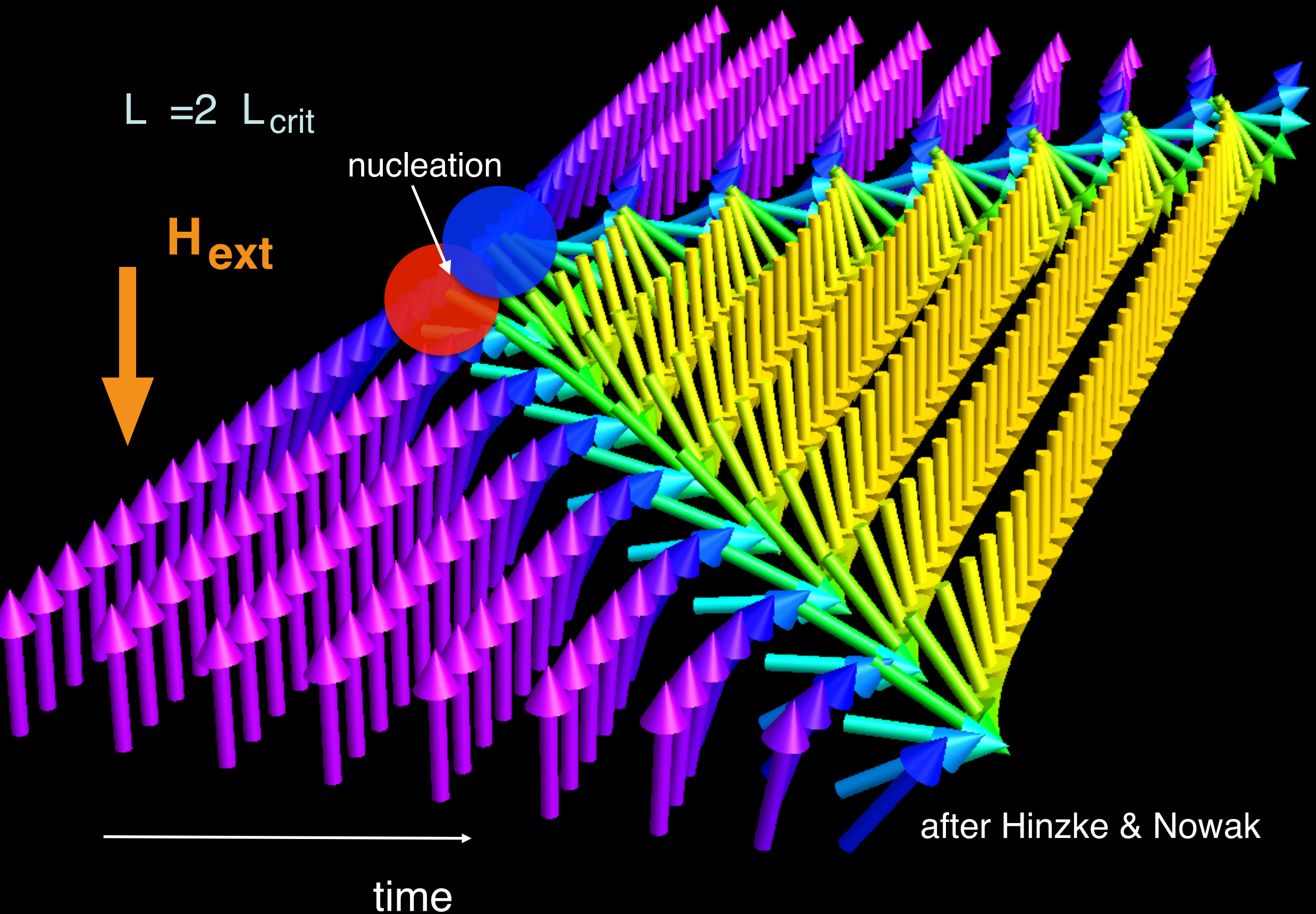


spin waves
(nucleus) -
cf. Winter but more
complicated

D. Hinzke & U. Nowak (PRB '00)

cf. B. Hillebrands' lecture (this School!)

Soliton-antisoliton nucleation



Application: 'Perpendicular Magnetic Recording' (PMR)

TOPICAL REVIEW

The transition from longitudinal to perpendicular

H J Richter

Seagate Technology, 47010 K

Received 7 February 20

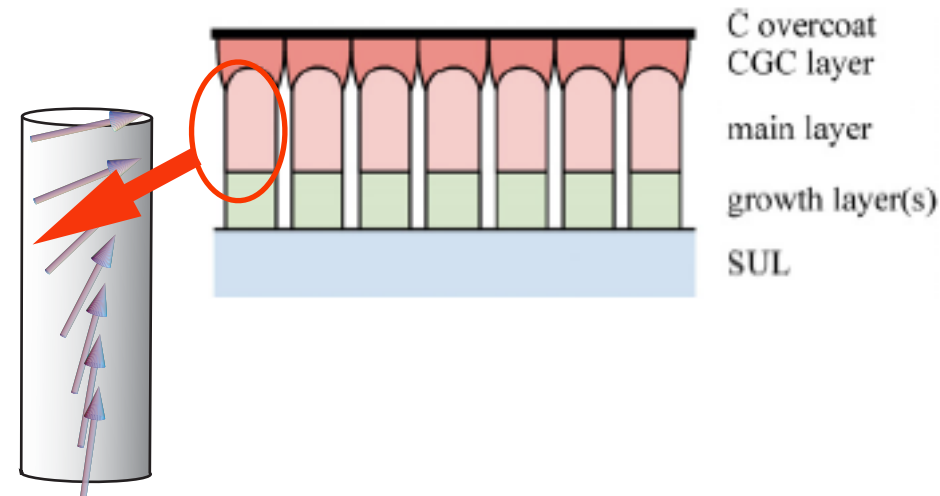
Published 19 April 2007

Online at stacks.iop.org

6. Perpendicular

6.1. Media structure

However, it also has to do with the tallness of the thermal energy barrier. The energy barrier is $4\sqrt{AK}$ where K is the stiffness. For a cylindrical barrier becomes



ℓ_T at which the reversal process transitions coherent:

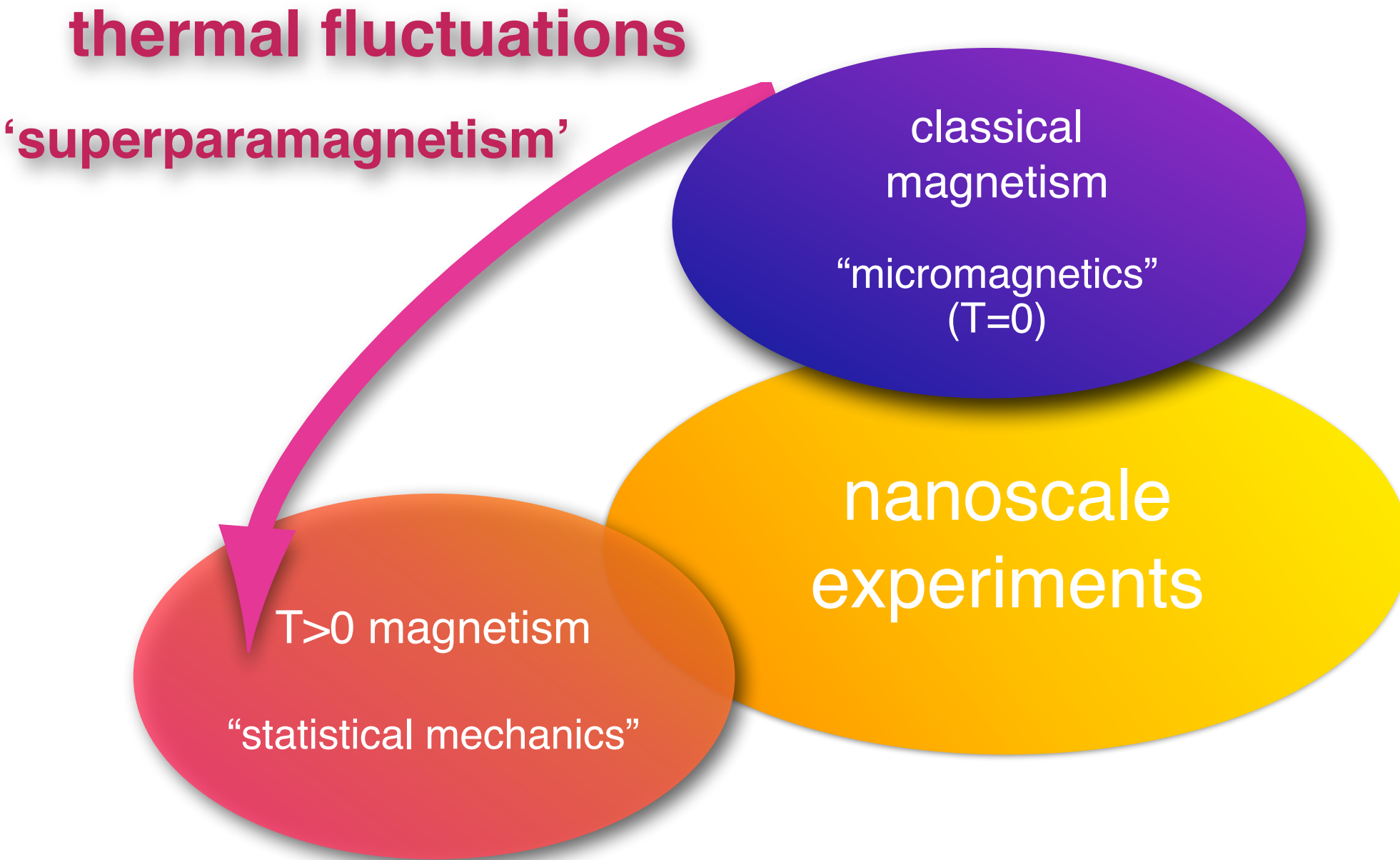
$$\ell_T = 4\sqrt{\frac{A}{K}} \quad (30)$$

Typical values for current perpendicular recording media are $K \approx 4 \times 10^5 \text{ Jm}^{-3}$ and $A \approx 10^{-11} \text{ Jm}^{-1}$ and hence $\ell_T \approx 20 \text{ nm}$, which has been confirmed experimentally [94]. It is important to note that materials with higher K lead to smaller transition thicknesses ℓ_T . This has an important implication for possible SNR gains that can be achieved by a reduction of grain size. As long as the medium thickness is less than the transition length ℓ_T , maintaining thermal stability (that is KV) requires that the anisotropy K has to be scaled according to $K \propto 1/D^2$. However, if the medium thickness is equal to ℓ_T ,

$$\Delta E_{DW} = \pi D^2 \sqrt{AK} \quad (29)$$

cf. J. Coker's lecture (this School!)

Theoretical descriptions of magnetism

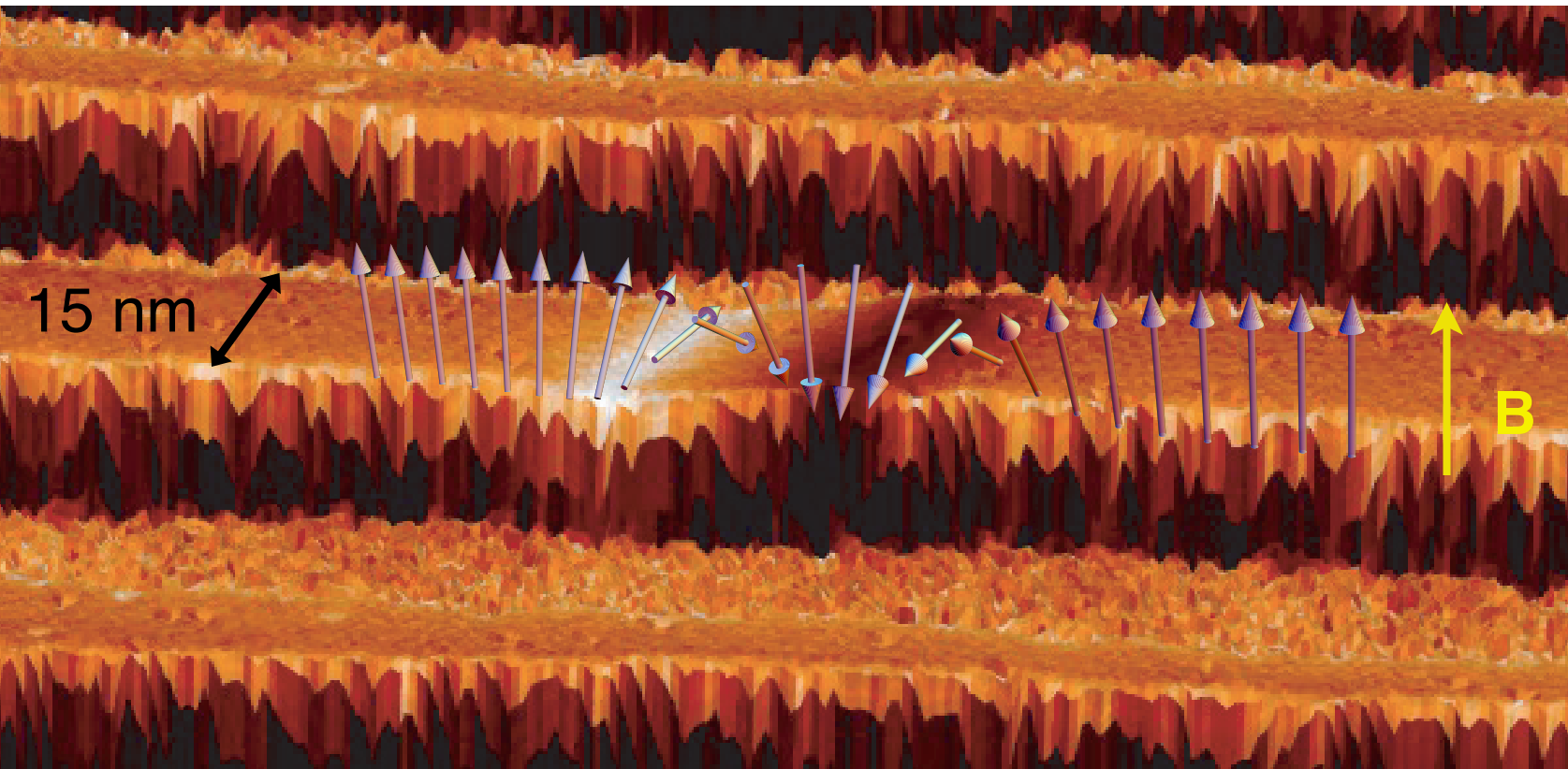


III. Quantization of micromagnetics

Semiclassical quantization of micromagnetics, Berry phase and topology

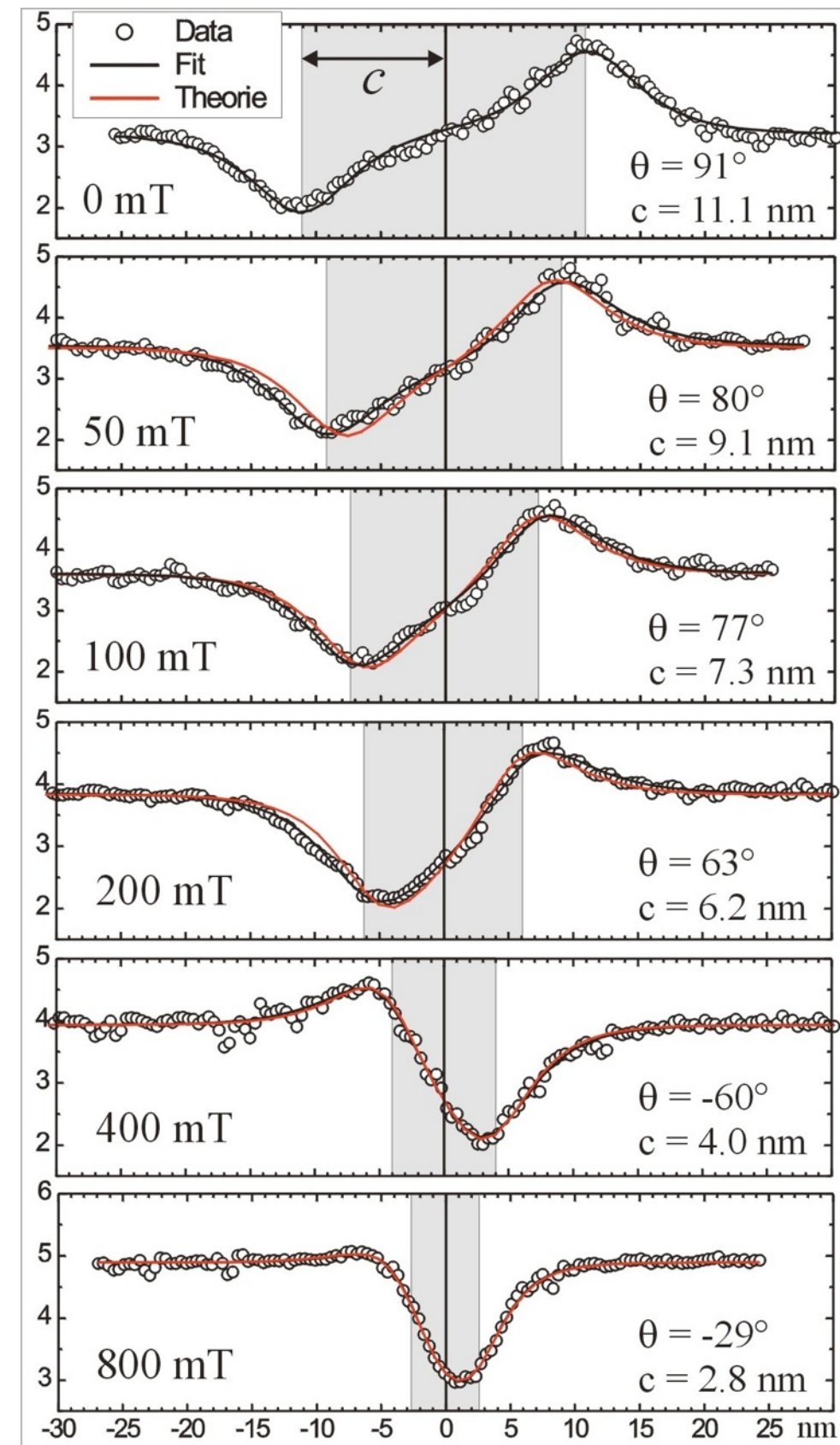
How to derive excitations of anisotropic XYZ-Heisenberg spin chains from micromagnetics

Soliton-soliton pairs in nanowires

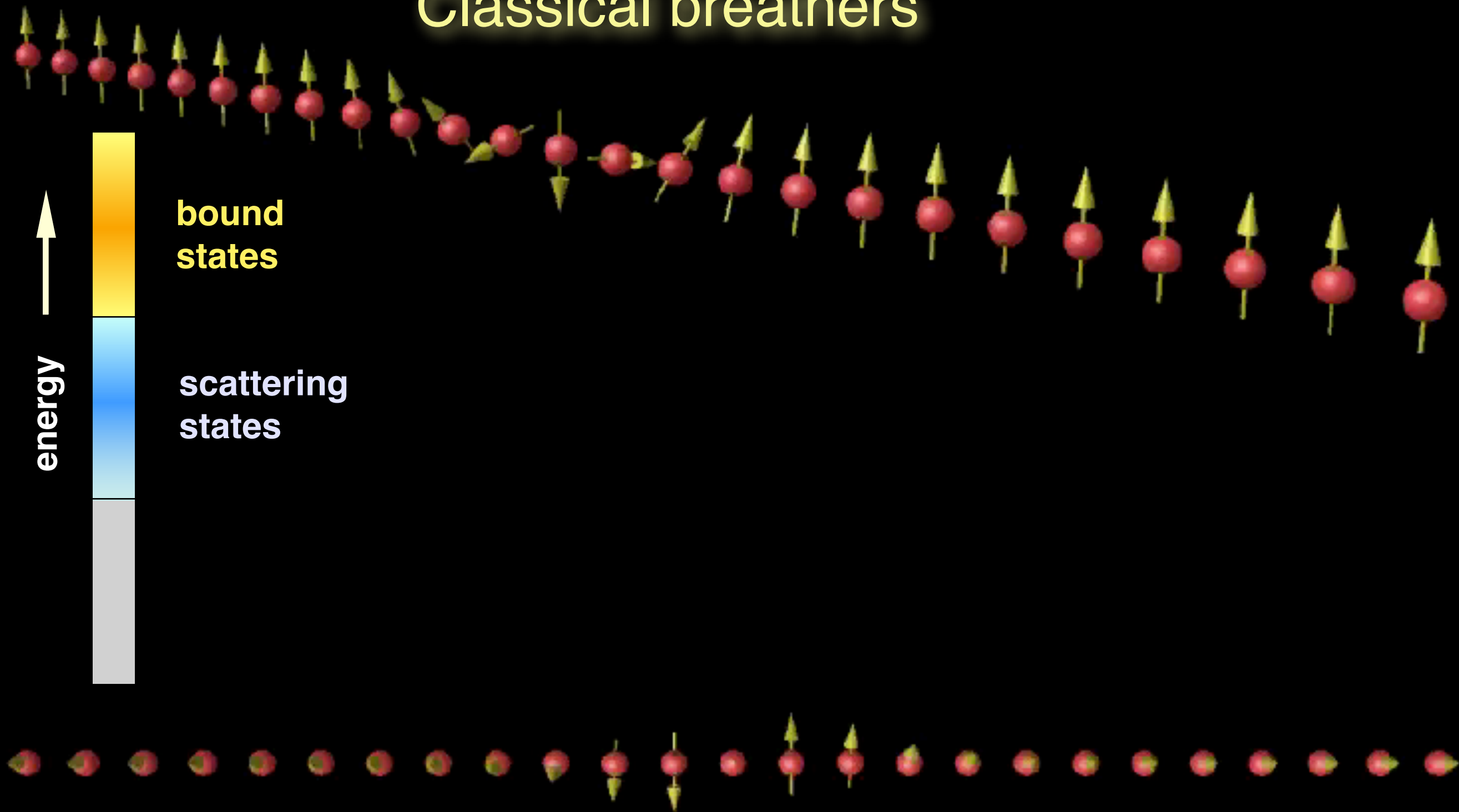


expts: Kubetzka, Pietzsch, Bode,
Wiesendanger (PRB '03)

theory: HBB (PRB '94)



Classical breathers

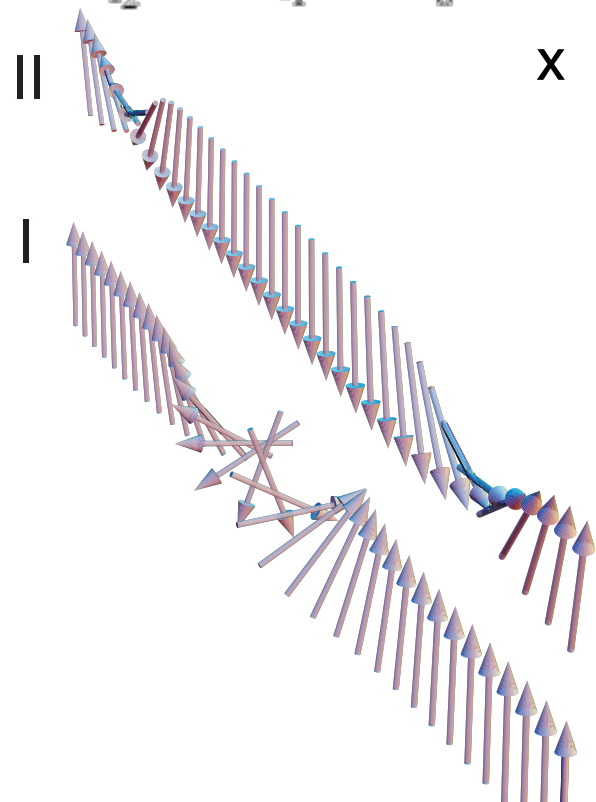
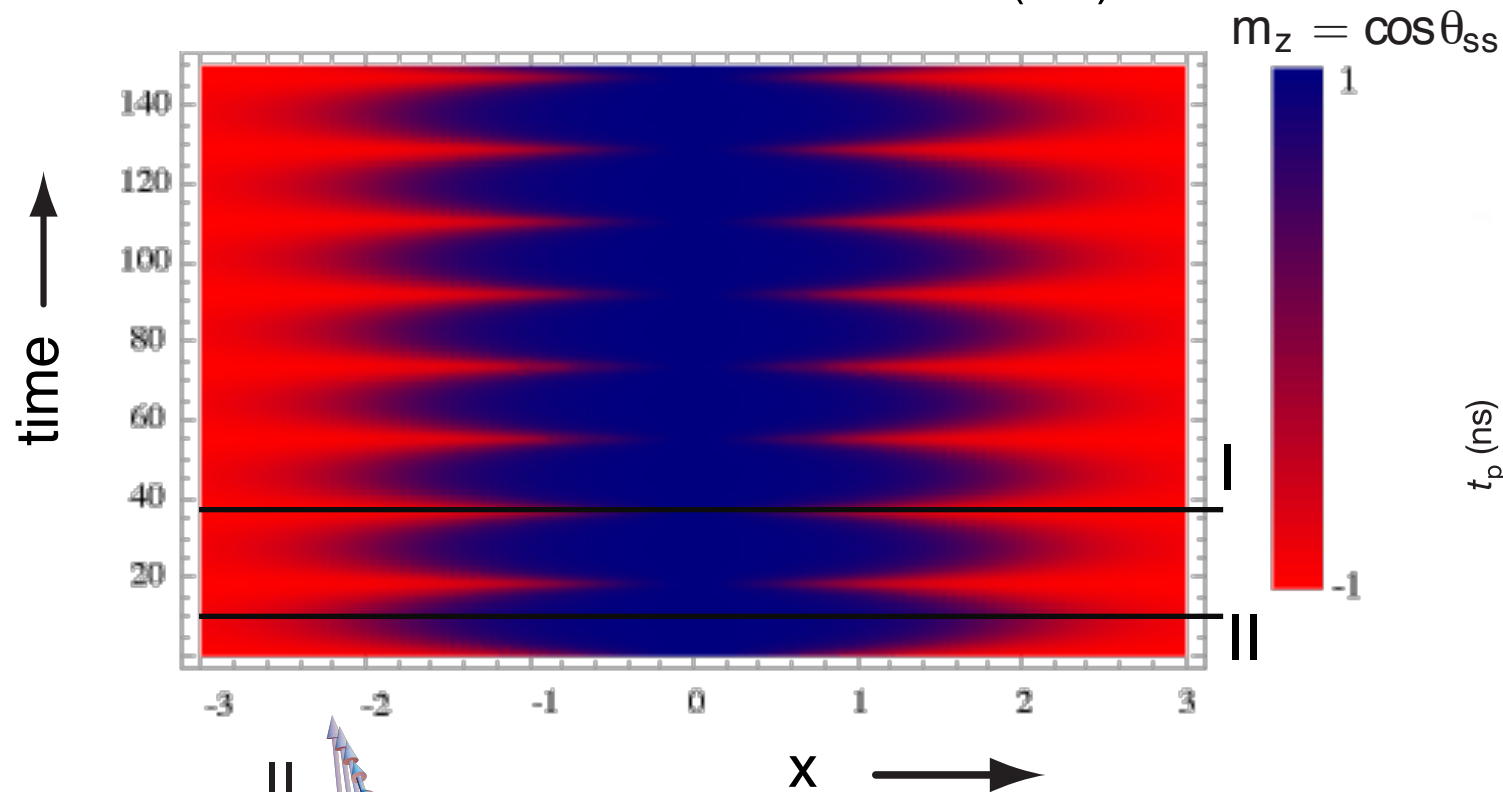


exact solutions: HBB & Brodbeck, PRL ('93),
J. Eves et al. ('10)

Are breathers observable ?

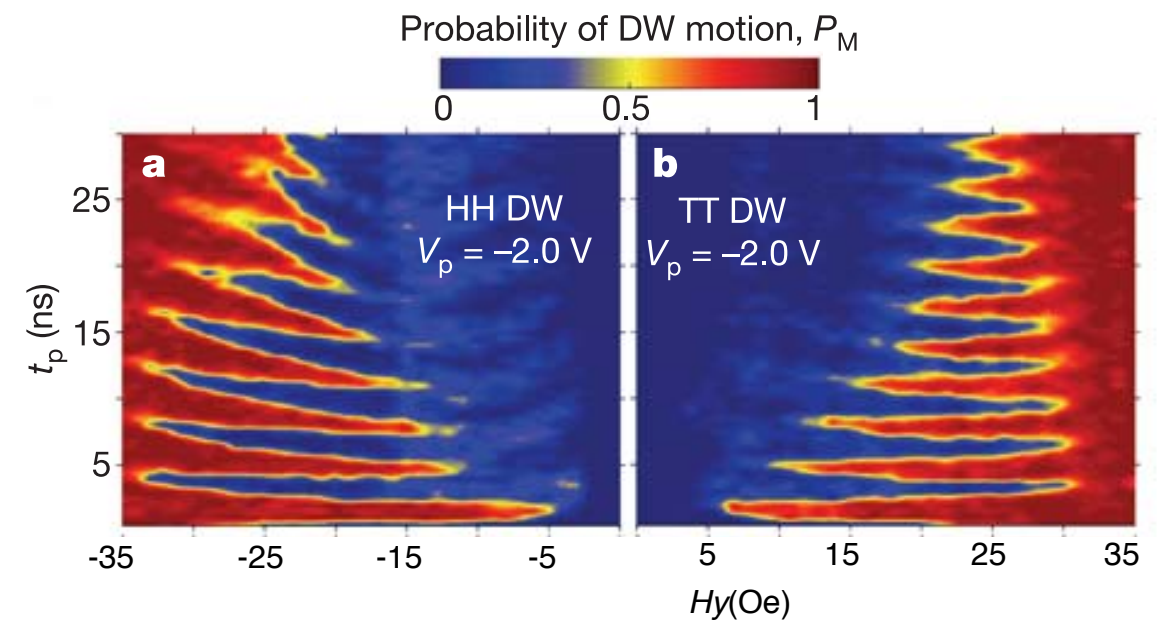
predicted breather oscillations

J. Eves et al. ('10)

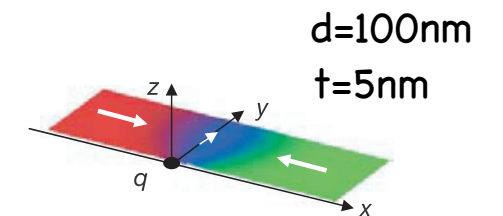
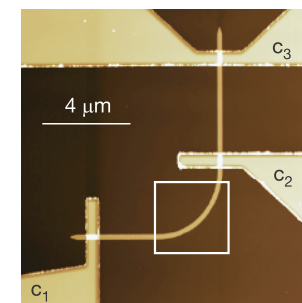


spin torque stabilization:
Iacocca et al. PRL 112, 047201 ('14)

Depinning und induced wall oscillations in nanowires

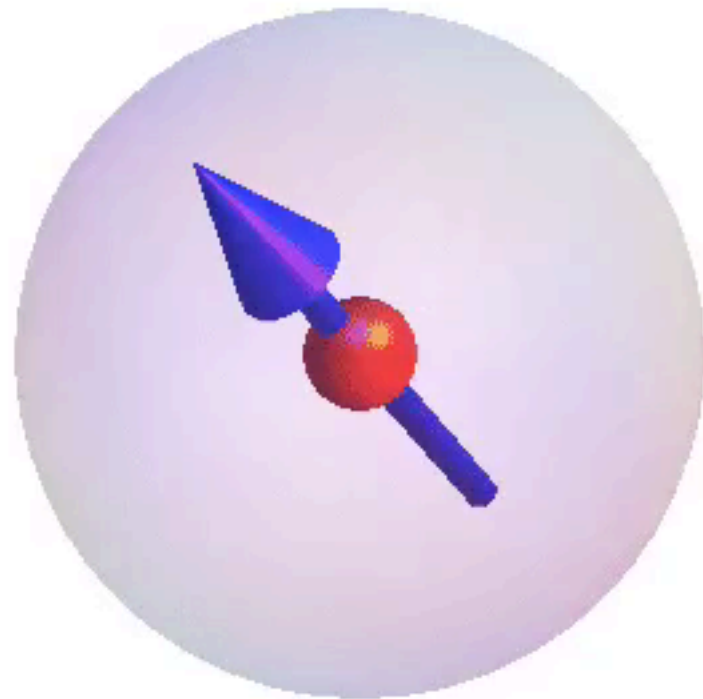


L. Thomas et al, Nature (06)



Importance of quantum effects

classical spin



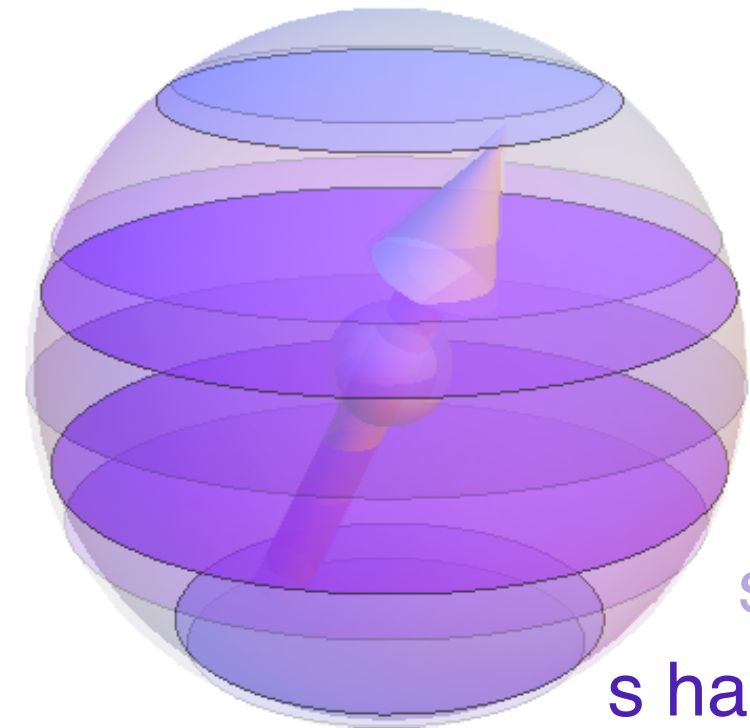
$$\mathbf{S} = s(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \phi < 2\pi$$

real

quantum spin



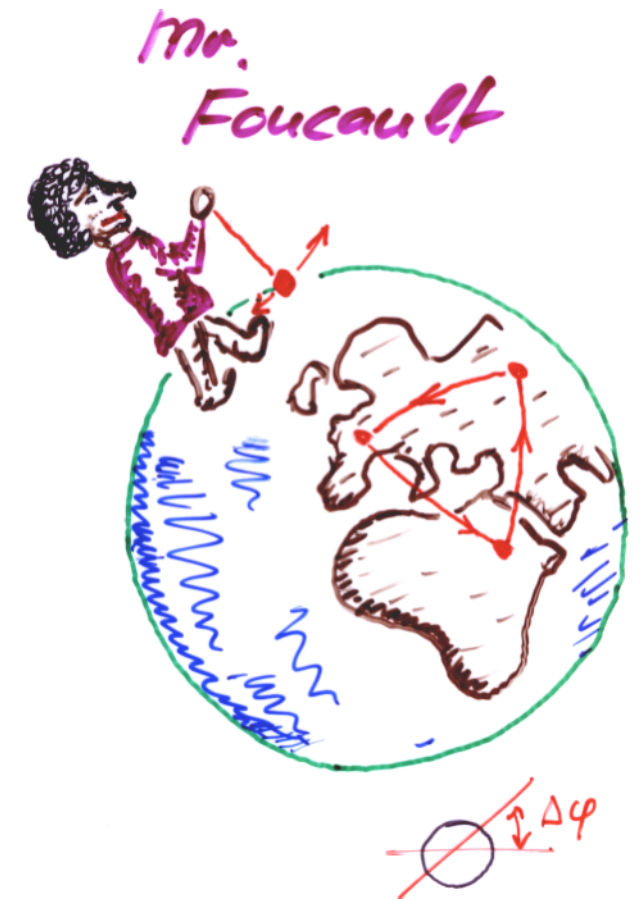
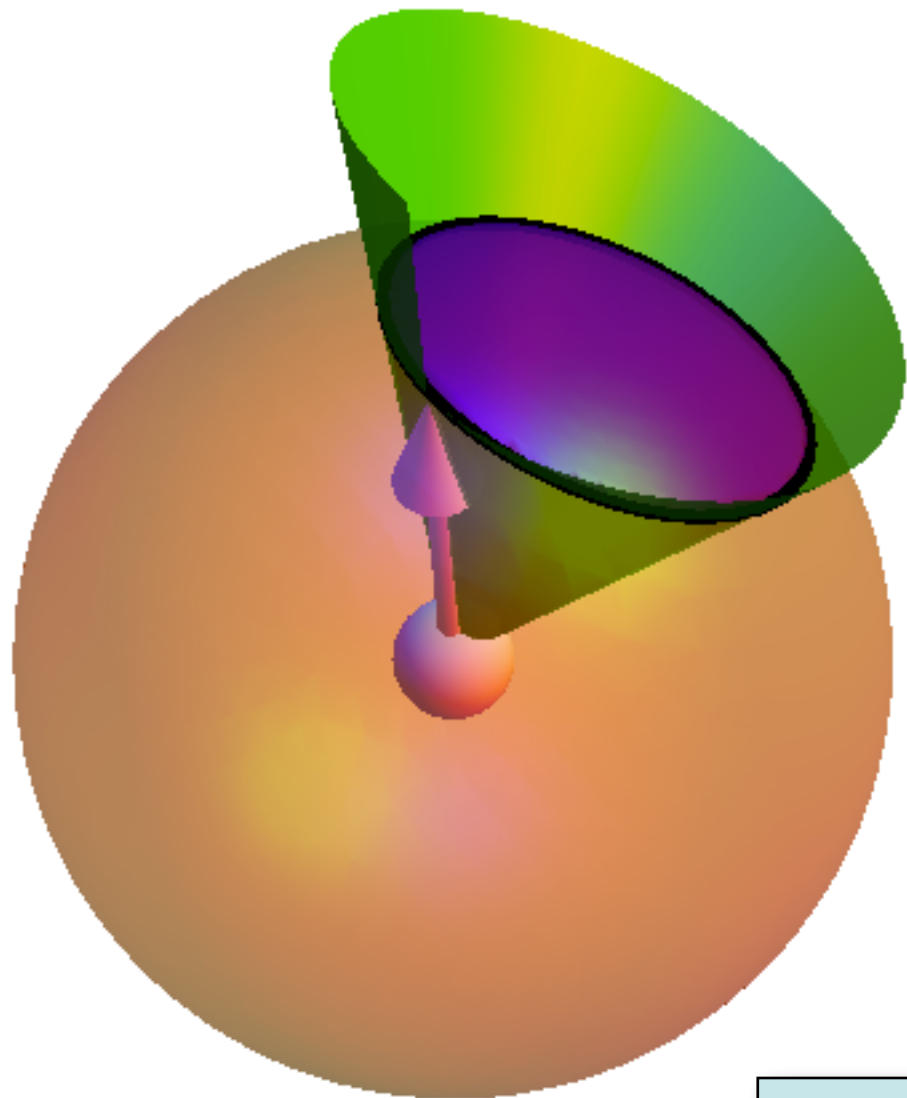
s integer
 s half integer

$$|s m_s\rangle$$

$$m_s = -s, \dots, s$$

discrete

Key concept - Berry phase



$$|\psi(T)\rangle = e^{i\gamma(T)} e^{-i \int_0^T dt E_n(t)} |\psi(0)\rangle$$

$$\gamma(T) = s \oint d\mathbf{n} \cdot \mathbf{A} = s \int_0^T dt \partial_t \phi (1 - \cos \theta) = s \text{ area}$$

monopole vector potential (!)

Quantized breathers

Quantization condition ($\hbar = 1$)

$$\mathcal{S}_B[\mathbf{n}_{ss, s\bar{s}}] = 2\pi N$$

Berry-phase

ss-breather

$$E_{ss}^{\text{bound}}(N)/2E_0 = 1/\{\tilde{m} \operatorname{sn}((N/2\tilde{s}\tilde{m}), \tilde{m})\}$$

$\bar{s}\bar{s}$ -breather

$$E_{\bar{s}\bar{s}}^{\text{bound}}(N)/2E_0 = \operatorname{sn}((N/2\tilde{s}\tilde{m}), \tilde{m})$$

cf. sine-Gordon model

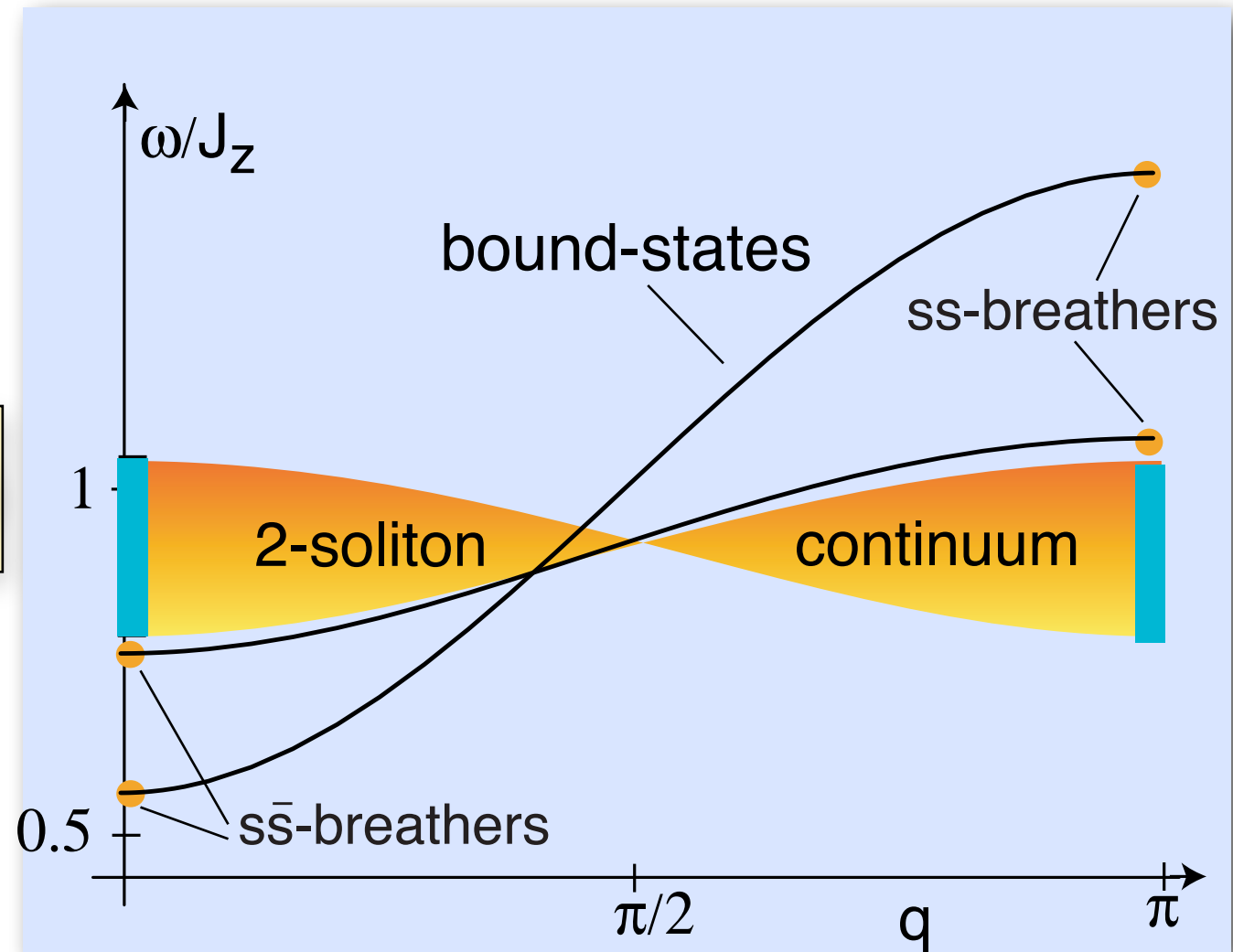
$$0 < N/2\tilde{s}\tilde{m} < K(\tilde{m})$$

easy-plane limit $\tilde{m} \rightarrow 0$

Ising limit $\tilde{m} \rightarrow 1$

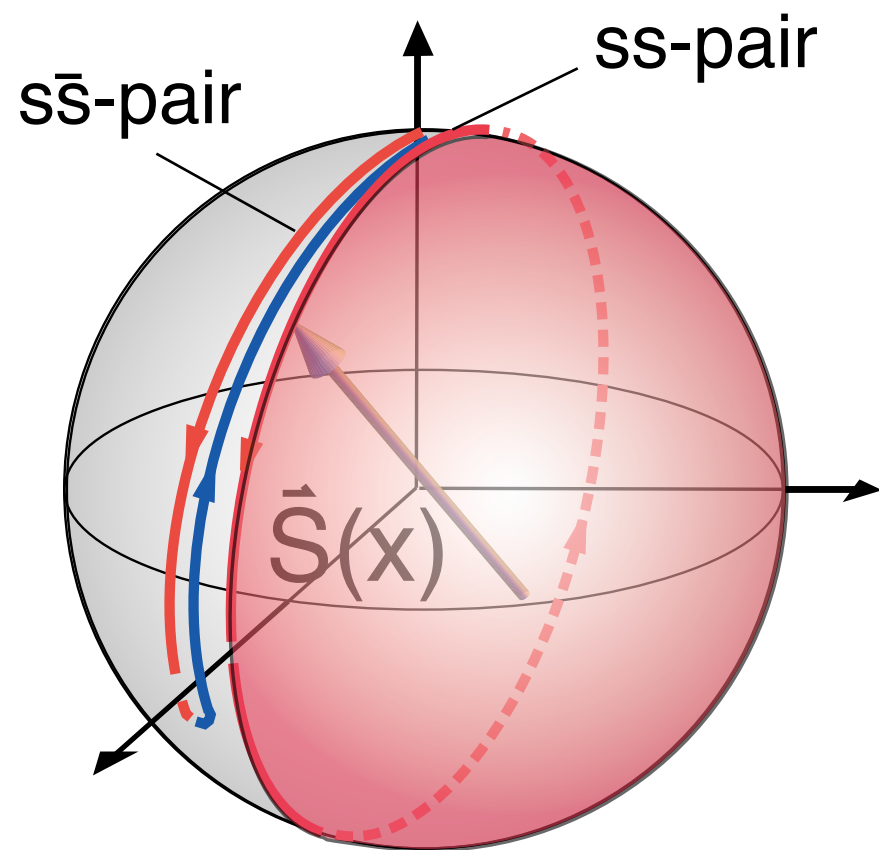
excitation spectrum agrees with that of discrete spin-1/2 xyz-chain:

$$H_{xyz} = \sum_i J_x S_i^x S_{i+1}^x + J_y S_i^y S_{i+1}^y + J_z S_i^z S_{i+1}^z$$



HBB & N. Fettes, JAP 85 ('99)
HBB & O. Brodbeck, PRL ('93)

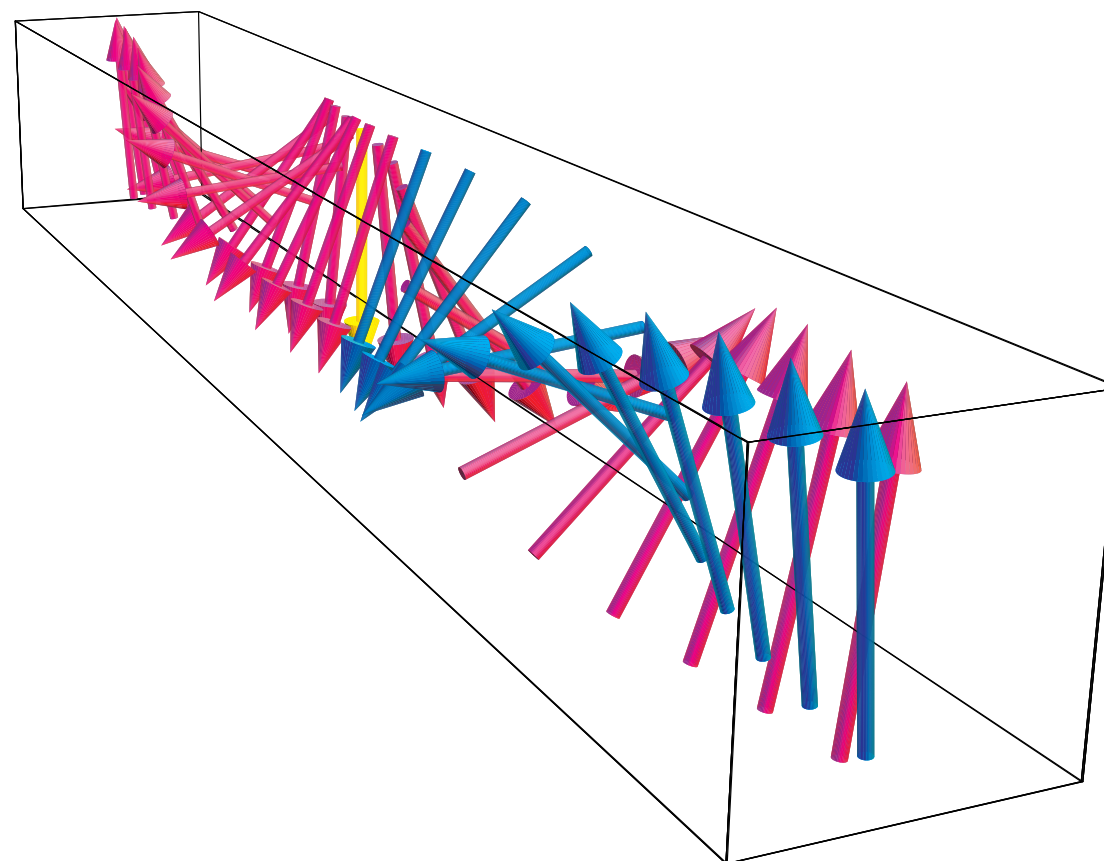
Spin momentum and solitons



momentum (Haldane)

$$\mathcal{P} = \oint dx \partial_x \mathbf{n} \cdot \mathbf{A}$$

monopole



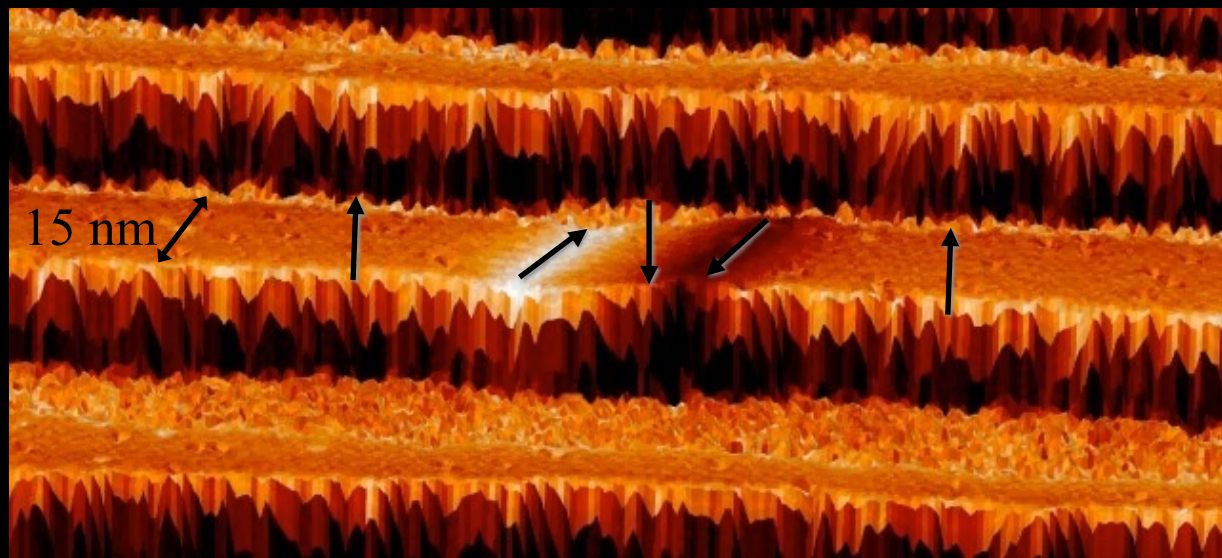
wavevector $k=0$

$$k = s\mathcal{P}$$

Relative wave vector of solitons with opposite chirality is π !

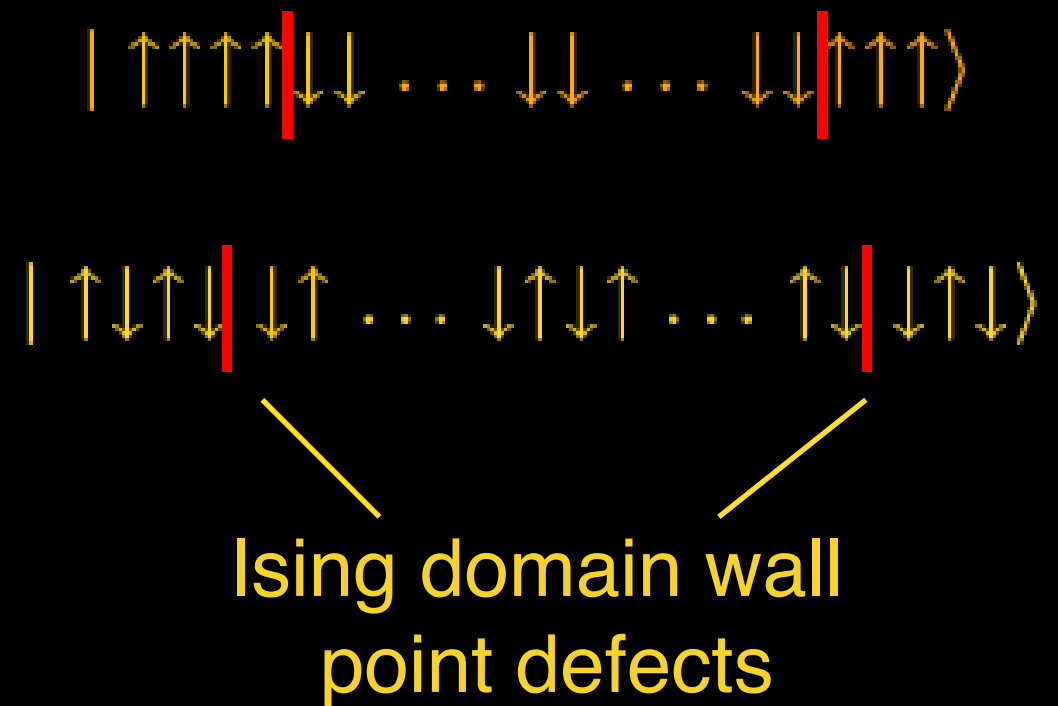
Quantum fluctuations, point defects & emergence of chirality

Fe-nanowires



Kubetzka et al. PRB ('03)

quantum spin-chains



CsCoBr₃ - a quasi 1D Heisenberg-Ising chain

$$H_{\text{xxz}} = \sum_i J_z S_i^z S_{i+1}^z + J_t (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)$$

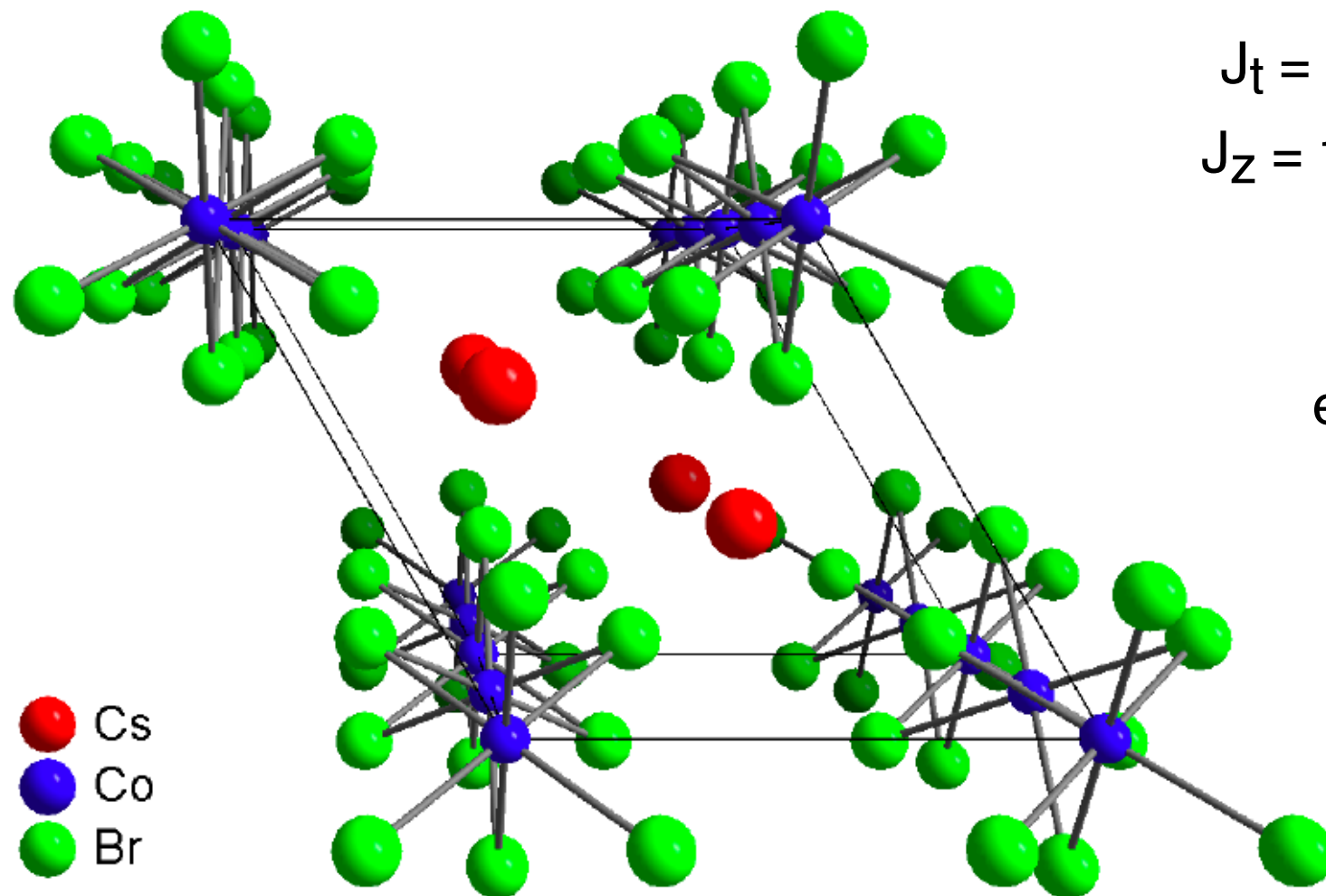
Ising-term XY-term

$$J_t = 1.9 \text{ meV}$$

$$T_N = 28 \text{ K}$$

$$J_z = 13.8 \text{ meV}$$

$$J'/J \sim 10^{-2}$$



extensively studied:

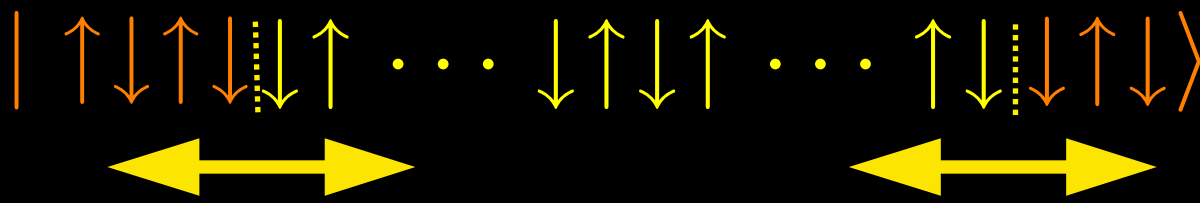
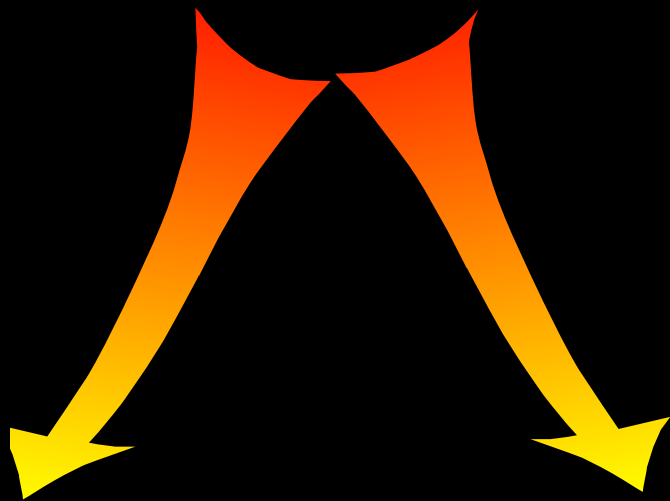
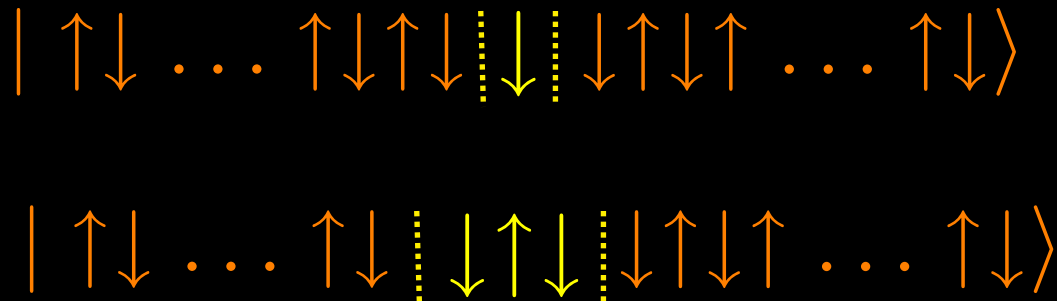
Shirane

Nagler, Tennant, Goff, Gaulin,

Cowley, Regnault, Boucher

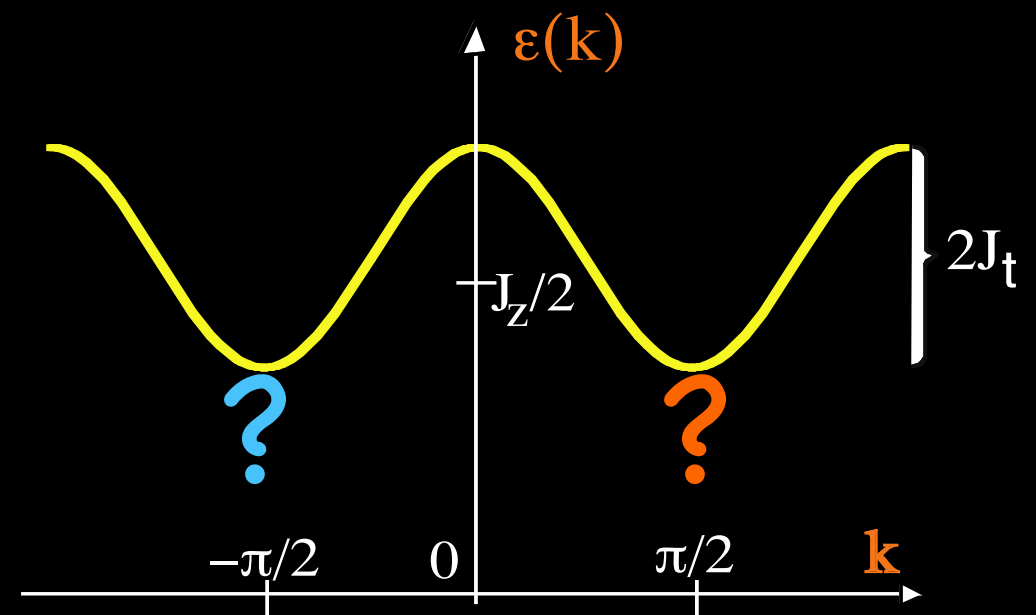
Chirality and Solitons

Ising limit,
no chirality



magnon decays into 2 solitons

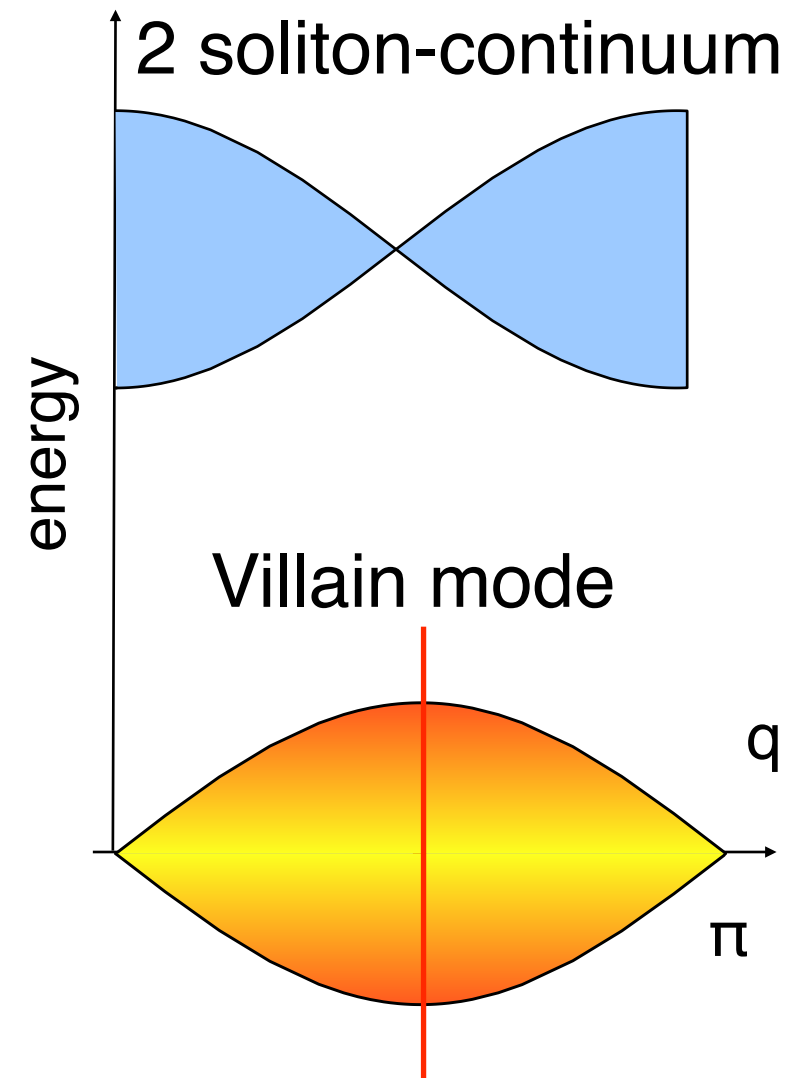
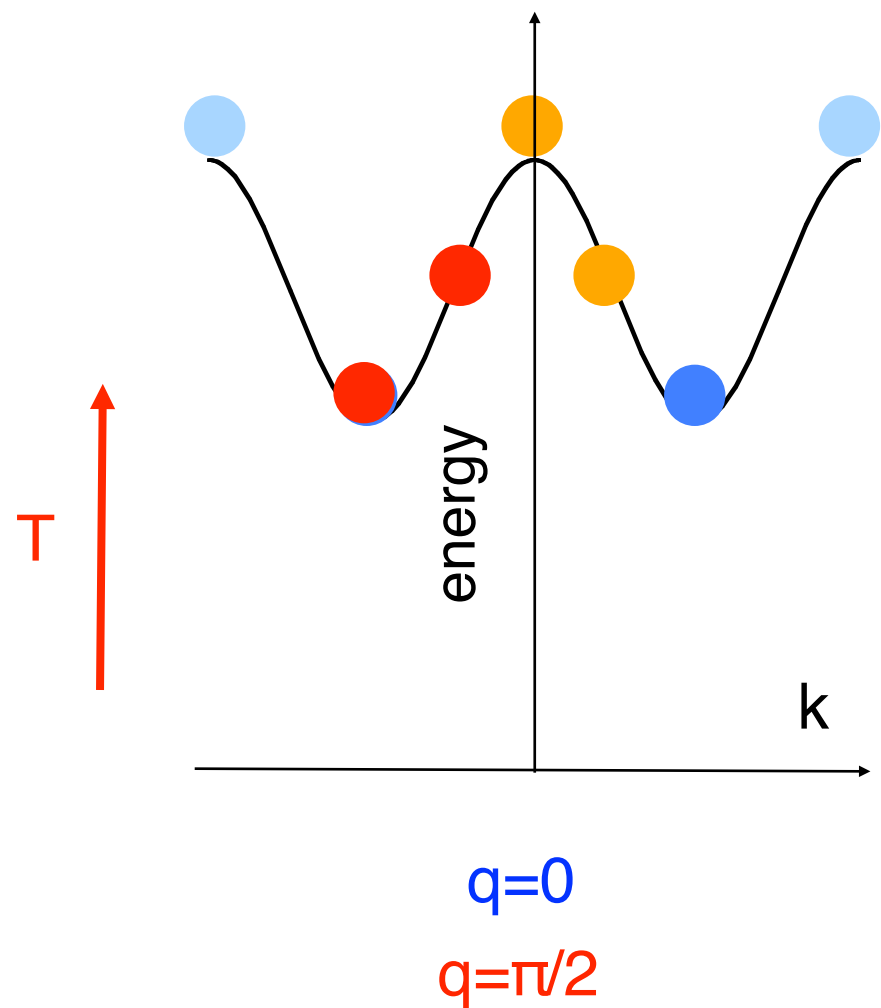
quantum fluctuations (XY-term)



Are the two bandminima
equivalent?

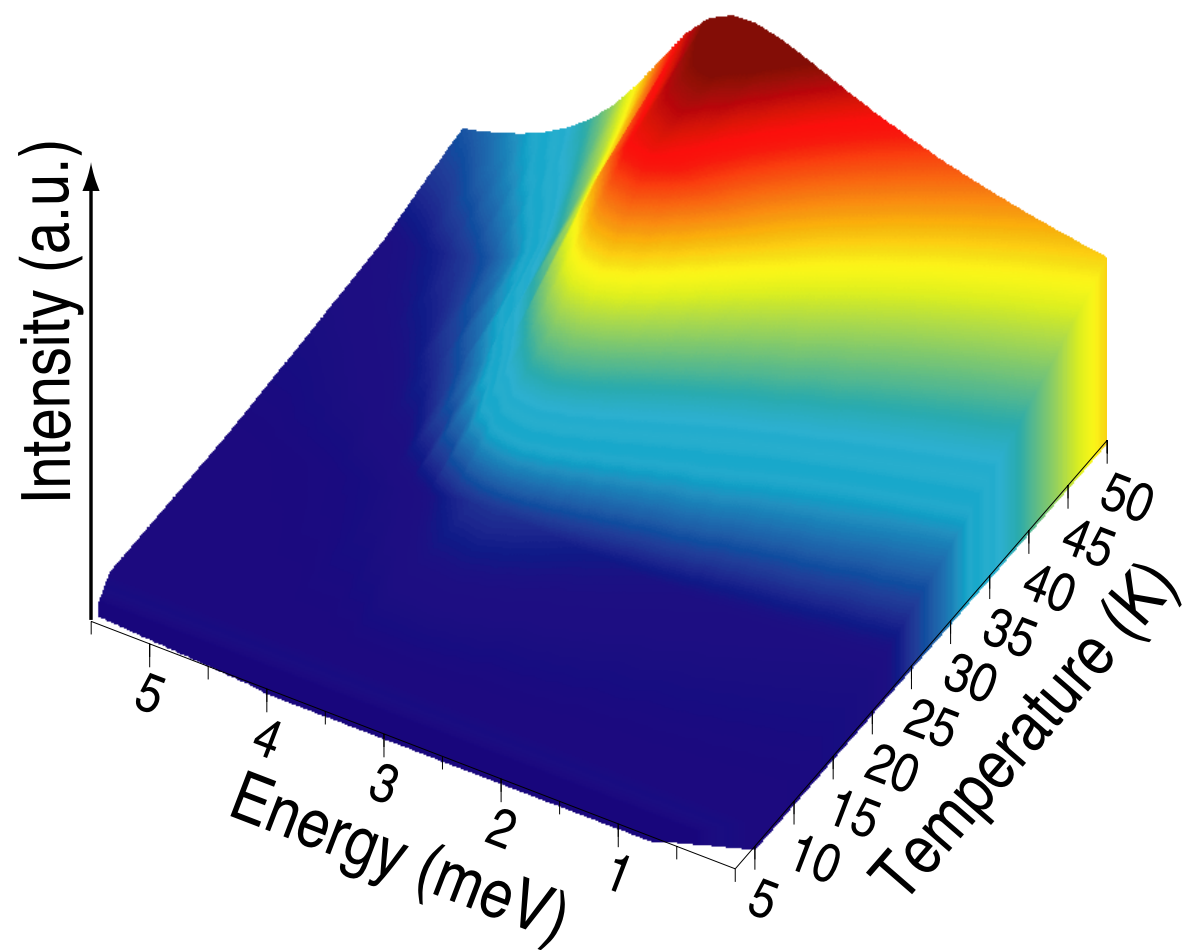
$$\left\langle \sum_i (\mathbf{S}_i \wedge \mathbf{S}_{i+1})_x \right\rangle = \pm 1$$

How neutrons couple to solitons

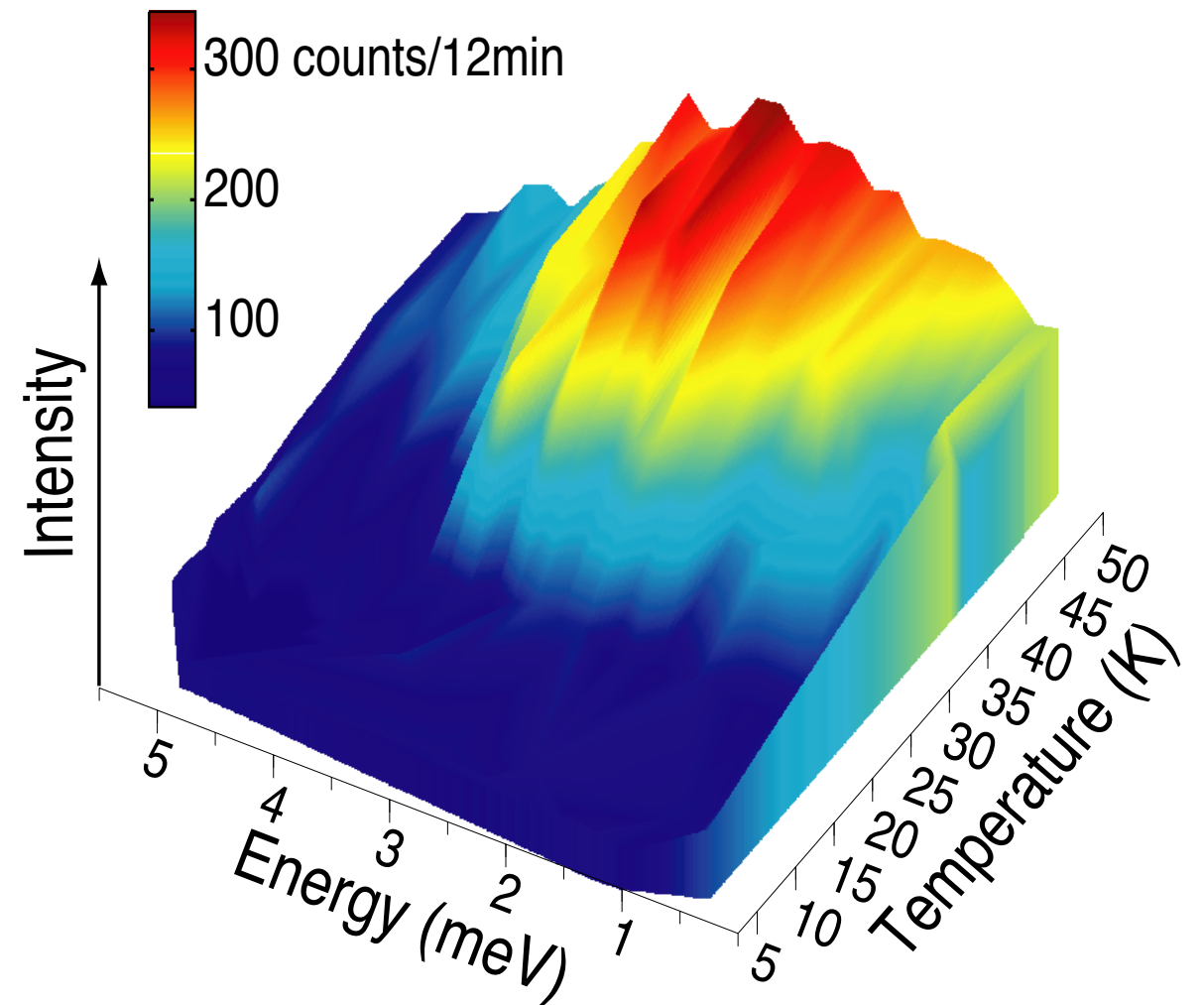


Villain mode

theory



experiment CsCoBr_3



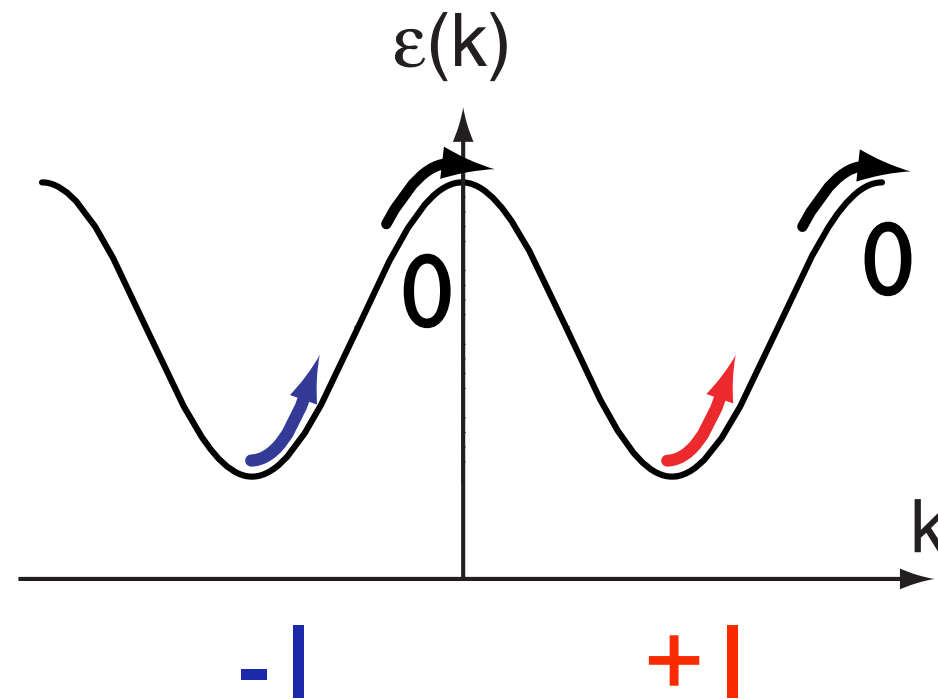
Polarized neutrons and chirality

$$I_+ - I_- \propto 2i \int dt e^{-i\omega t} \sum_k e^{-\beta\epsilon_k} \langle k | S_{-Q}^\perp \wedge S_Q^\perp(t) | k \rangle$$

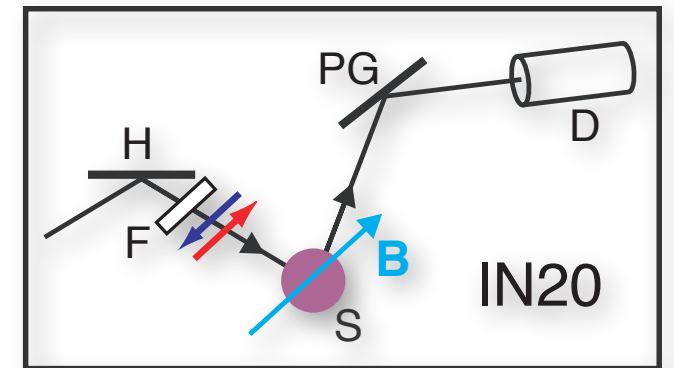
$$I_P = \dots + i \mathbf{P} \cdot \int dt e^{-i\omega t} \langle S_{-Q}^\perp \times S_Q^\perp(t) \rangle$$

$$\propto \frac{1}{\cos(q/2)} \sin\left(\frac{k+k'}{2}\right) \delta(\hbar\omega - \epsilon_k + \epsilon_{k'})$$

Maleyev, Blume, ...



Chirality is hidden !!



$$k' = k + q - \pi$$

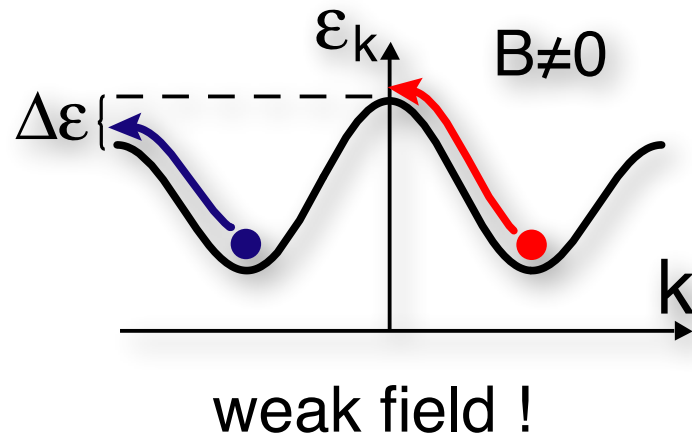
Institute Laue Langevin Grenoble



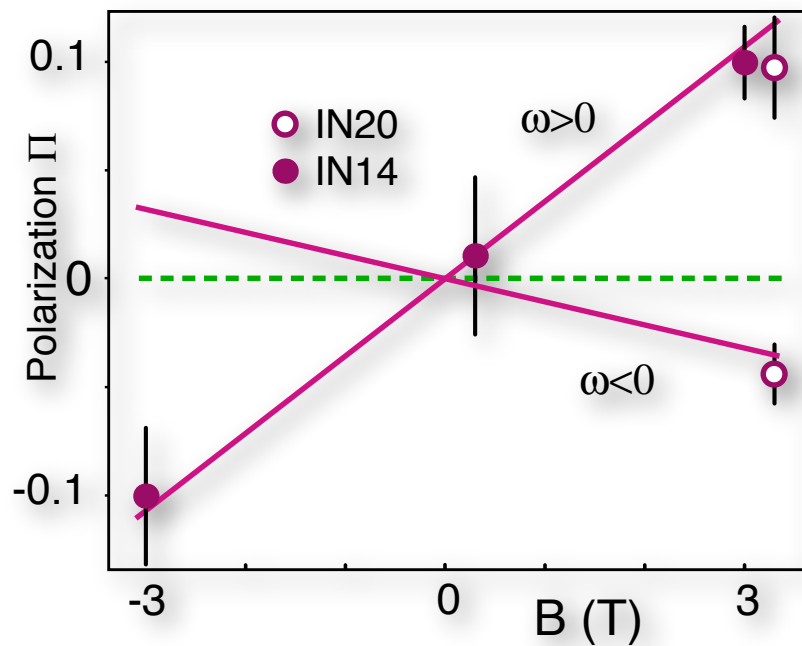
ESRF

ILL

Chirality and spin-currents in CsCoBr₃



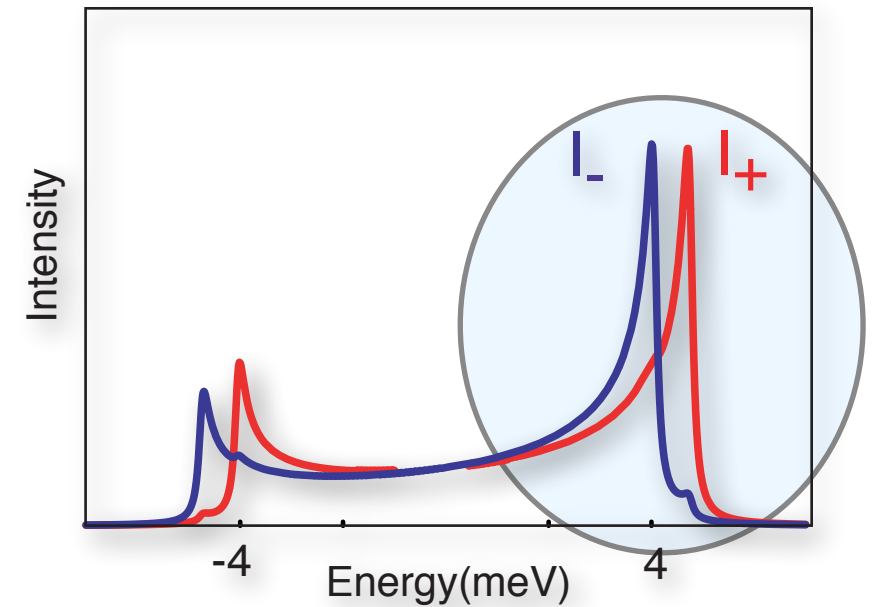
magnetic field



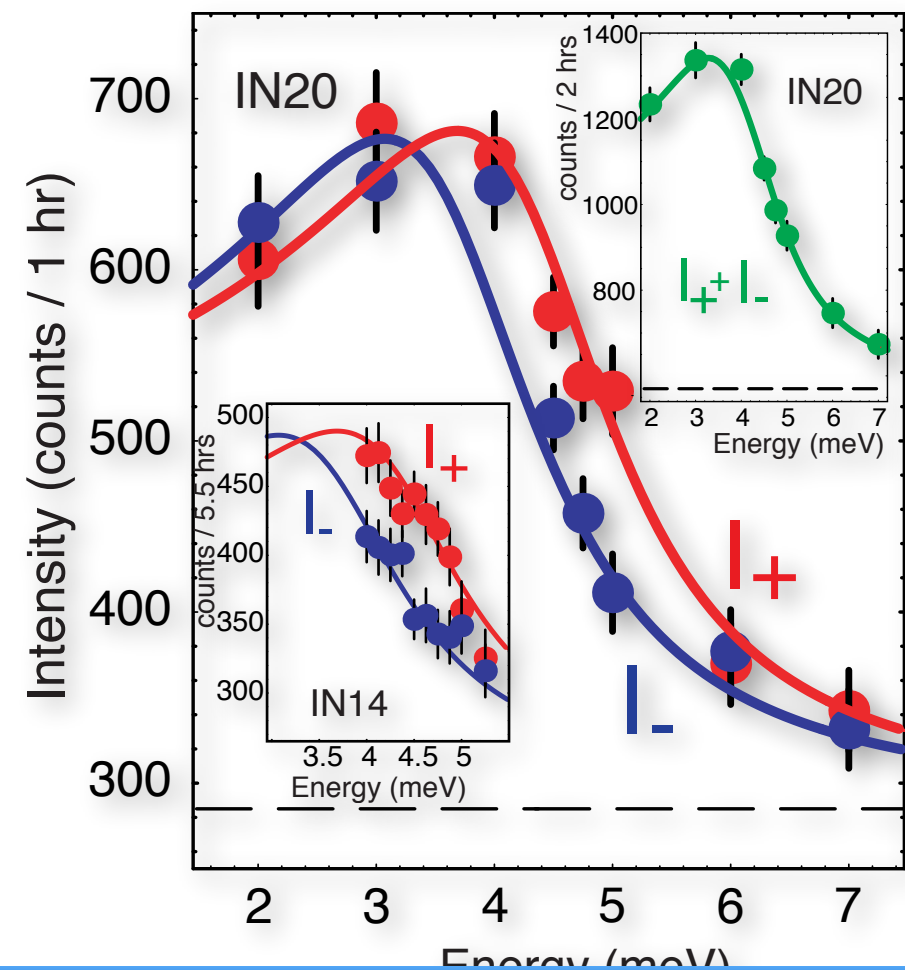
First detection of (chargeless) spin currents due to solitons:

HBB et al. *Nature Phys.* **1**, 159 ('05)

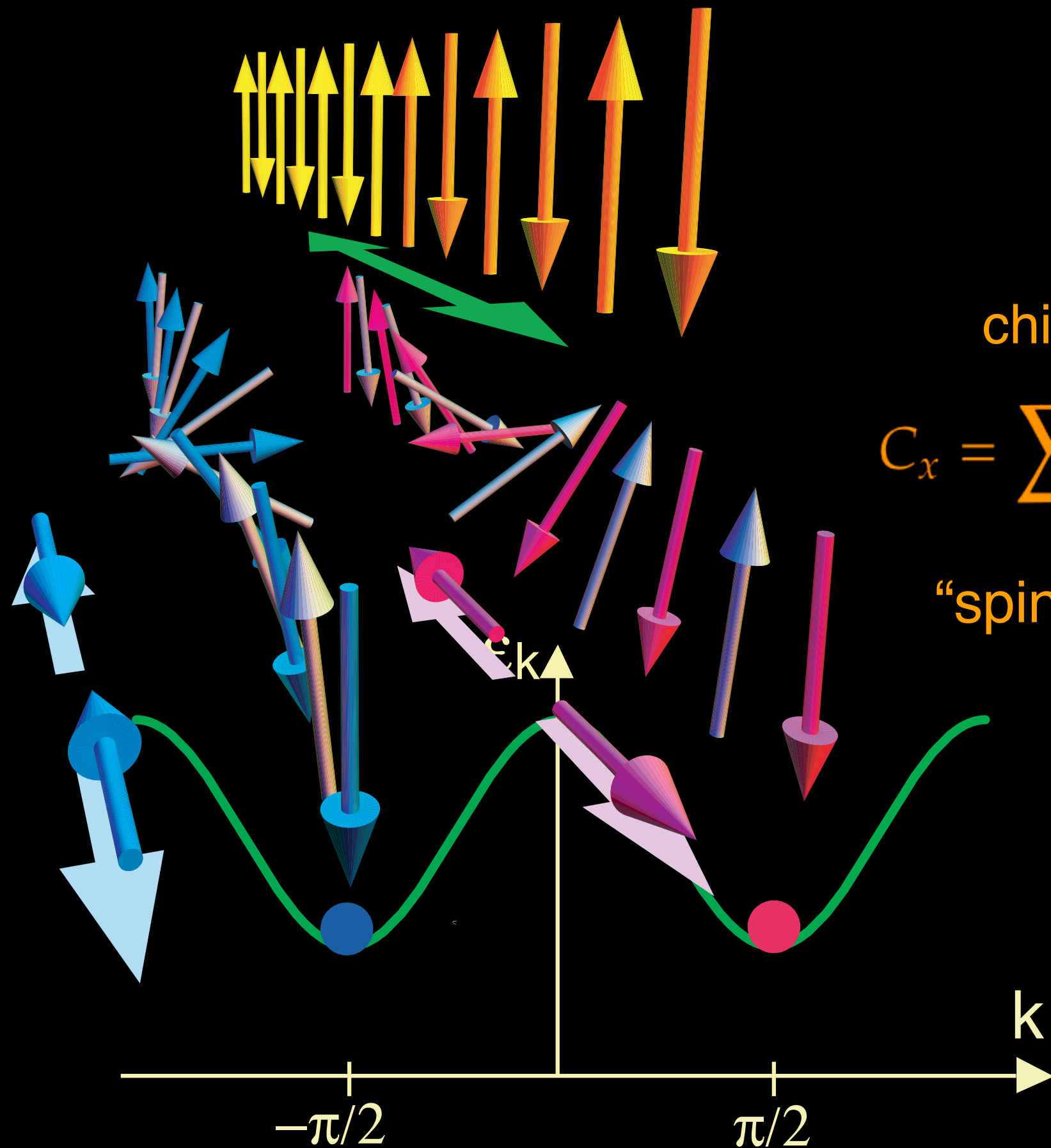
theory



expts (ILL)



Emergence of soliton chirality



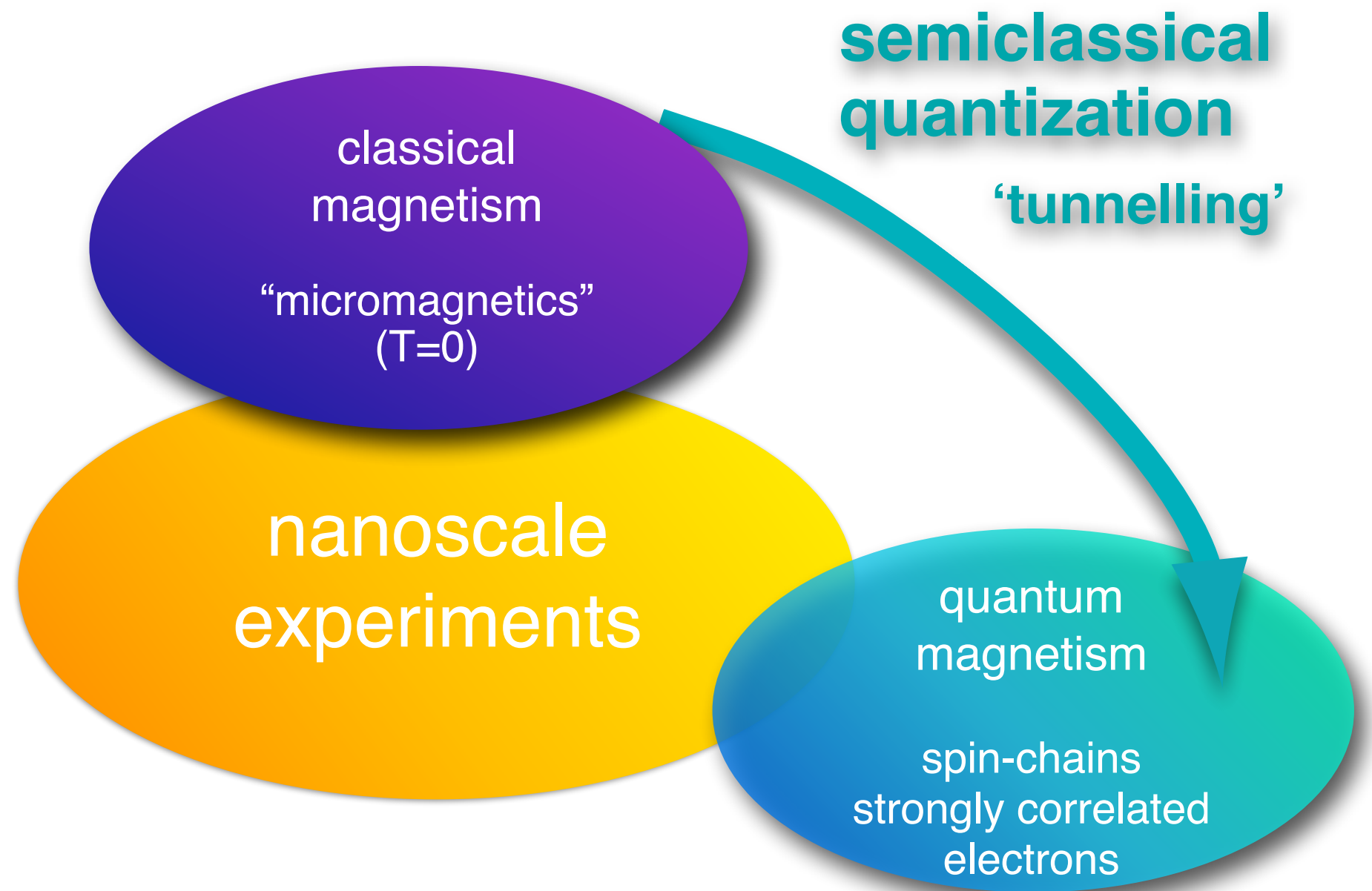
chirality

$$C_x = \sum_i (\mathbf{S}_i \wedge \mathbf{S}_{i+1})_x$$

“spin current”

cf. R. Jansen,
this School
(spin currents
with charge)

Theoretical descriptions of magnetism

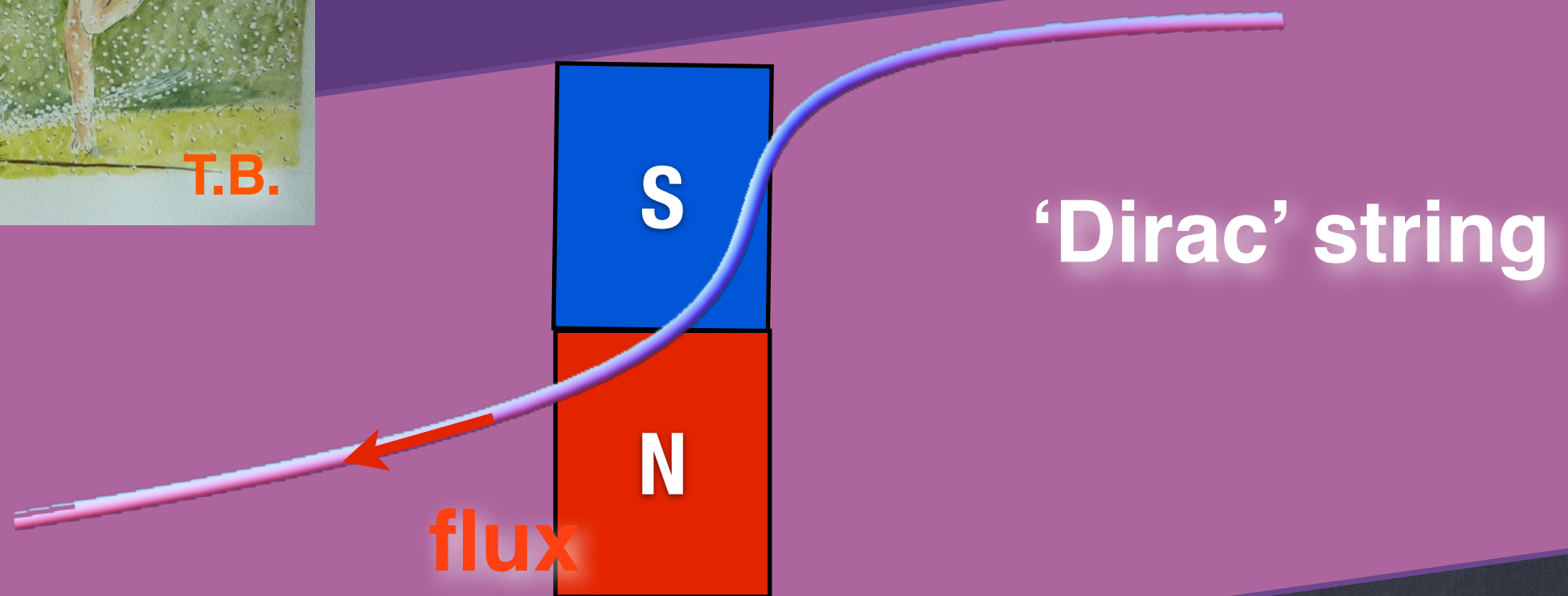


IV. Dipolar interactions in nanomagnetic arrays

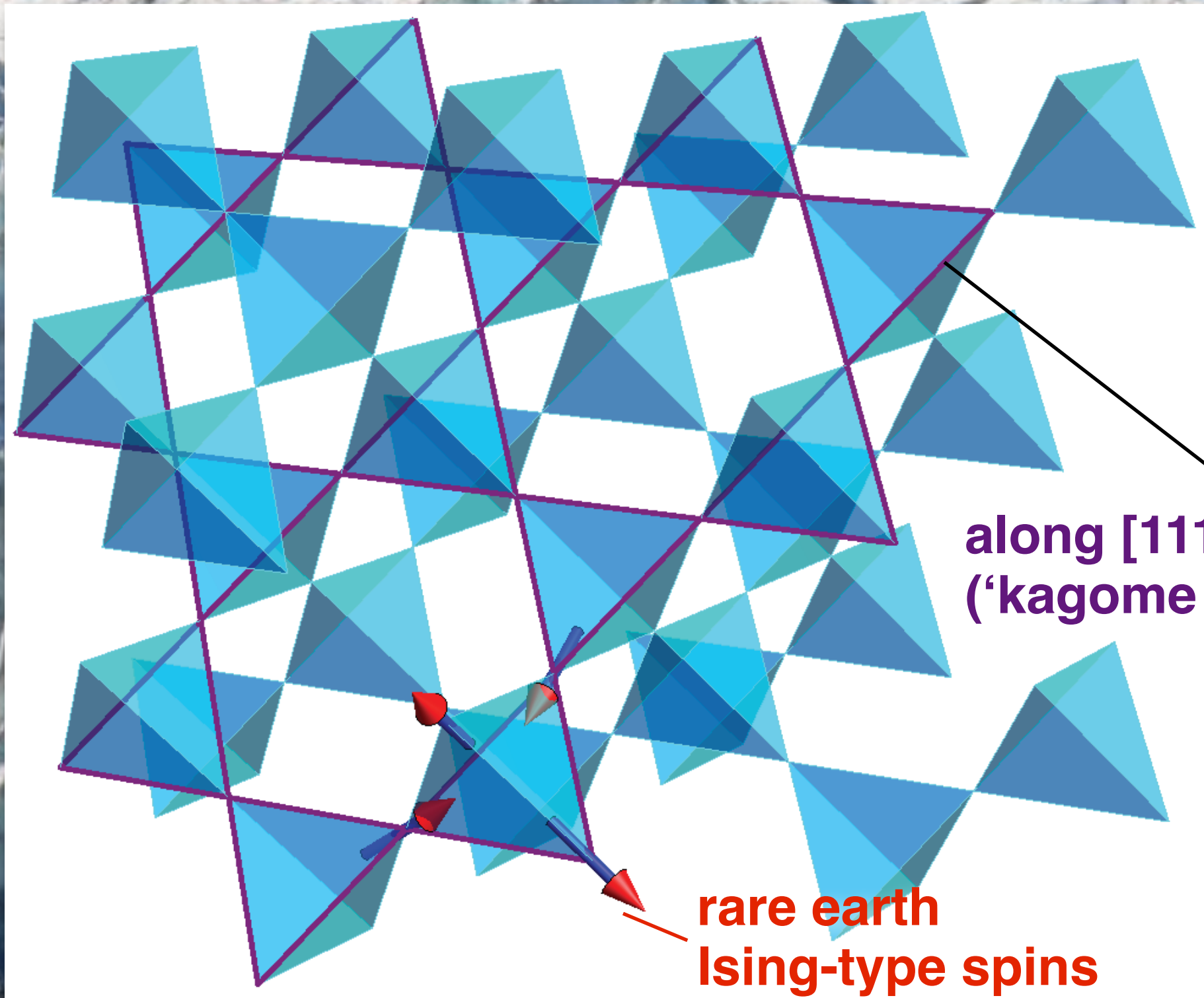
Emergent 'monopoles' & 'Dirac strings' in pyrochlore spin ice

Emergent 'monopoles' and Dirac string avalanches in artificial spin ice - nanolithographic arrays of nanomagnets

Magnetic Monopoles - Can they exist as emergent quasiparticles?



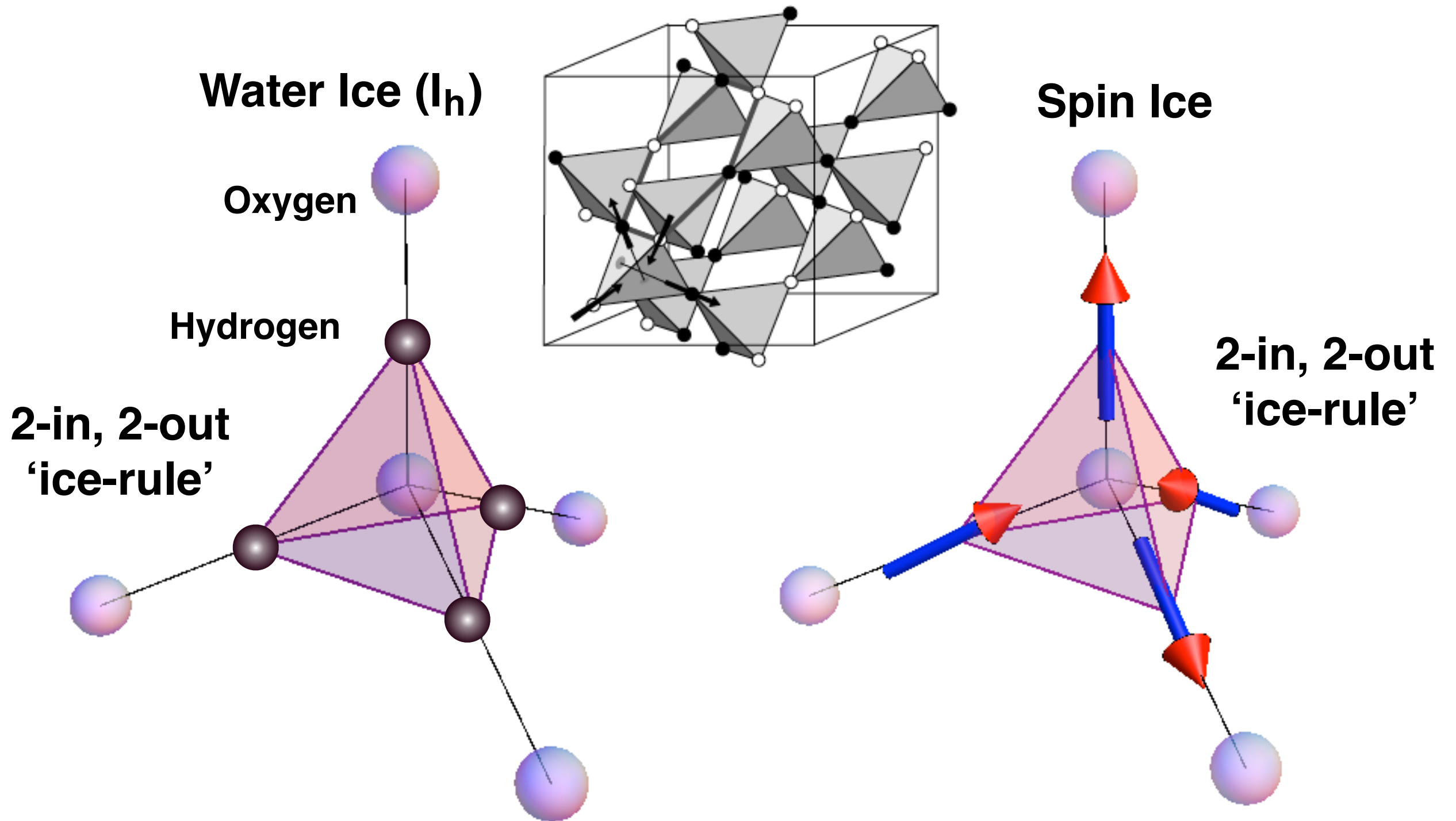
Pyrochlore spin-ice



along [111] direction
(‘kagome lattice’)

rare earth
Ising-type spins

Why 'spin-ice' ?

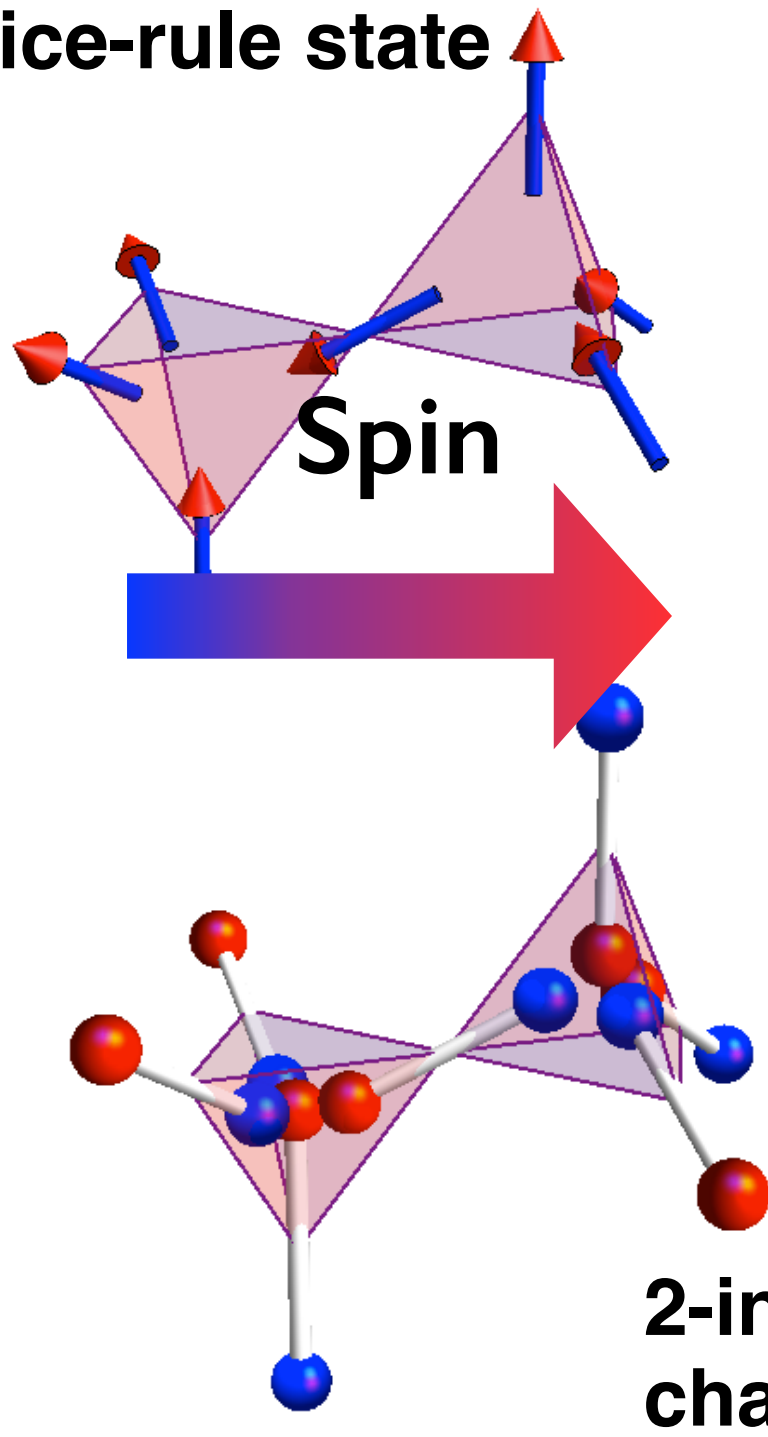


Excitations - dipoles as charge dumbbells

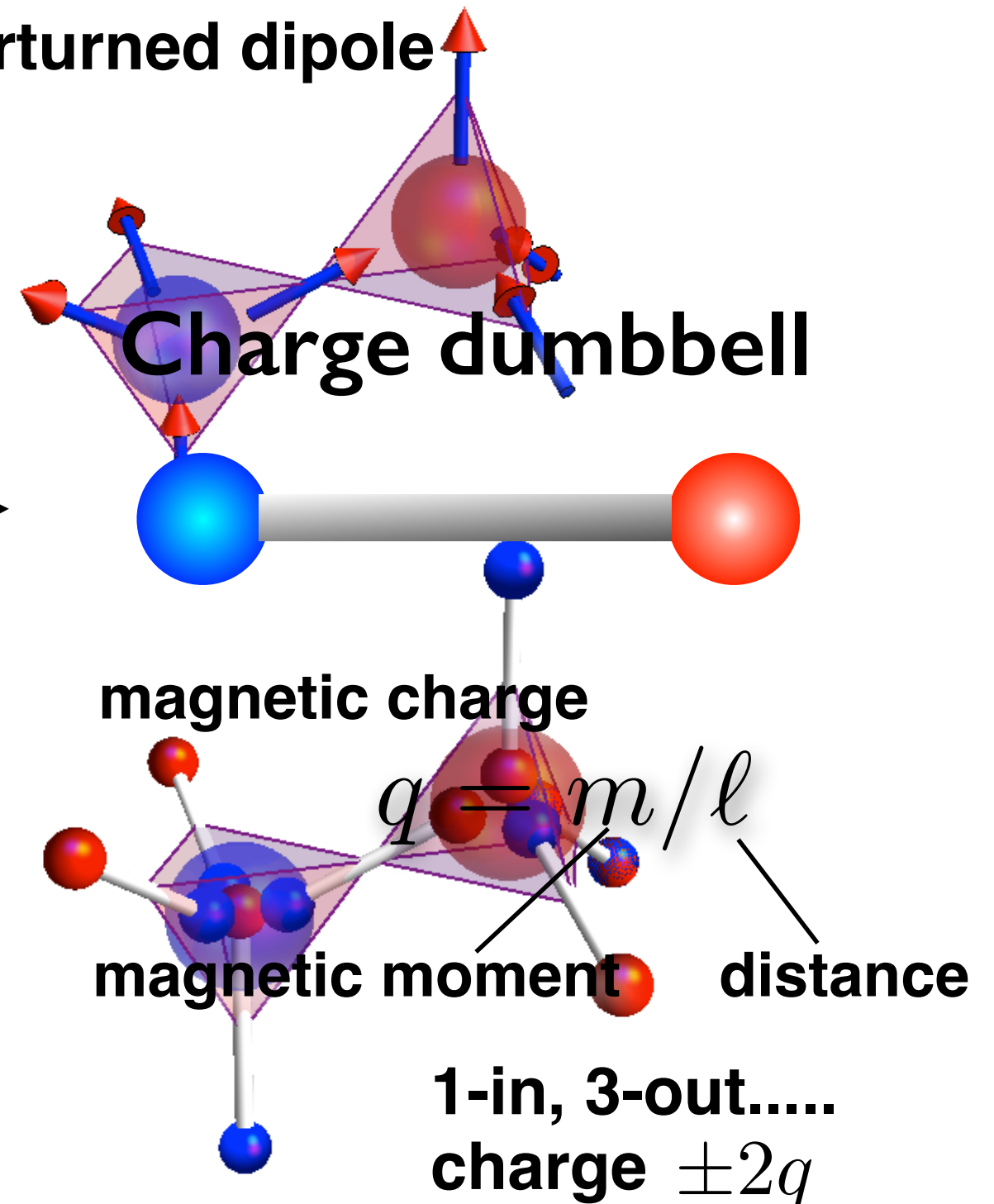
Castelnovo, Moessner, Sondhi, Nature ('08)

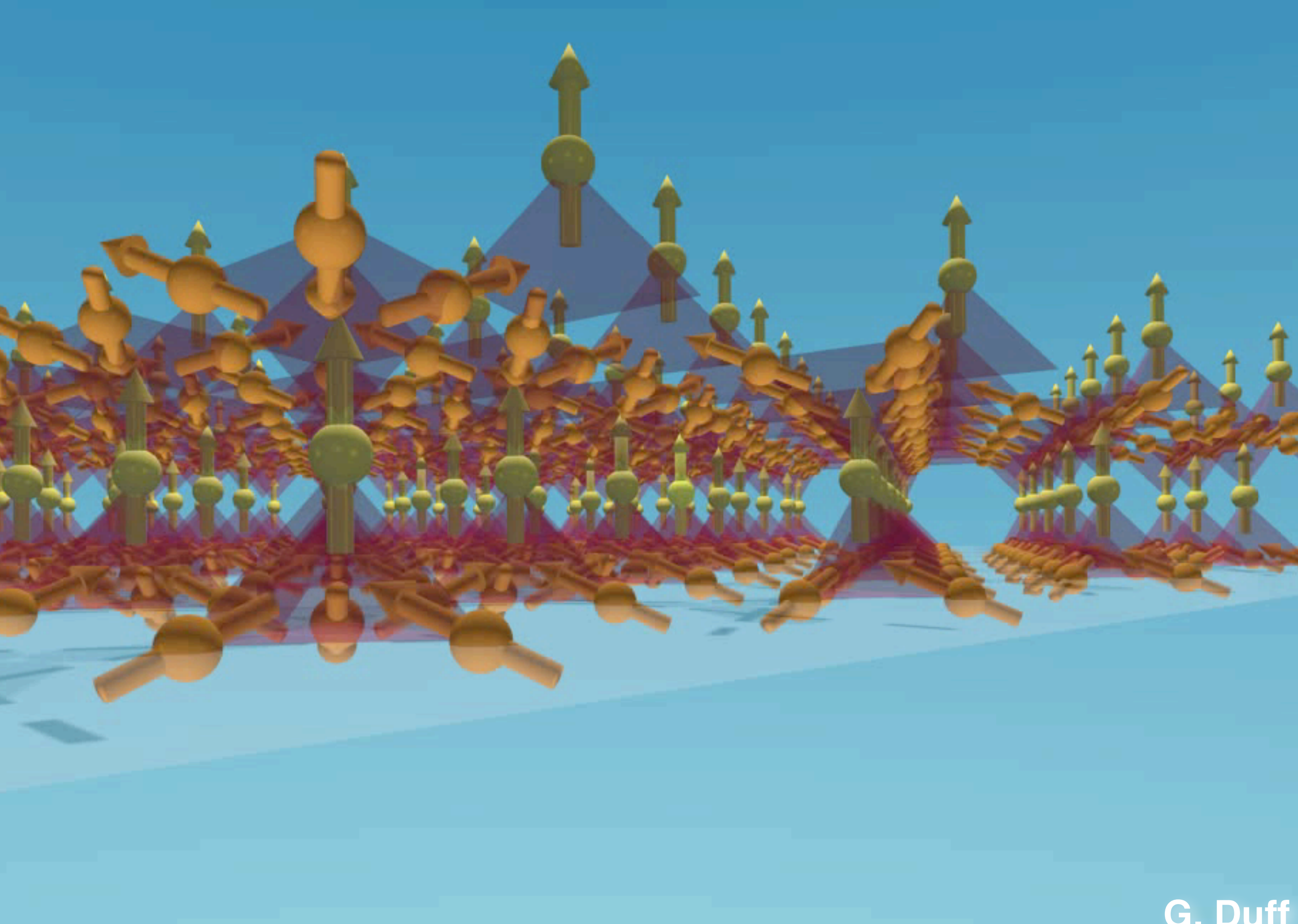
Ryzhkin ('05)

ground state
ice-rule state

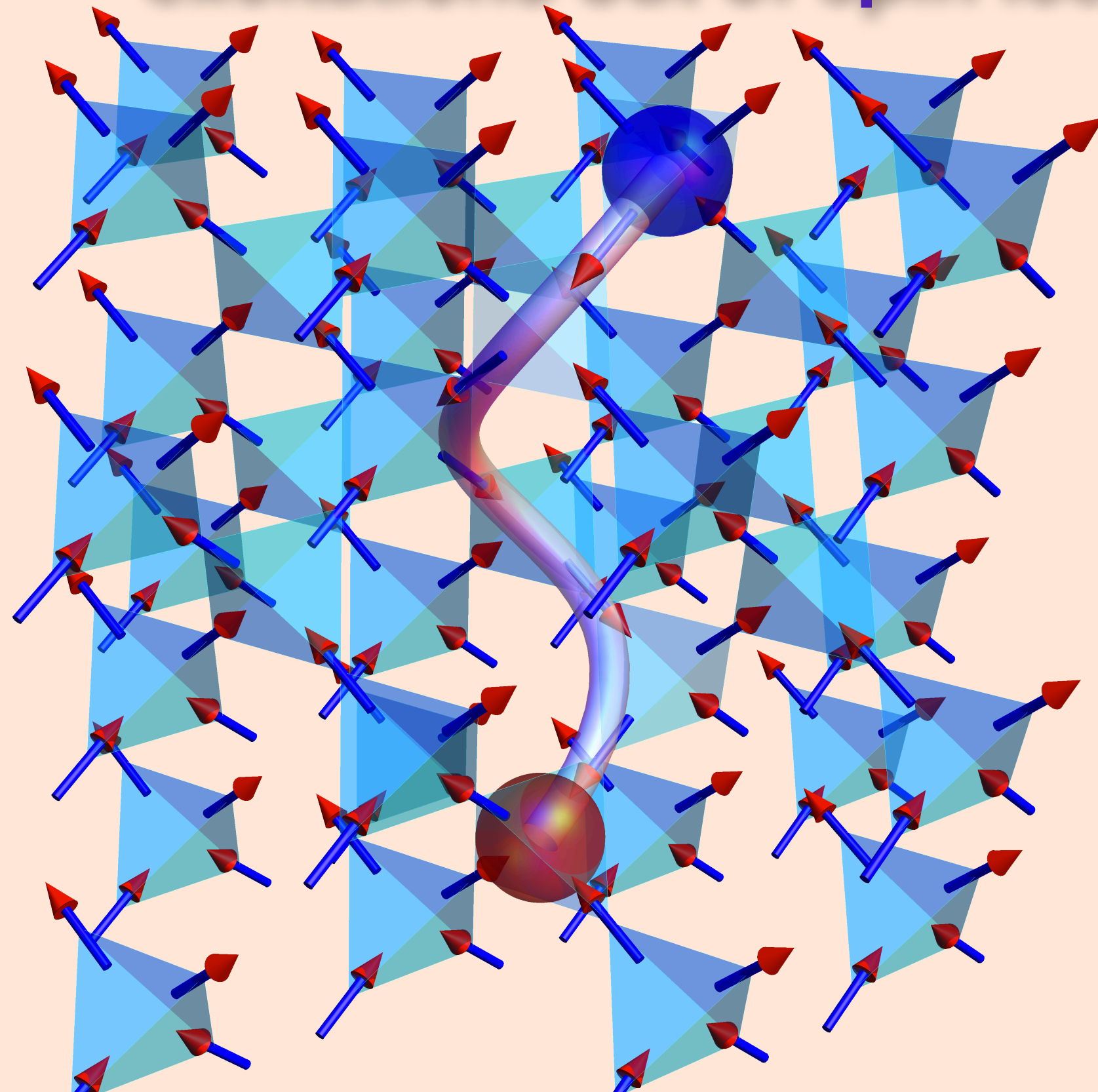


excited state
overturned dipole





Monopoles and (unquantized) Dirac strings as excitations out of spin ice ground state

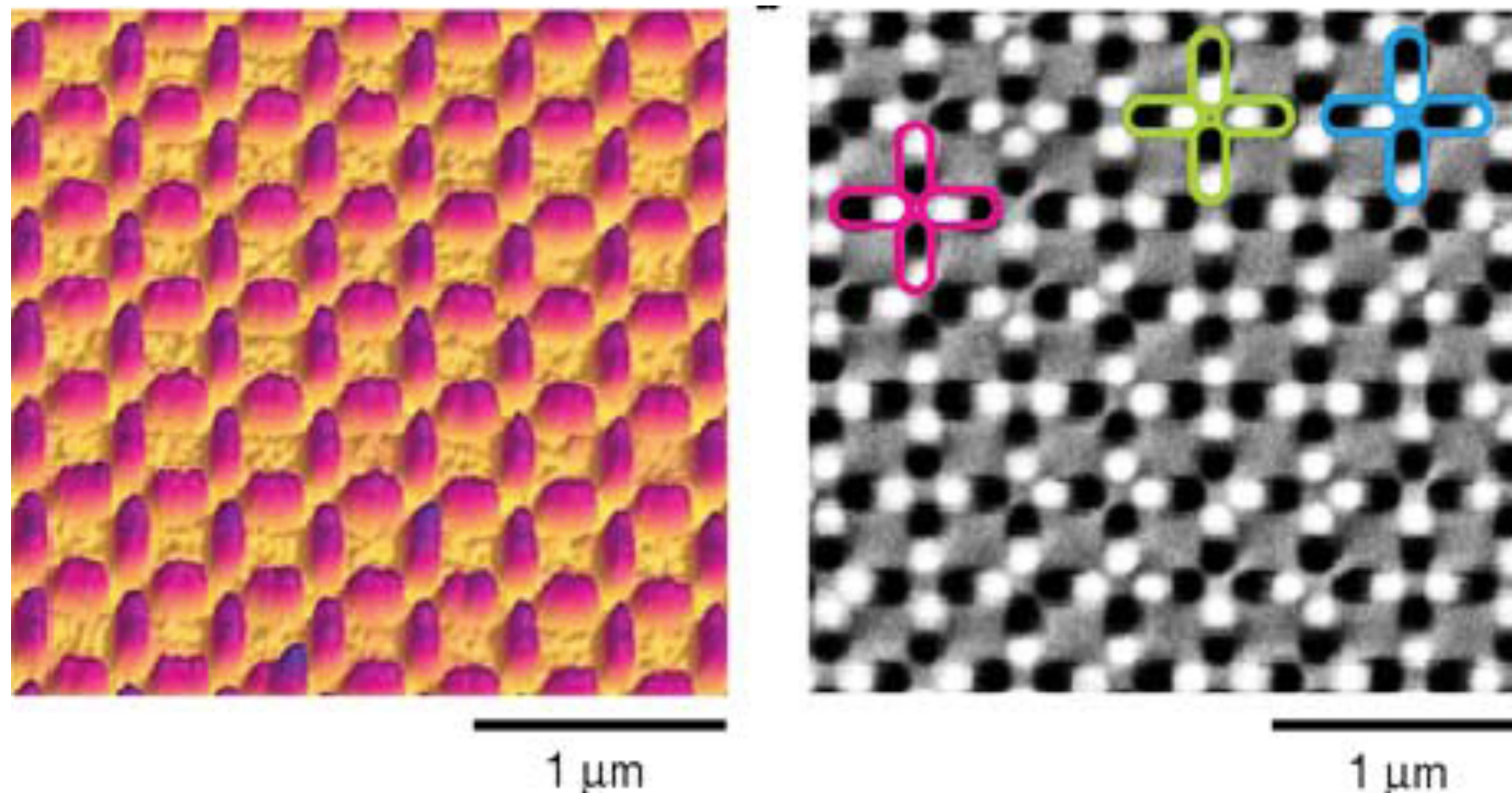


Neutron scattering expts:
Morris et al, Science ('09)
Kadowaki et al. ('09)
Fennell et al. ('09)

$$T \lesssim 1\text{K}!$$

Low T and reciprocal space - can one do better?

Artificial spin ice - dipolar coupled array of isolated nanoislands

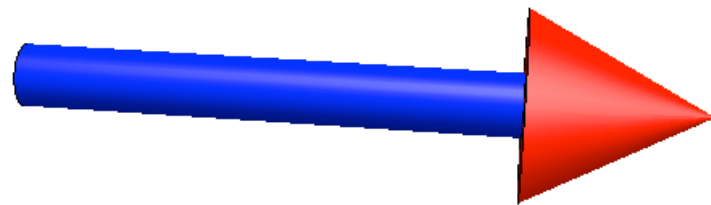


Wang et al. Nature ('06)

Isolated nanoislands as macrospins

Magnetic moments in (artificial) spin ice

Pyrochlore spin ice

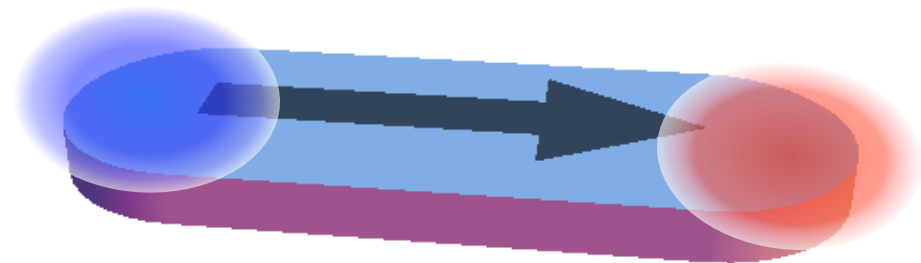


rare earth spin

moment m



Artificial spin ice



permalloy island
'macrospin'

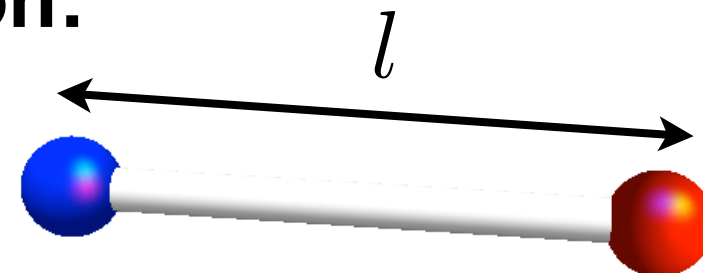
effective description:



charge
dumbbell



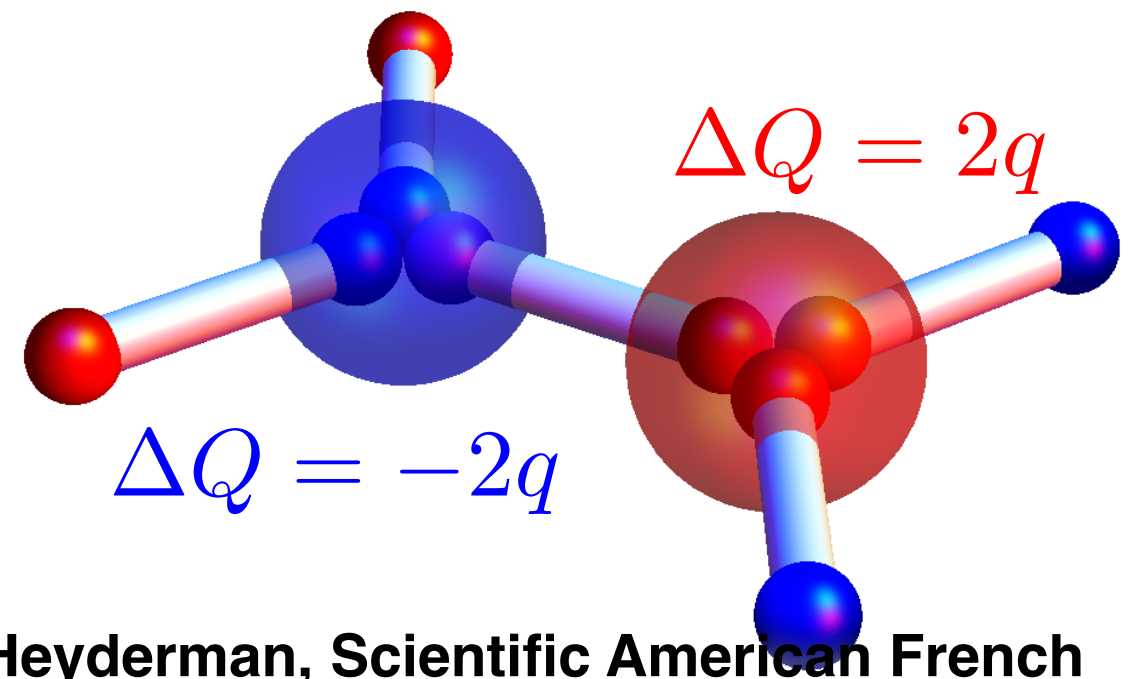
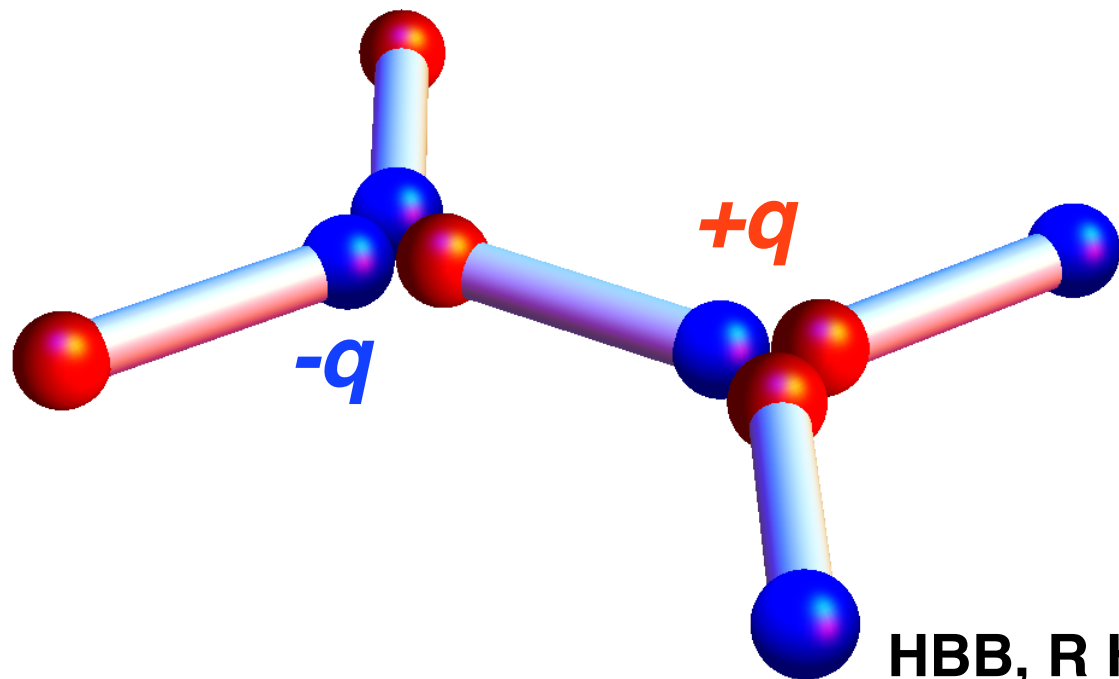
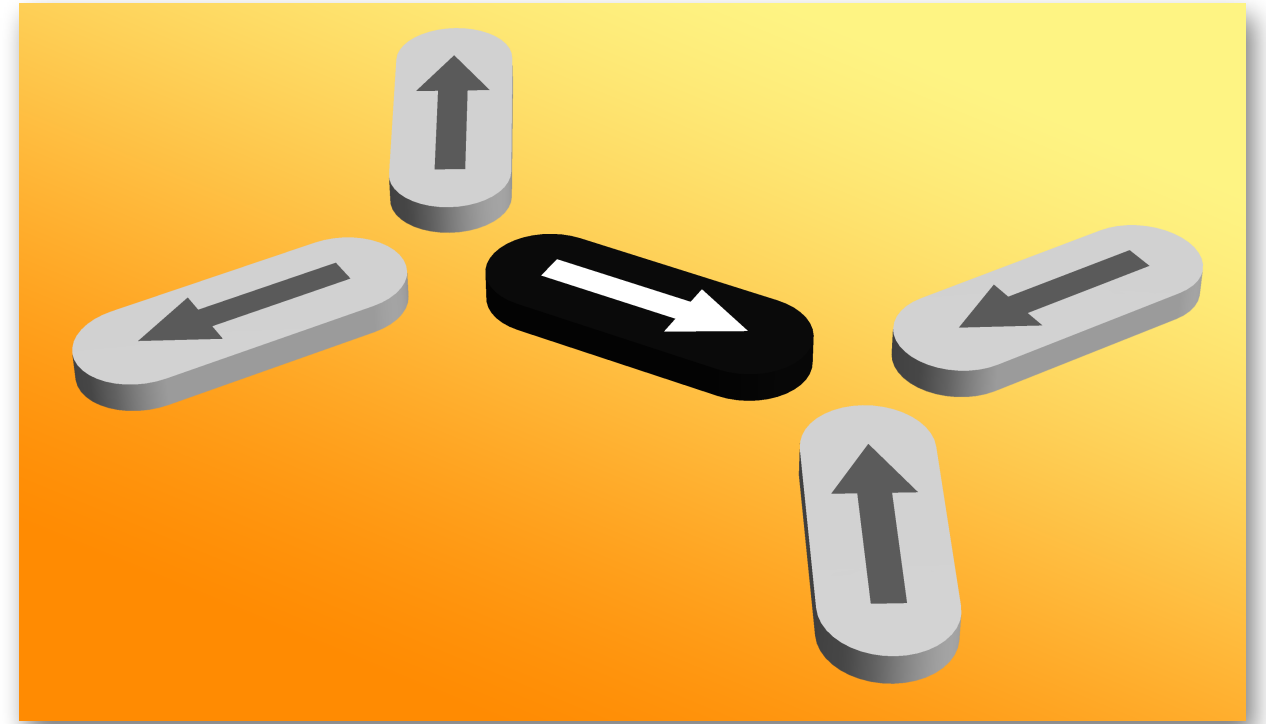
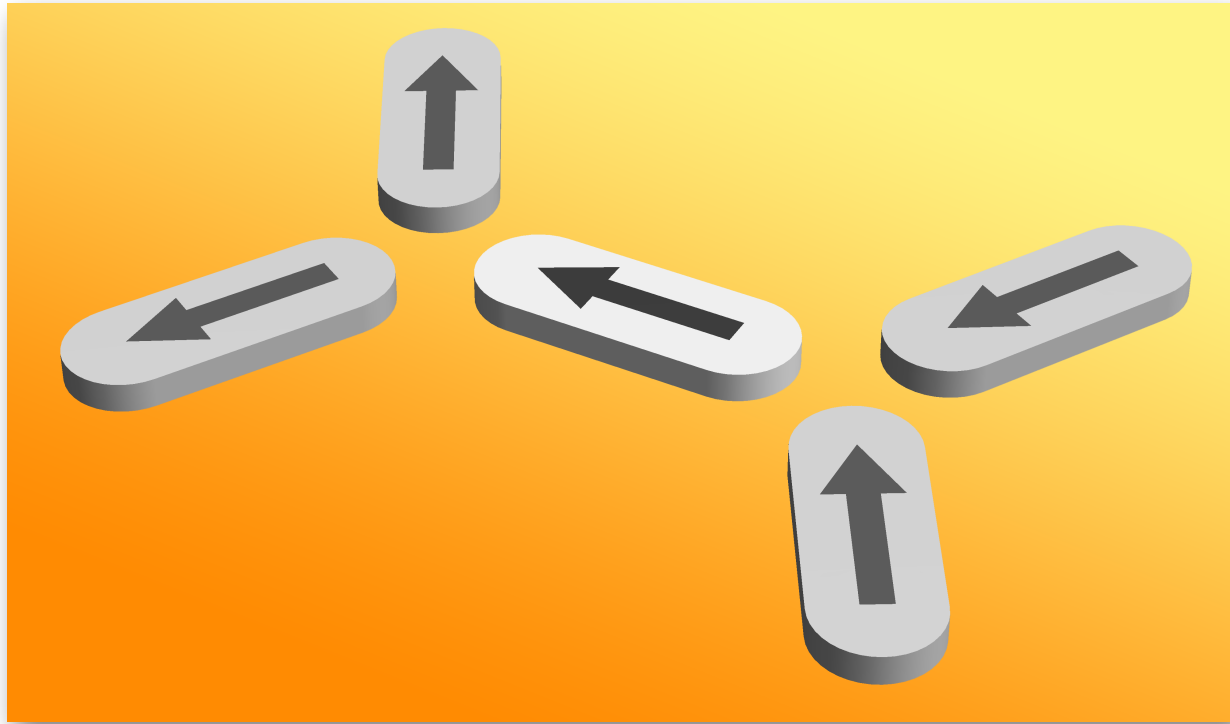
charge



charge
dumbbell

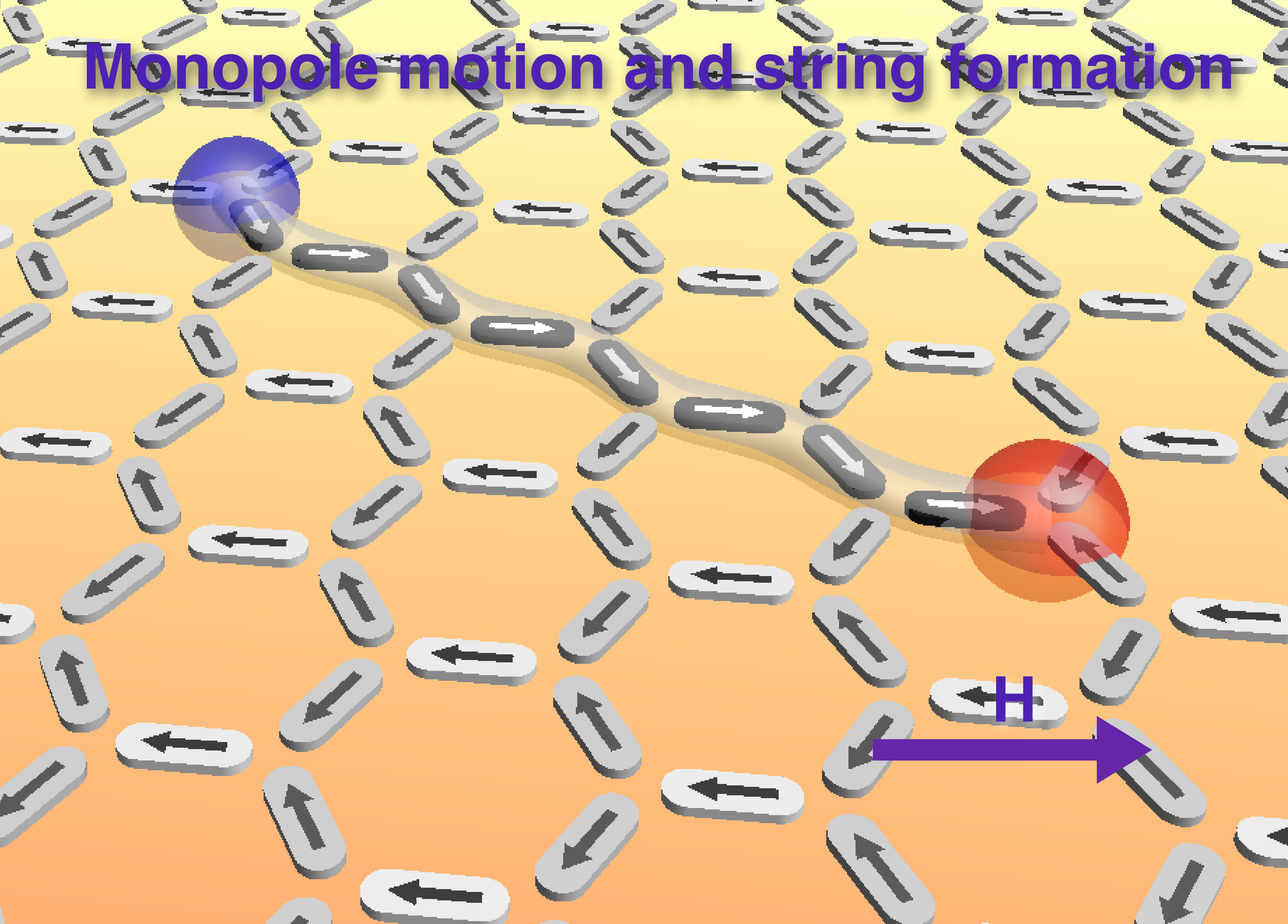
$$q = m/l$$

Dumbbell picture



HBB, R Hügli, L Heyderman, Scientific American French Edition ('Pour La Science'), April-June ('13)

Monopole motion and string formation



PSI

PAUL SCHERRER INSTITUT

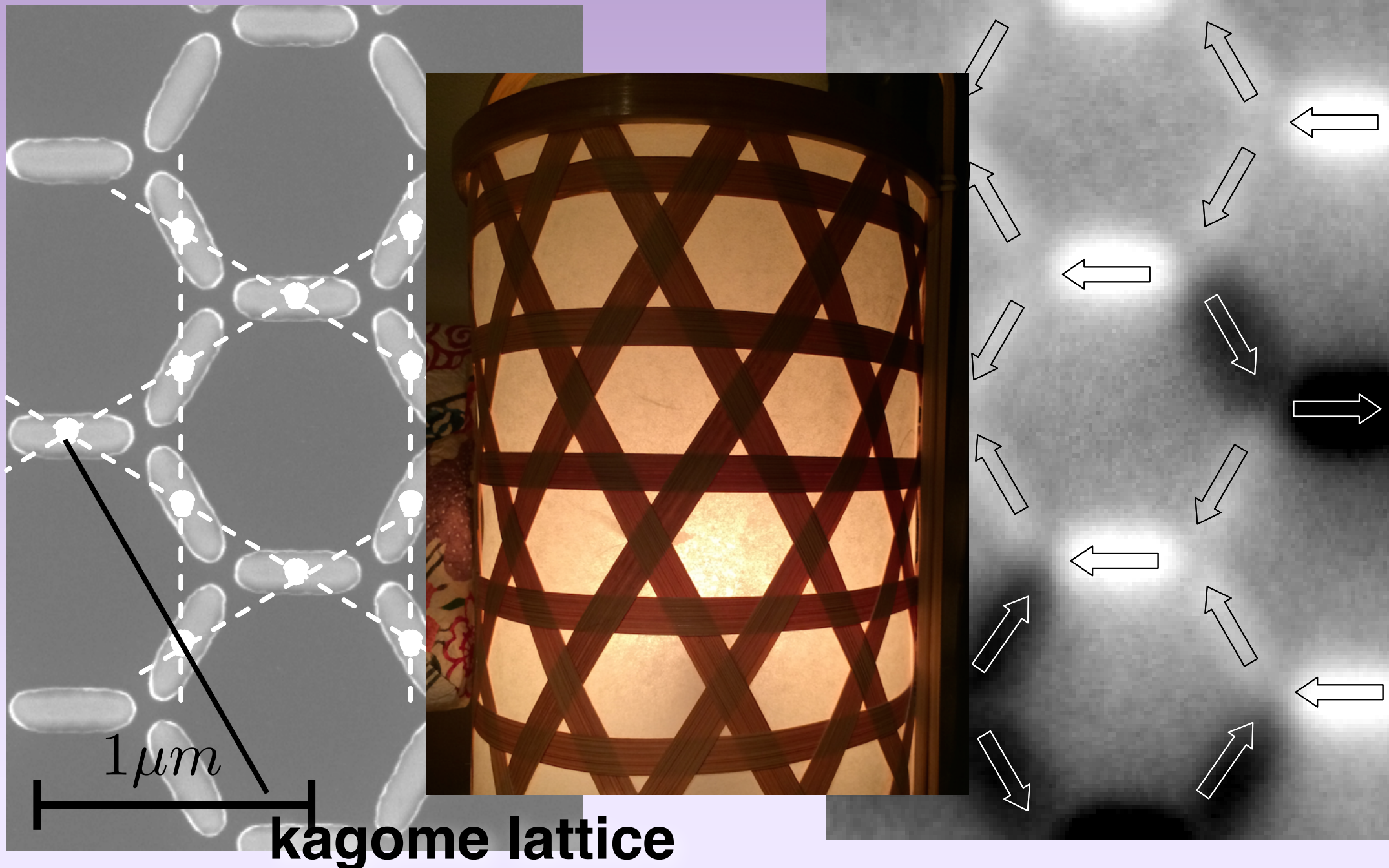


Islands on a kagome lattice

with L. Heyderman, F. Nolting, R. Hügli, G. Duff

SEM image

PEEM image
(SLS)



kagome lattice

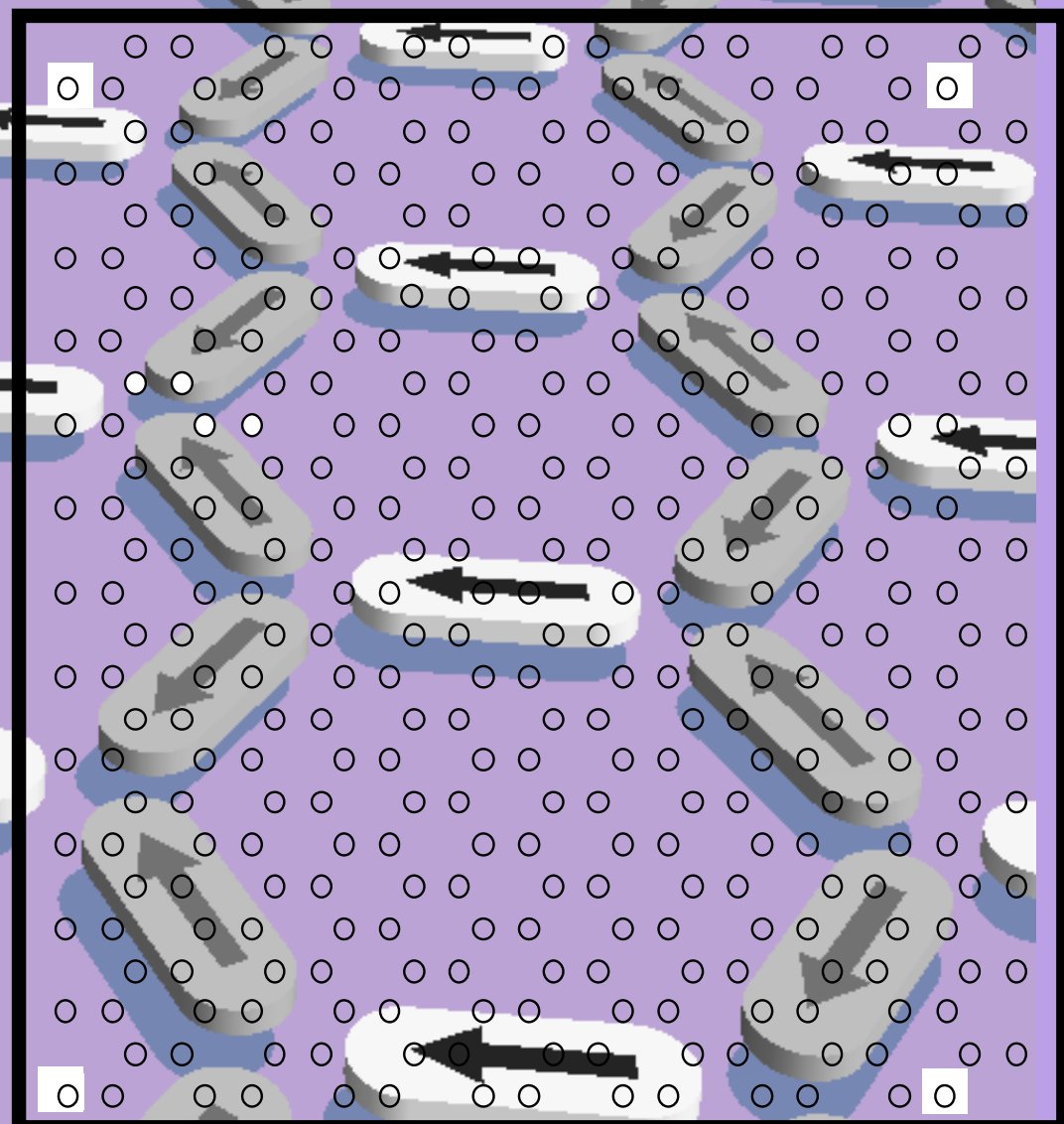
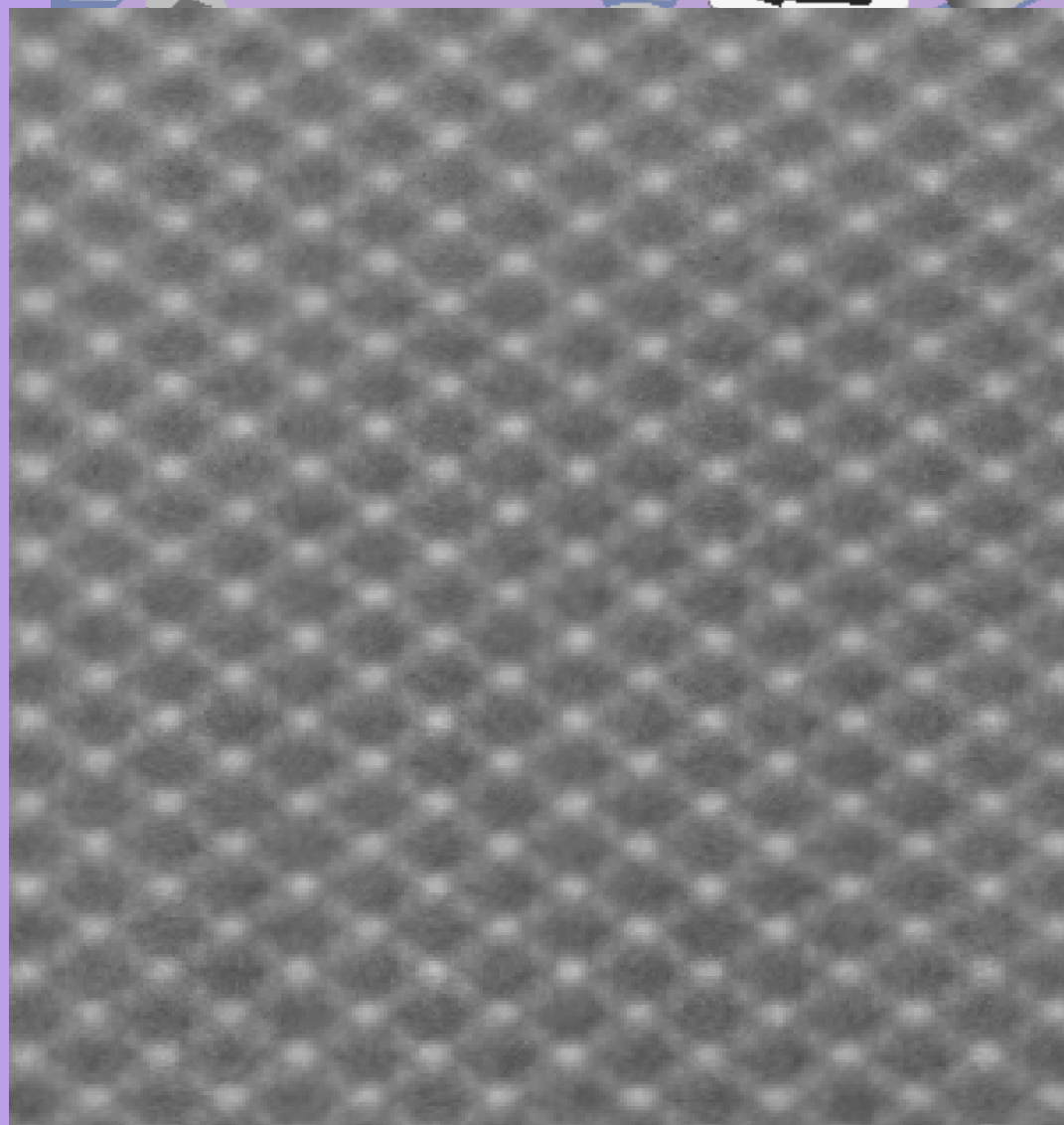
contrast depends on orientation

Initial saturation

$$H < 0.82 H_c$$

PEEM image

Charge map

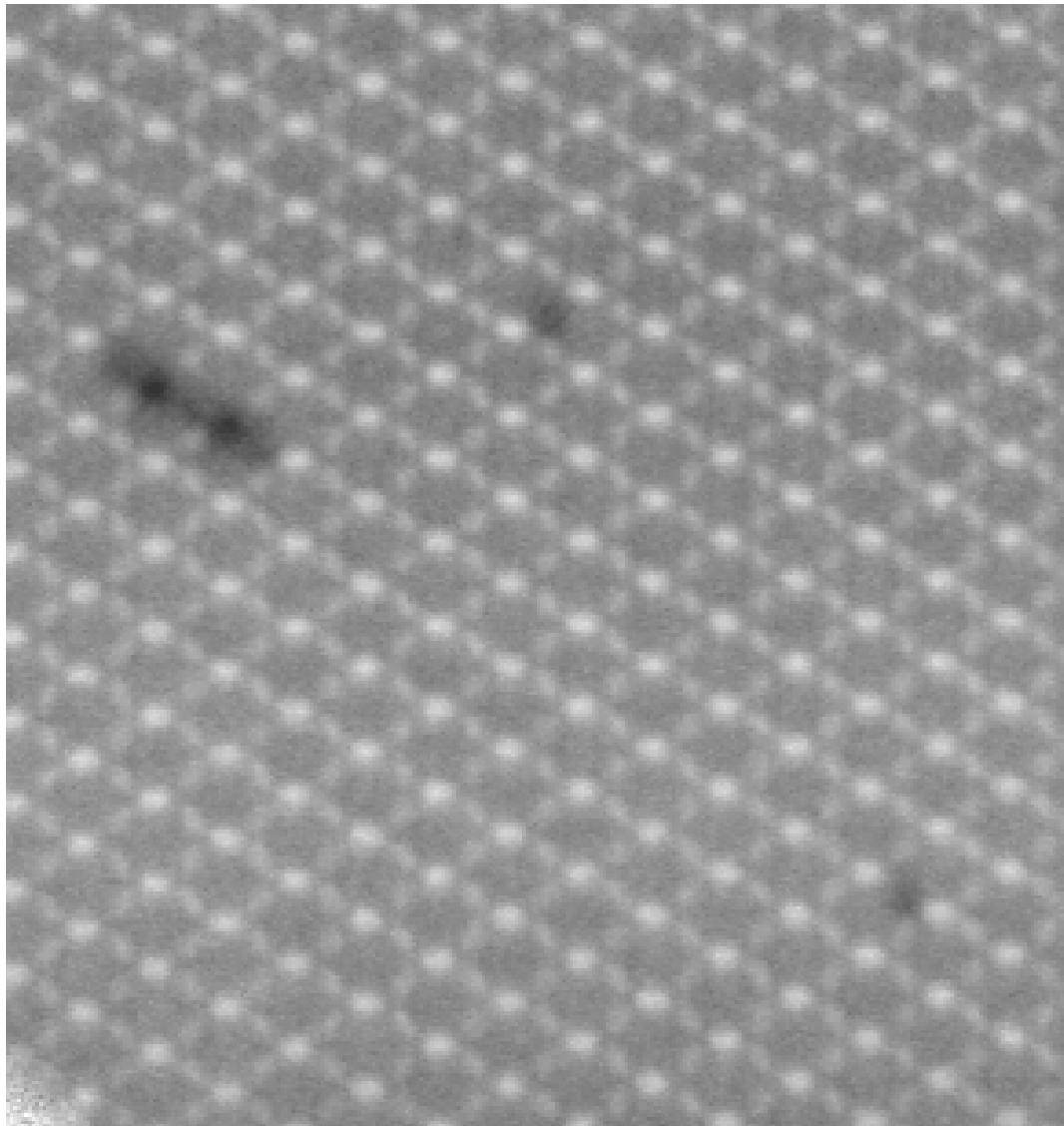


ΔQ map

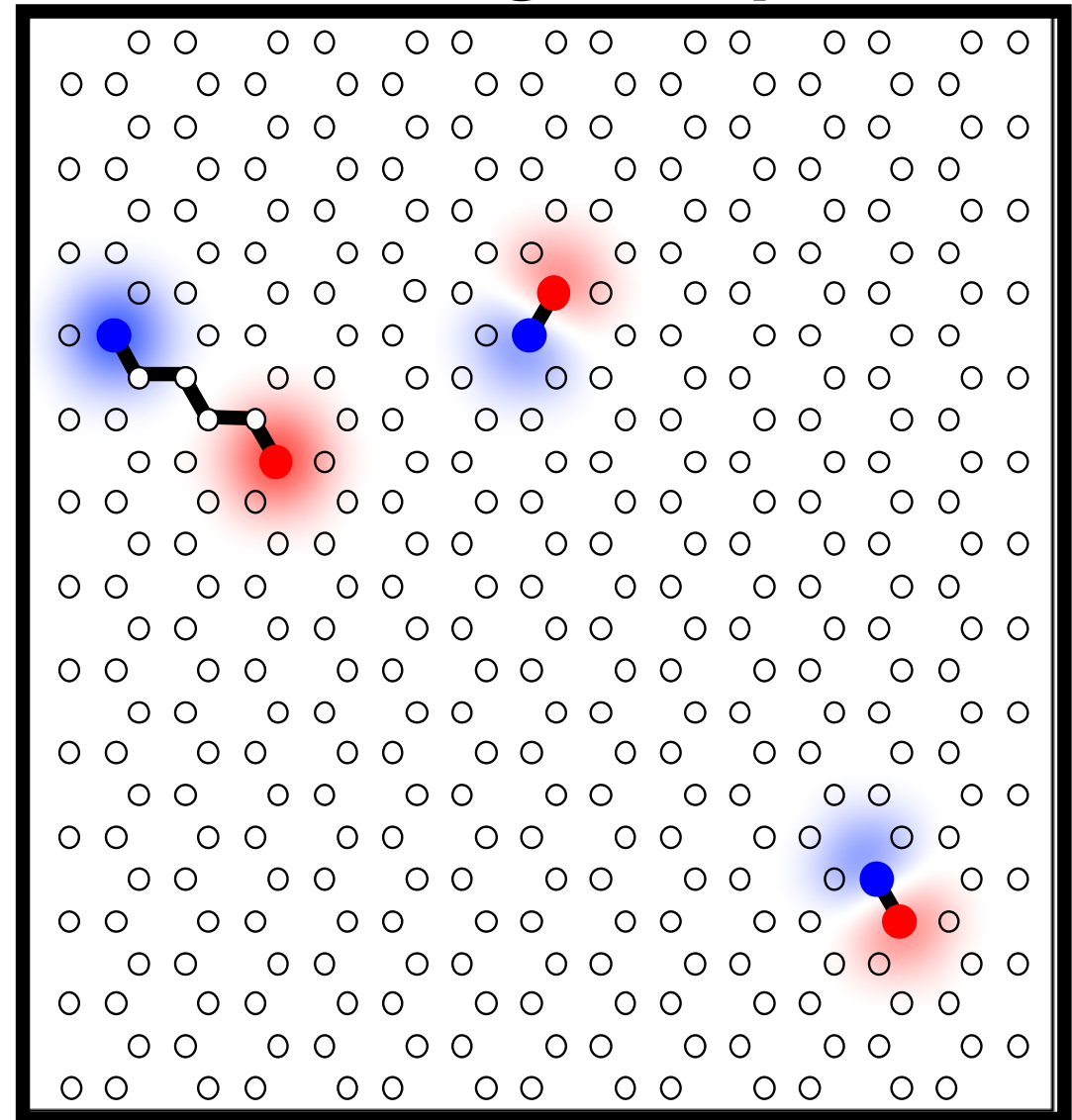
H=0.85 H_c

E. Mengotti *et al.*, Nature Phys. 7, 68 (2011)

PEEM image



charge map



smearred charge density ('MFM')
Gaussian

$$\rho_m(\mathbf{r}) = \int d^2r' f_G(\mathbf{r} - \mathbf{r}') \rho_Q(\mathbf{r}')$$

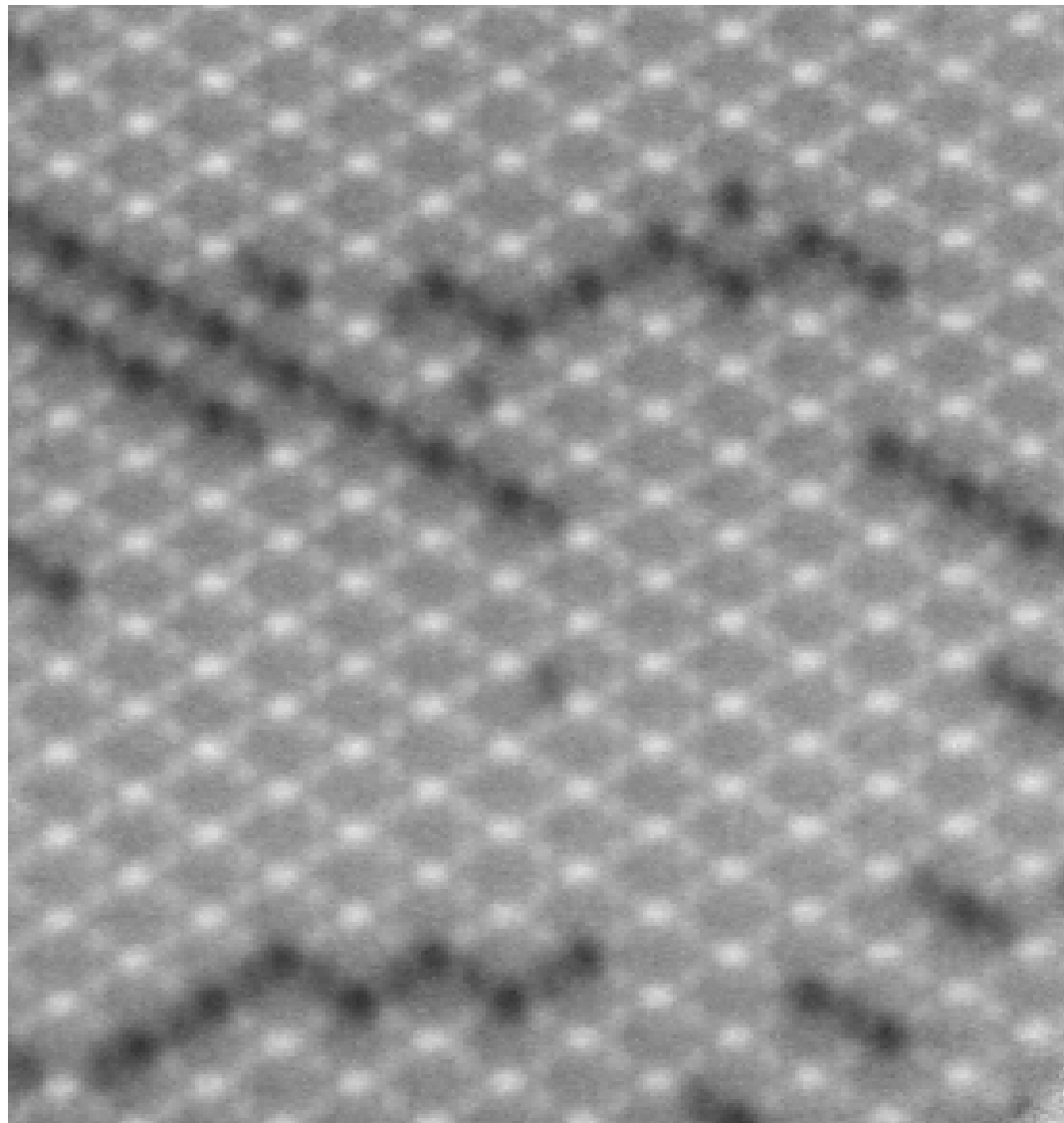
ΔQ

map
total charge

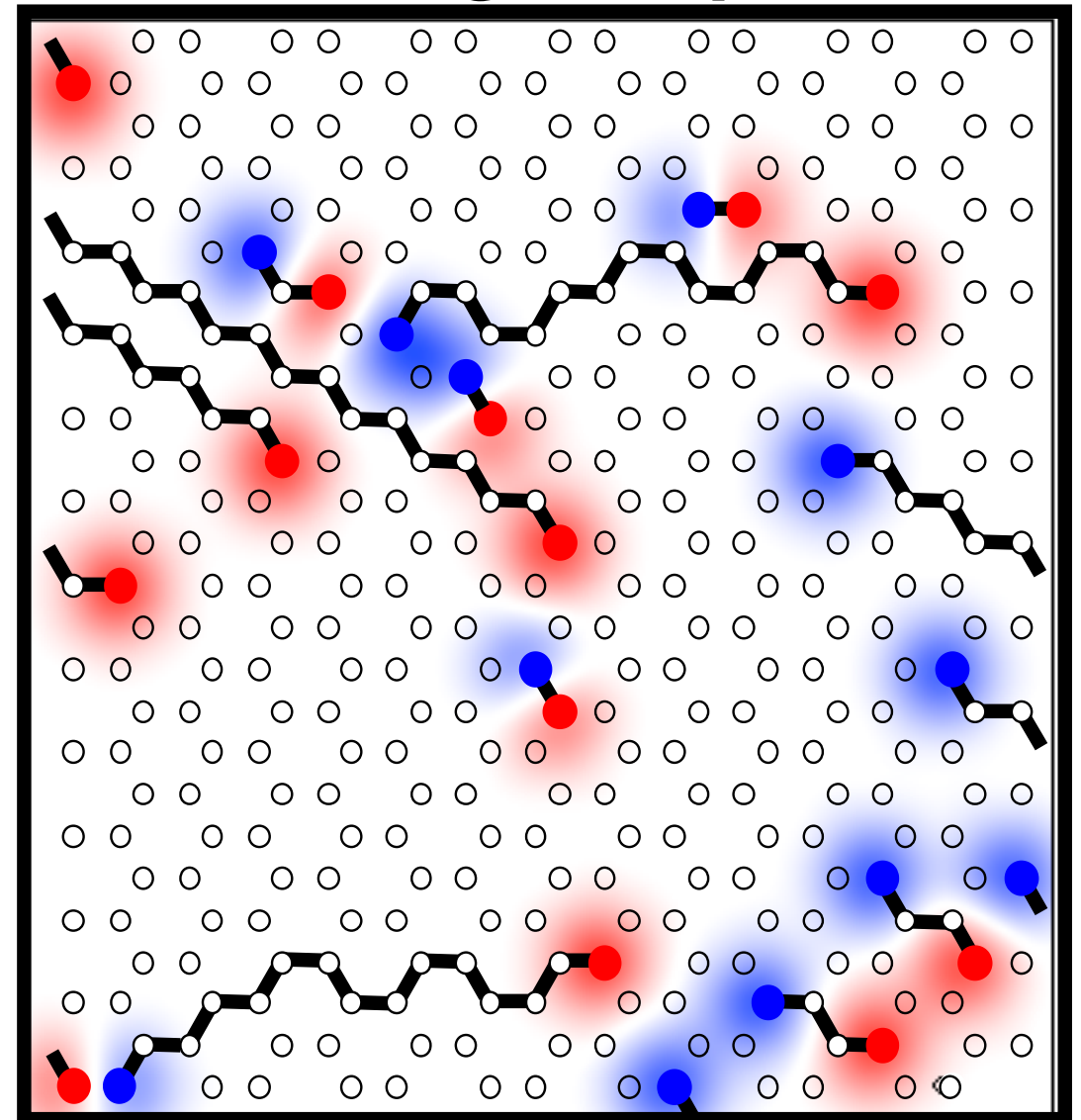
$$\rho_Q(\mathbf{r}) = \sum Q_\alpha \delta(\mathbf{r} - \mathbf{R}_\alpha)$$

$$H=0.92 H_c$$

PEEM image

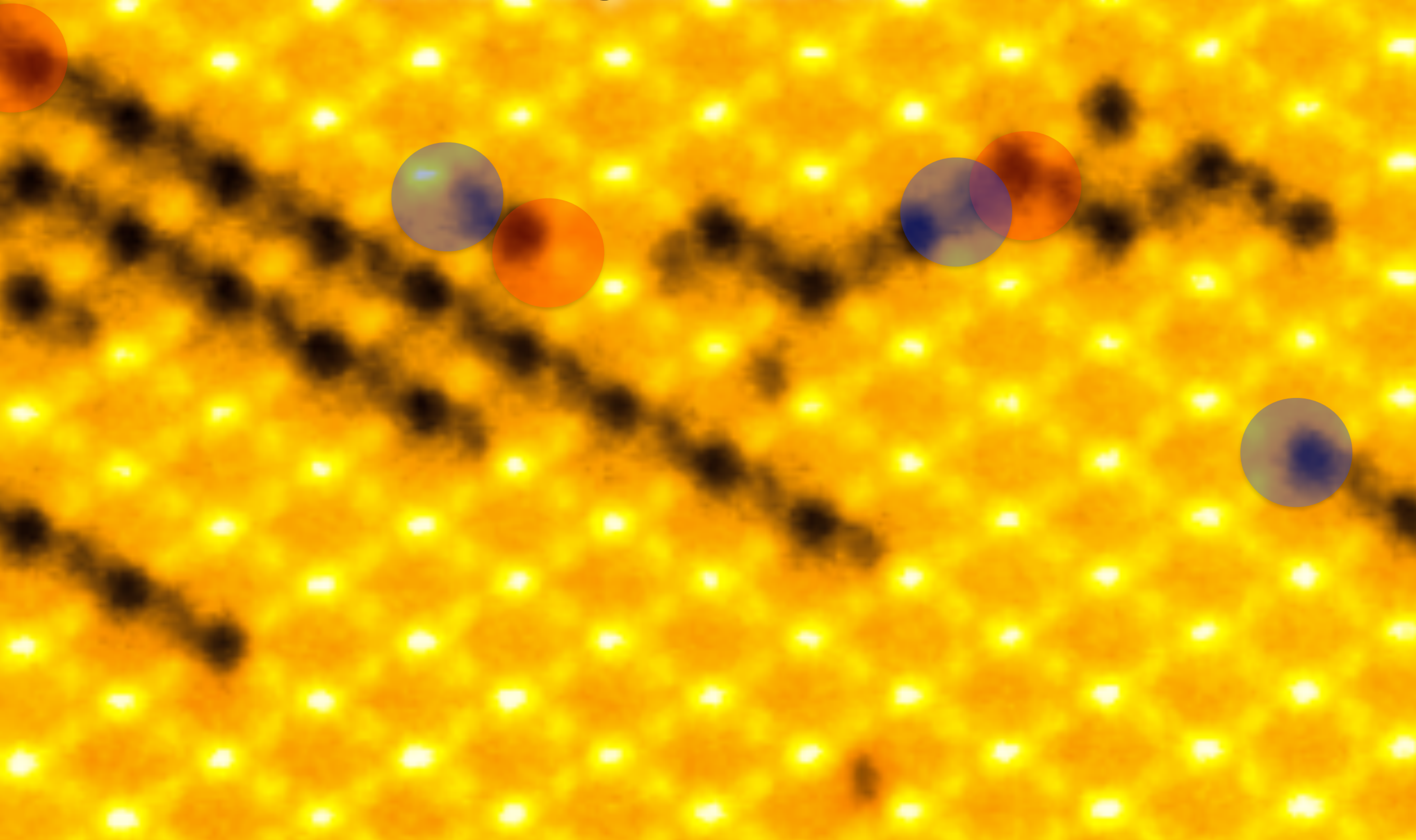


charge map



ΔQ map

XMCD-PEEM Images taken at Swiss Light Source

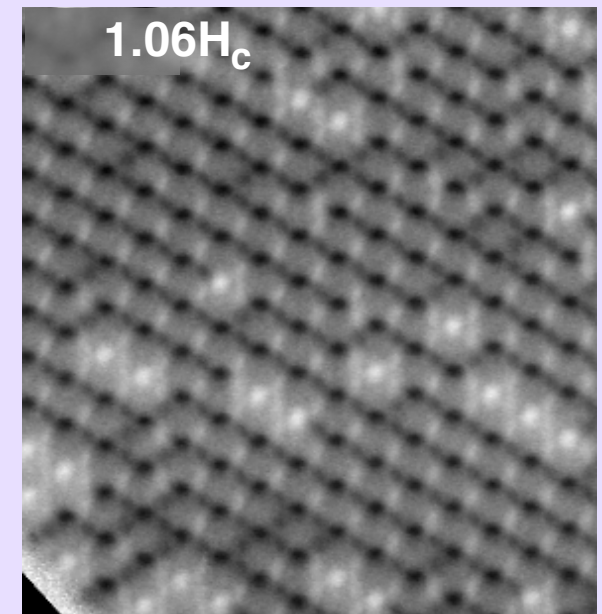
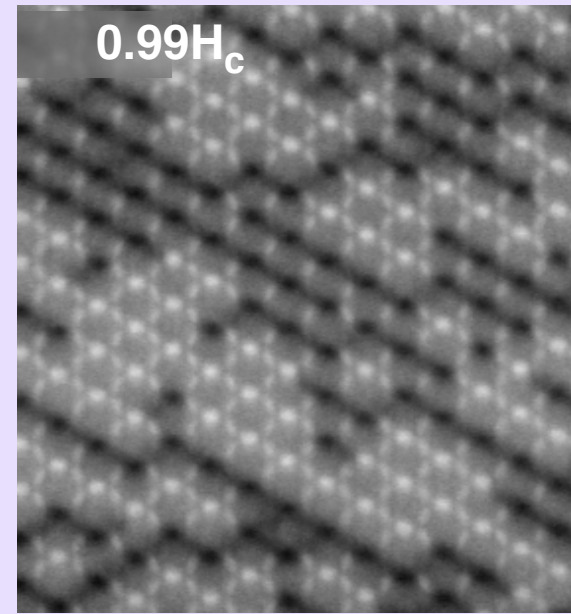
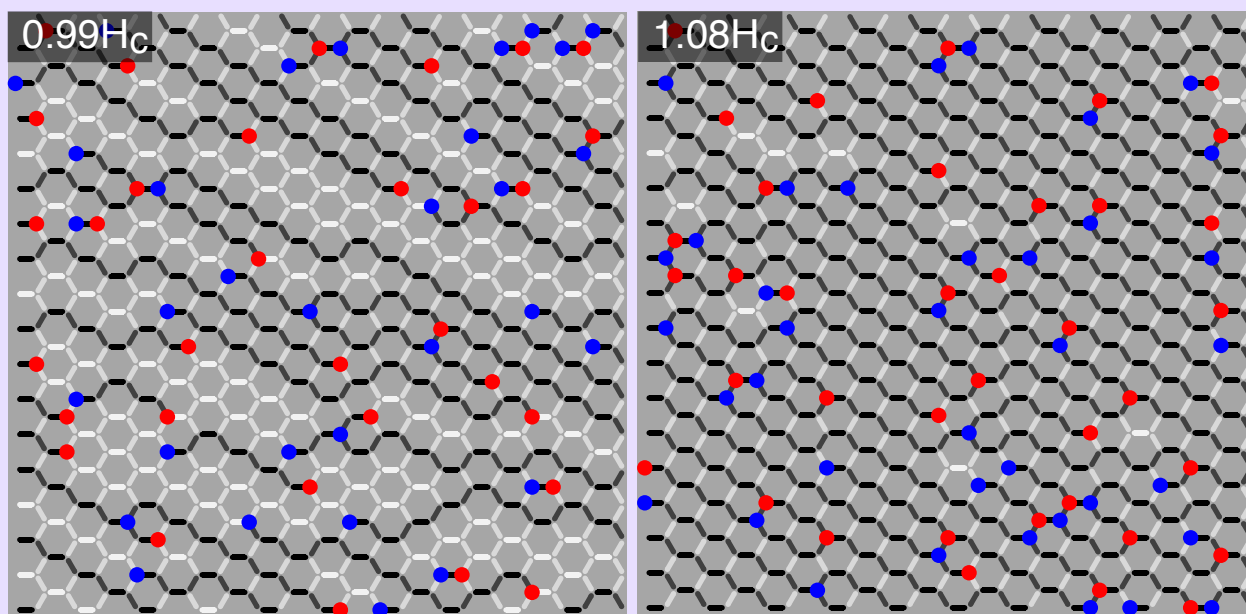
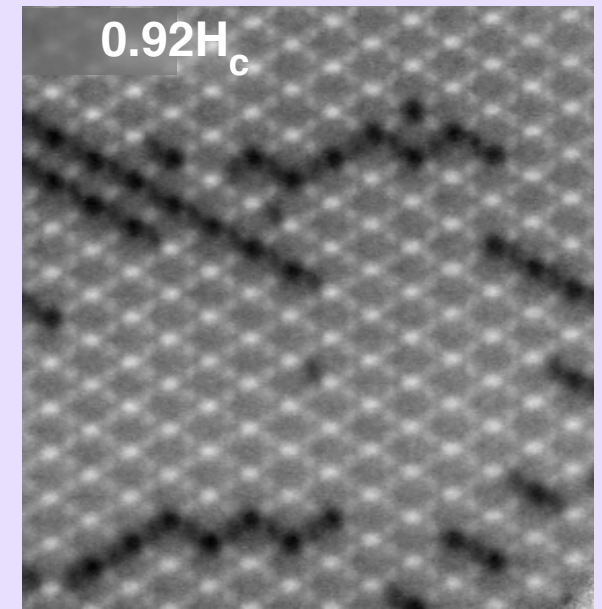
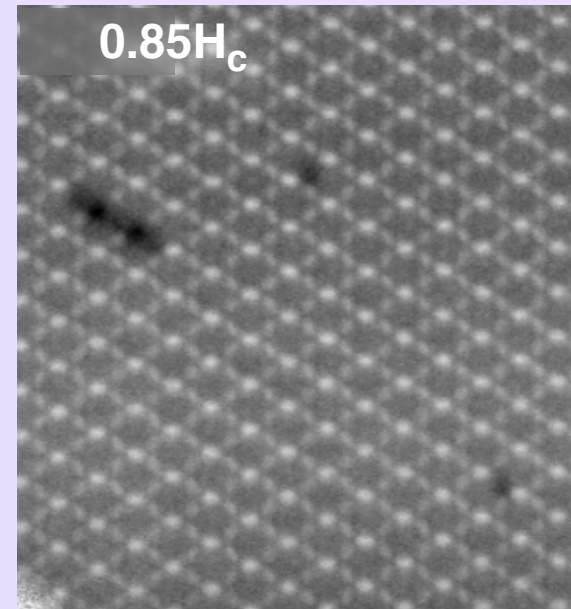
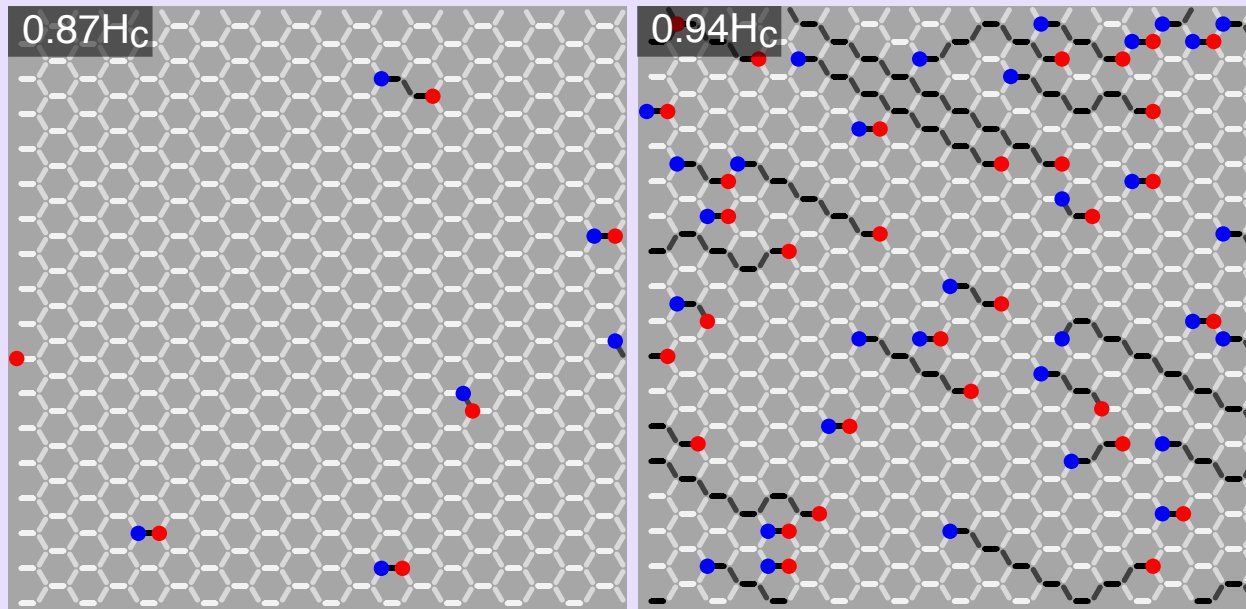


Dirac strings and monopoles

Simulations

E. Mengotti *et al.*, Nature Phys. 7, 68 (2011)

PEEM images

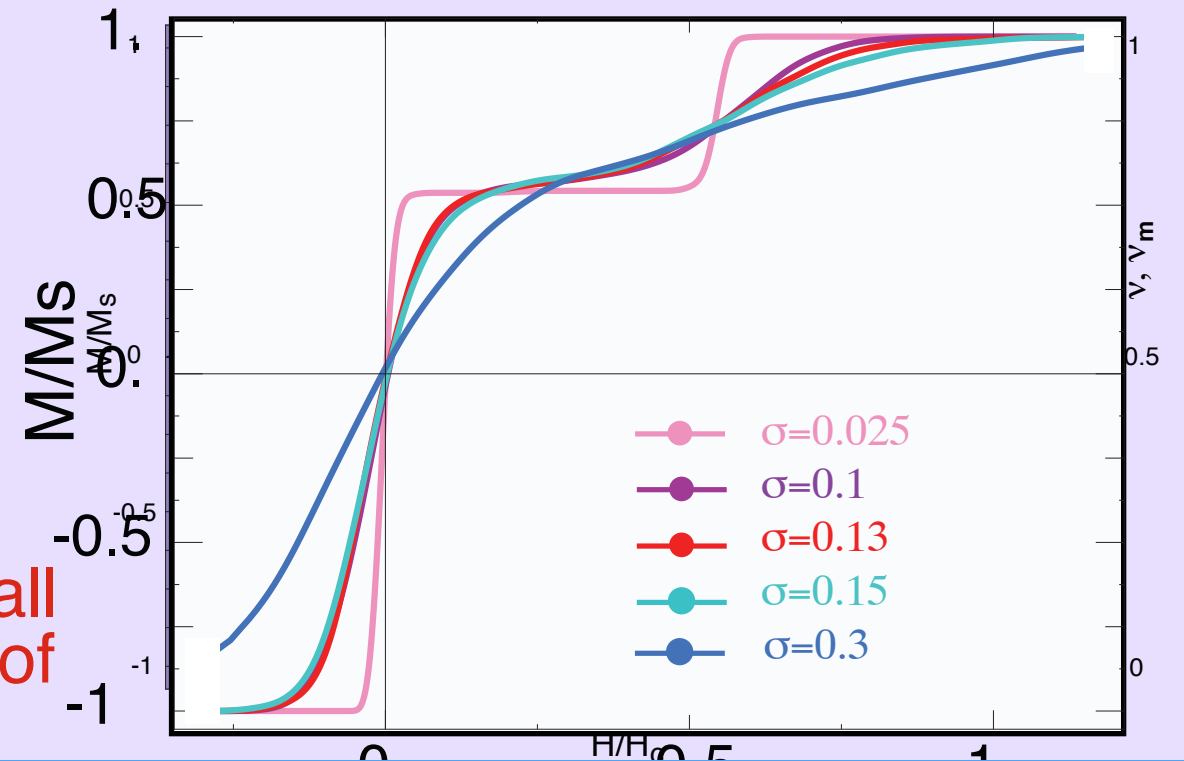
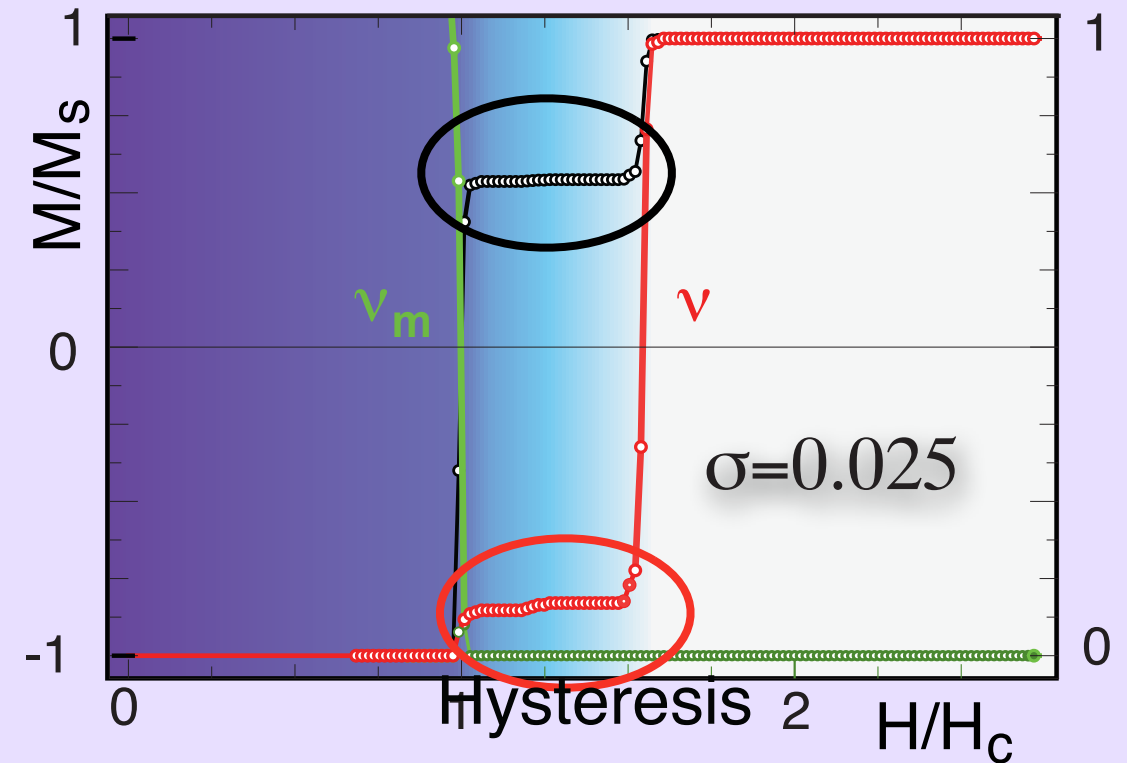
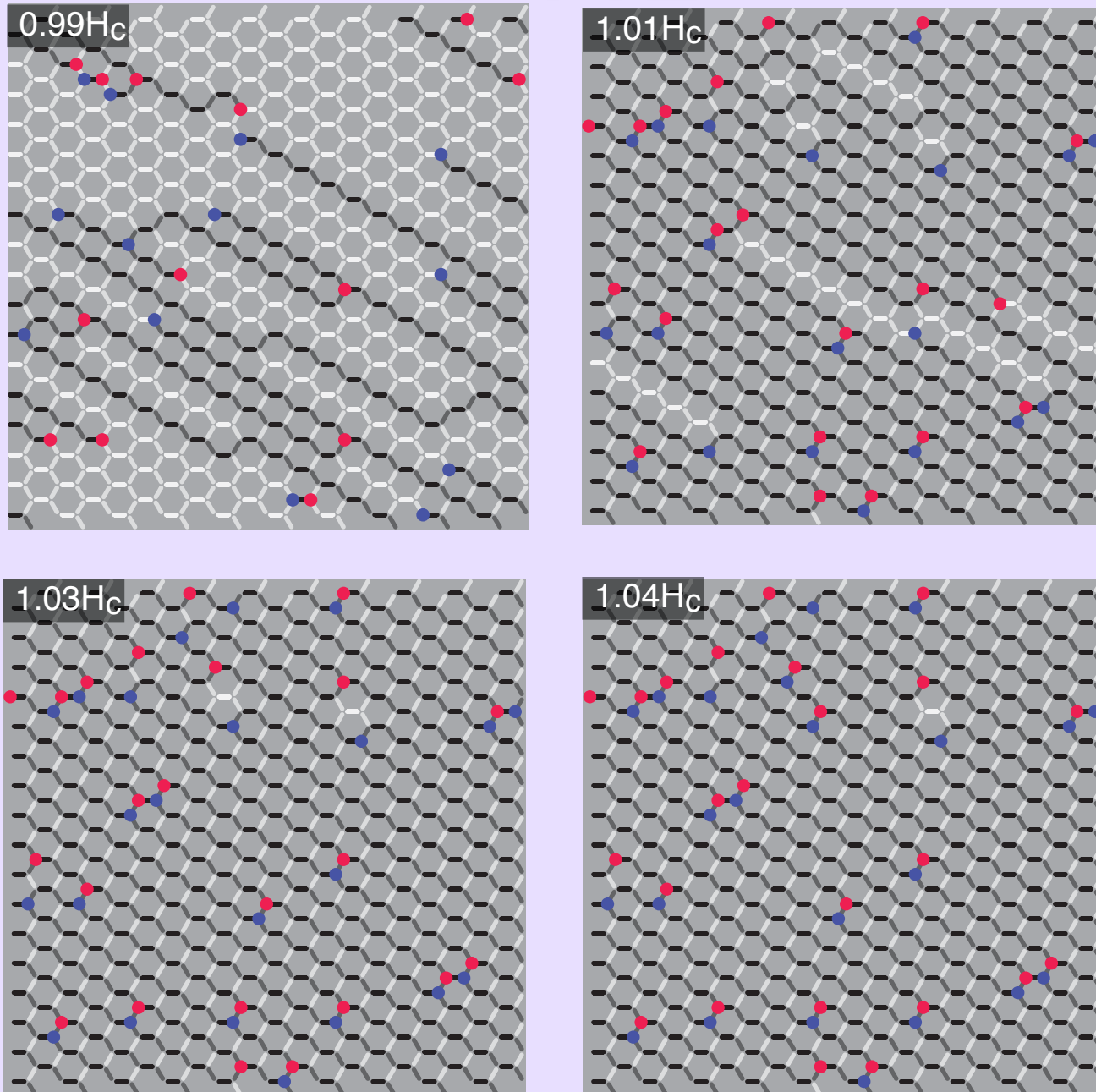


Low Disorder (simulations)

R. Hügli *et al.*, Phil. Trans. R. Soc. A 370, 5767 (2012)

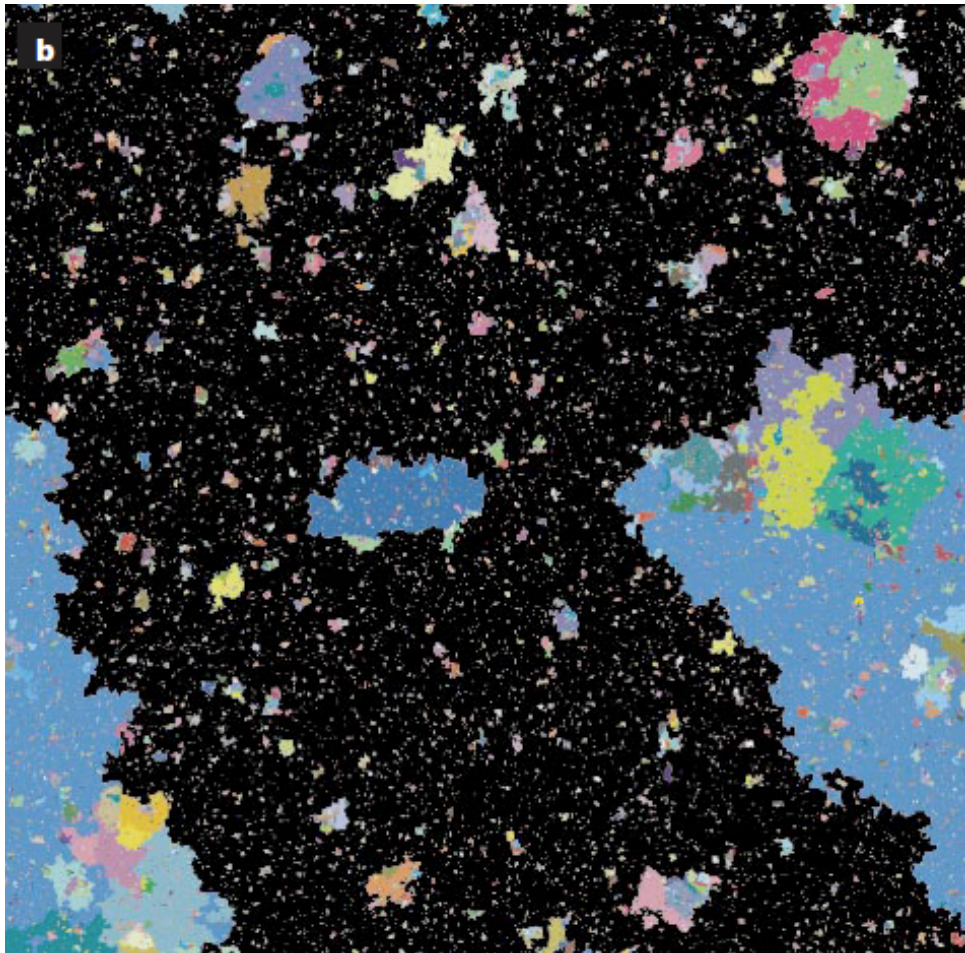
$\sigma=0.025$

Distinct stripe phase plateau



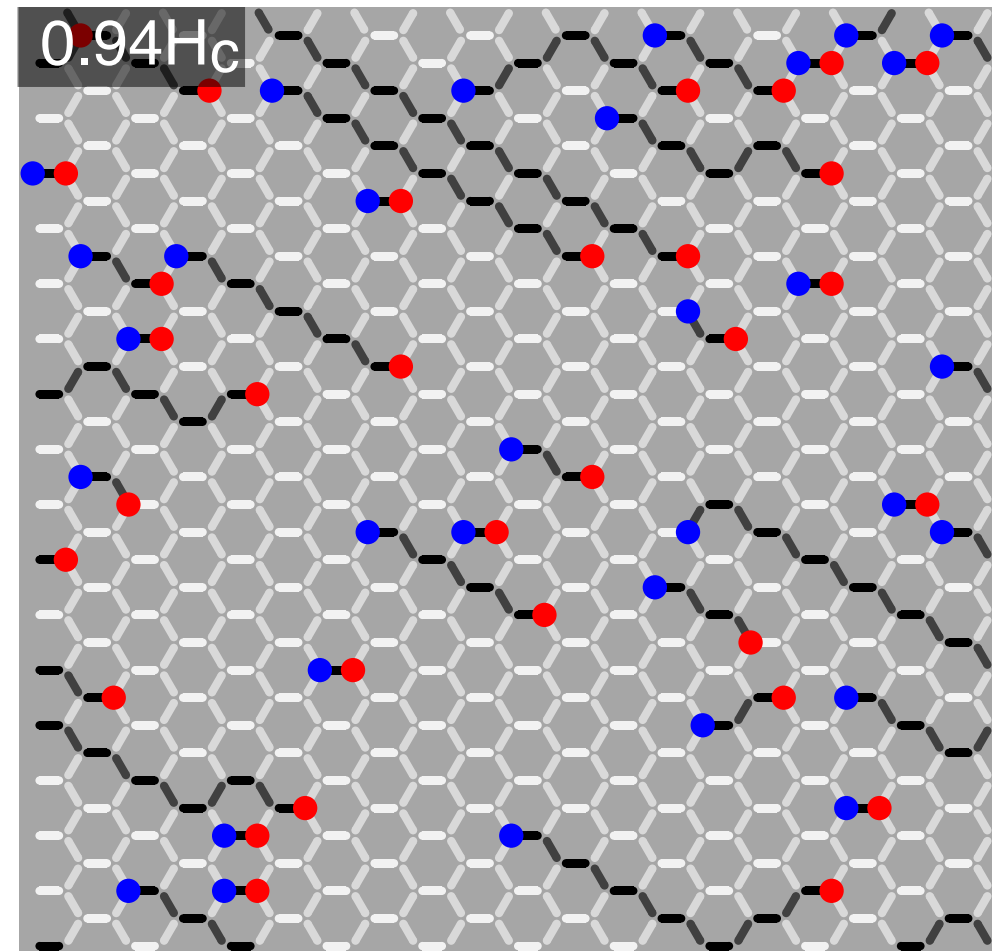
Avalanches grow over large distances \rightarrow small monopole concentrations over large regions of hysteresis

Avalanches and dimensional reduction due to frustration



conventional:
2D avalanches
in 2D system
(Sethna, Dahmen et al.)

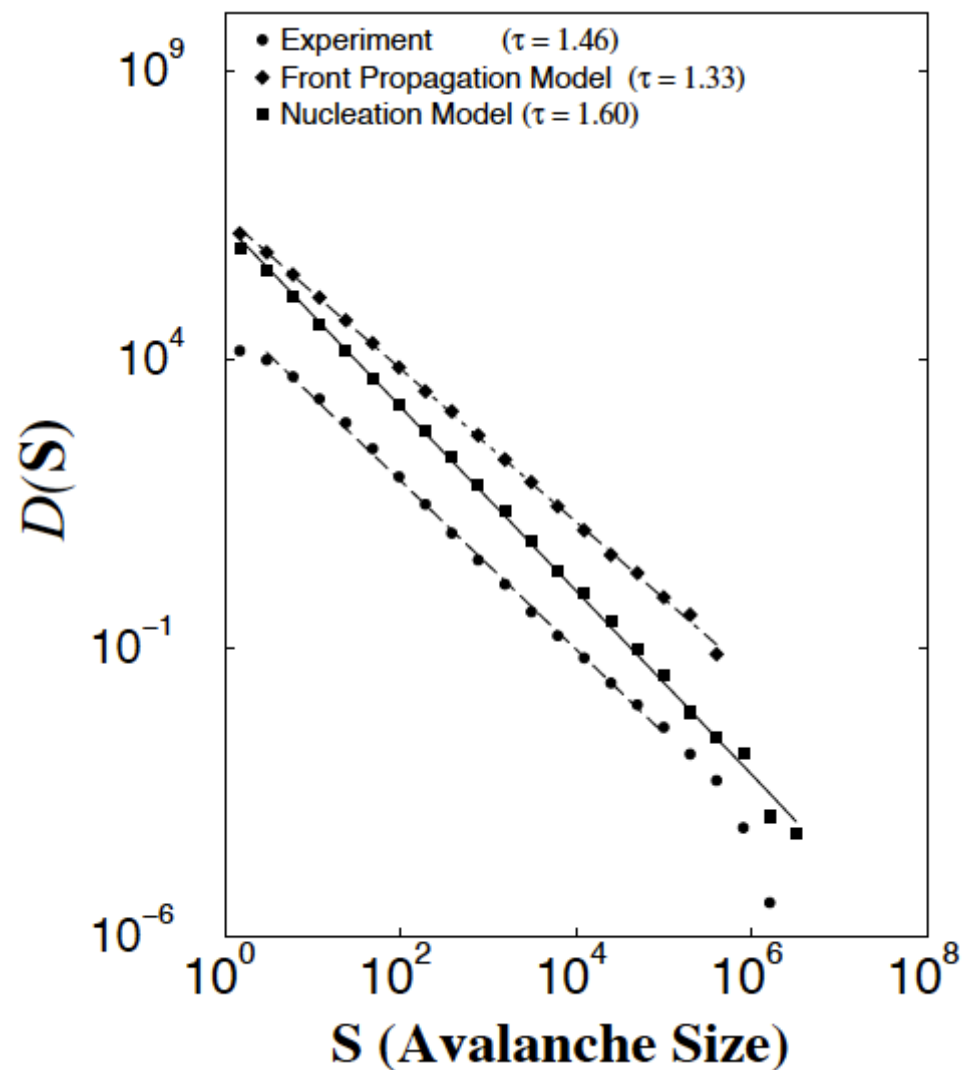
Random Field Ising Model
(RFIM)



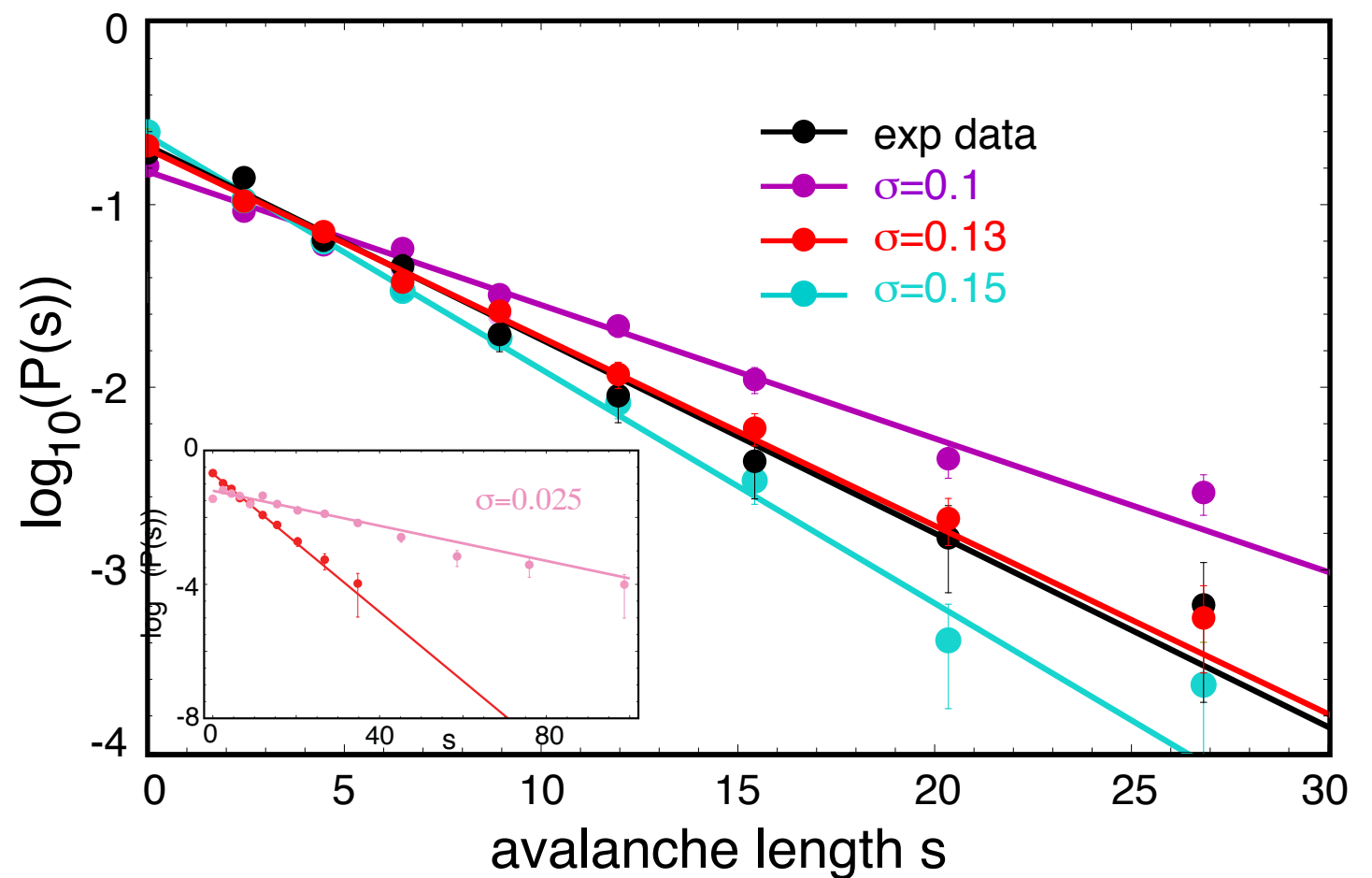
Here:
1D avalanches in 2D system
'**dimensional reduction due to
frustration**'

Random field Ising model vs artificial kagome spin ice

conventional (RFIM):
(power law scaling)



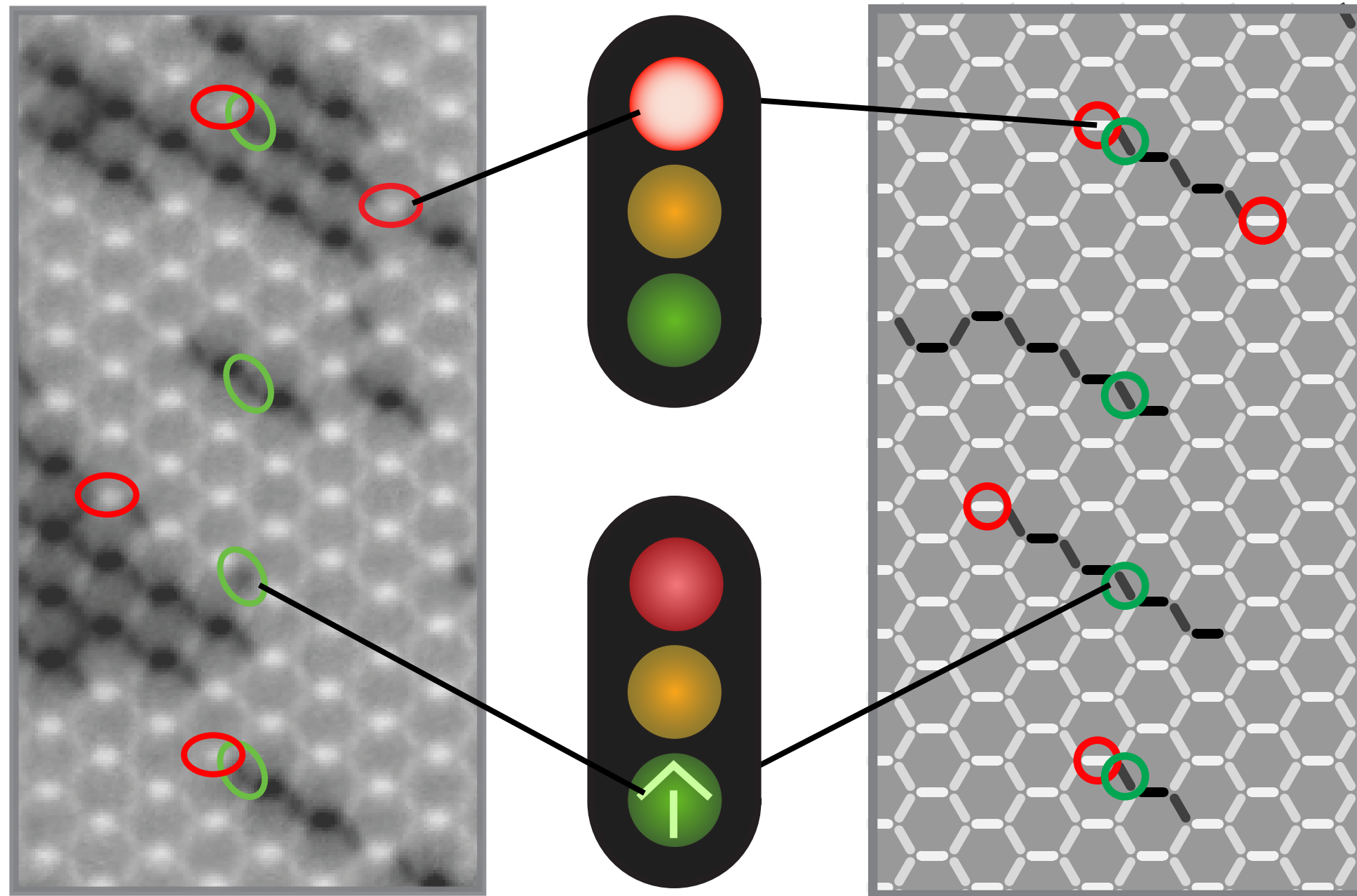
avalanche statistics



$$P(s) \propto p^s$$

cf. avalanches in 1D model
O. Chubykalo et al. (1998)

Control of monopole dynamics - experiments & simulations



R. Hügli *et al.*, *Phil. Trans. R. Soc. A* 370, 5767 (2012)

Acknowledgements

UCD:

Remo Hügli

Gerard Duff

Naoise Grisewood

Leonard English

B. O'Conchuir (now U.
Cambridge)

P. Böni (TUM)

B. Roessli (PSI)

J. Kulda (ILL)

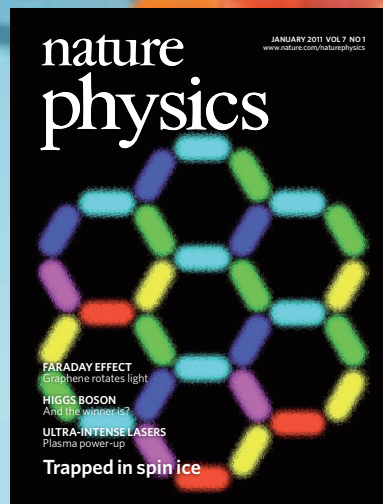
K. Krämer (Bern)

E. Mengotti (ABB)

L. Heyderman (ETH)

F. Nolting (PSI)

A. Fraile Rodriguez
(Barcelona)



SciAm (French)
('13)



SciAm (German)
('14)

